$$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-2)-2(3)-4(-2) & 2(-4)-2(4)-4(-3) \\ -1(2)+3(-1)+4(1) & -1(-2)+3(3)+4(-2) & -1(-4)+3(4)+4(-3) \\ 1(2)-2(-1)-3(1) & 1(-2)-2(3)-3(-2) & 1(-4)-2(4)-3(-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

11. Question

If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, show that $A^2 = I$.

Answer

Given : $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$,

Matrix A is of order 3 × 3

To show : $A^2 = I$

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

$$A^2 = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4(4)-1(3)-4(3) & 4(-1)-1(0)-4(-1) & 4(-4)-1(-4)-4(-3) \\ 3(4)+0(3)-4(3) & 3(-1)+0(0)-4(-1) & 3(-4)+0(-4)-4(-3) \\ 3(4)-1(3)-3(3) & 3(-1)-1(0)-3(-1) & 3(-4)-1(-4)-3(-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 16-3-12 & -4+0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

12. Question

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $(3A^2 - 2B + I)$.

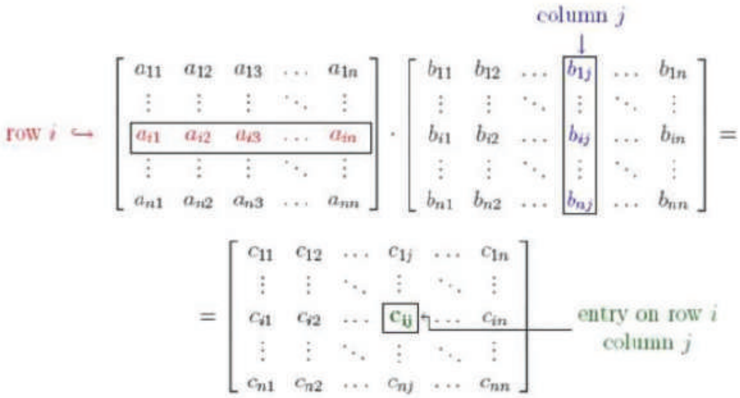
Answer

Given : $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$,

Matrix A is of order 2 × 2, Matrix B is of order 2 × 2

To find : $3A^2 - 2B + I$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2(2)-1(3) & 2(-1)-1(2) \\ 3(2)+2(3) & 3(-1)+2(2) \end{bmatrix} = \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$

$3A^2 = 3 \times \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix}$

$3A^2 = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix}$

$2B = 2 \times \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$2B = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-0+1 & -12-8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

13. Question

If $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$ then find $(-A^2 + 6A)$.

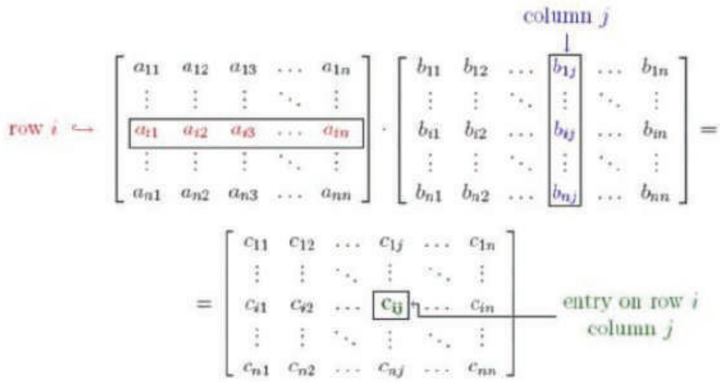
Answer

Given : $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$

Matrix A is of order 2 $\times$ 2.

To find : $-A^2 + 6A$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots\dots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A^2 = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2(2)-2(-3) & 2(-2)-2(4) \\ -3(2)+4(-3) & -3(-2)+4(4) \end{bmatrix} = \begin{bmatrix} 4+6 & -4-8 \\ -6-12 & 6+16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix}$

$-A^2 = -\begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix} = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix}$

$6A = 6 \times \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$

$6A = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$

$-A^2 + 6A = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix} + \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix} = \begin{bmatrix} -10+12 & 12-12 \\ 18-18 & -22+24 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$-A^2 + 6A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

14. Question

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that (A2 - 5A + 7I) = O.

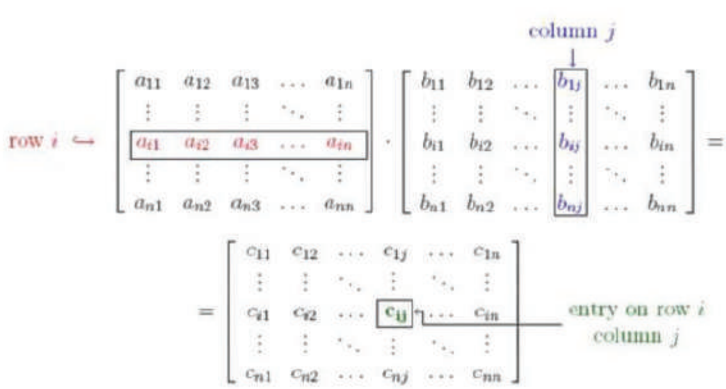
Answer

Given : $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$,

Matrix A is of order 2 $\times$ 2.

To show : $A^2 - 5A + 7I = 0$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots\dots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b =

c

$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = 5 \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 - 5A + 7I = 0$

15. Question

Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^3 - 4A^2 + A = O$.

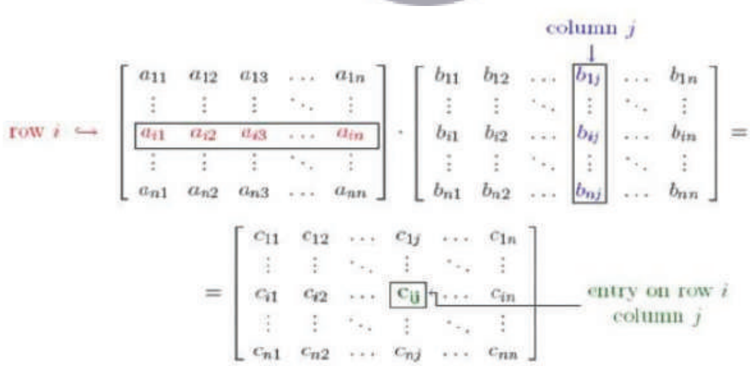
Answer

Given : $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Matrix A is of order 2 × 2.

To show : $A^3 - 4A^2 + A = 0$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 and A^3 are matrices of order 2 x 2.

$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2(2)+3(1) & 2(3)+3(2) \\ 1(2)+2(1) & 1(3)+2(2) \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

$A^3 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7(2)+12(1) & 7(3)+12(2) \\ 4(2)+7(1) & 4(3)+7(2) \end{bmatrix} = \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$

$A^3 = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$

$4A^2 = 4 \times \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}$

$4A^2 = \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}$

$A^3 - 4A^2 + A = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^3 - 4A^2 + A = 0$

16. Question

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

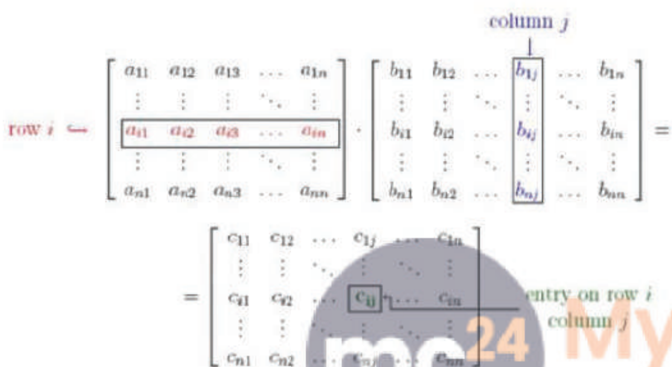
Answer

Given : $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, $A^2 = kA - 2I$.

Matrix A is of order 2 $\times$ 2.

To find : k

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3(3)-2(4) & 3(-2)-2(-2) \\ 4(3)-2(4) & 4(-2)-2(-2) \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$kA = k \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$

$kA - 2I = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - 2 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

It is the given that $A^2 = kA - 2I$

$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

Equating like terms,

$3k - 2 = 1$

$3k = 1 + 2 = 3$

$3k = 3$

$k = \frac{3}{3} = 1$

$k = 1$

17. Question

If $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, find f(A), where $f(x) = x^2 - 2x + 3$.

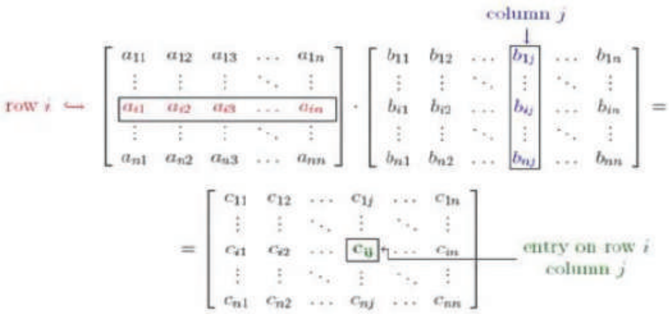
Answer

Given : $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, and f(x) = $x^2 - 2x + 3$.

Matrix A is of order 2 × 2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

$f(x) = x^2 - 2x + 3$

$f(A) = A^2 - 2A + 3I$

$A^2 = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -1(-1) + 2(3) & -1(2) + 2(1) \\ 3(-1) + 1(3) & 3(2) + 1(1) \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & -2 + 2 \\ -3 + 3 & 6 + 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$2A = 2 \times \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$

$2A = \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$

$3I = 3 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7 + 2 + 3 & -4 + 0 \\ 0 - 6 + 0 & 7 - 2 + 3 \end{bmatrix}$

$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$

$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$

18. Question

If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and f(x) = $2x^3 + 4x + 5$, find f(A).

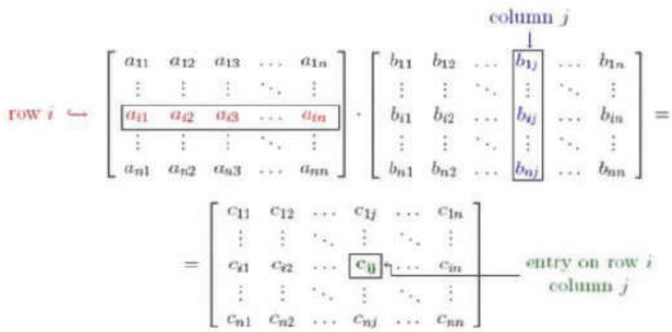
Answer

Given : $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and f(x) = $2x^3 + 4x + 5$

Matrix A is of order 2 × 2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^3 is a matrix of order 2 x 2.

$f(x) = 2x^3 + 4x + 5$

$f(A) = 2A^3 + 4A + 5I$

$A^2 = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1(1)+2(4) & 1(2)+2(-3) \\ 4(1)-3(4) & 4(2)-3(-3) \end{bmatrix} = \begin{bmatrix} 1+8 & 2-6 \\ 4-12 & 8+9 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$

$A^3 = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 9(1)-4(4) & 9(2)-4(-3) \\ -8(1)+17(4) & -8(2)+17(-3) \end{bmatrix}$

$A^3 = \begin{bmatrix} 9-16 & 18+12 \\ -8+68 & -16-51 \end{bmatrix} = \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}$

$A^3 = \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}$

$2A^3 = 2 \times \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix} = \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}$

$2A^3 = \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}$

$4A = 4 \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix}$

$4A = \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix}$

$5I = 5 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$5I = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$2A^3 + 4A + 5I = \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -14+4+5 & 60+8+0 \\ 120+16+0 & -134-12+5 \end{bmatrix}$

$f(A) = 2A^3 + 4A + 5I = \begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix}$

$f(A) = 2A^3 + 4A + 5I = \begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix}$

19. Question

Find the values of x and y, when

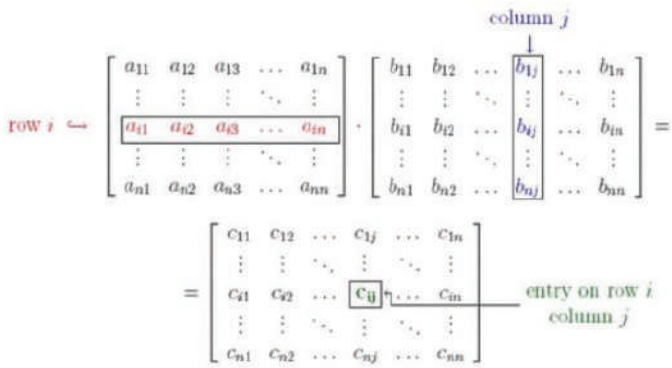
$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Answer

Given : $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

To find : x and y

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

The resulting matrix obtained on multiplying $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is of order 2 × 1

$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Equating similar terms,

2x - 3y = 1 equation 1

x + y = 3 equation 2

equation 1 + 3(equation 2) and solving the above equations,

$$\begin{array}{l} 2x - 3y = 1 \\ + \\ 3x + 3y = 9 \\ \hline 5x = 10 \end{array}$$

$x = \frac{10}{5} = 2$

x = 2 , substituting x = 2 in equation 2,

2 + y = 3

y = 3 - 2 = 1

x = 2 and y = 1

20. Question

Solve for x and y, when

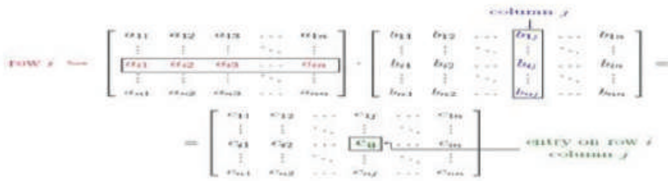
$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}.$

Answer

Given : $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}.$

To find : x and y

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

The resulting matrix obtained on multiplying $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is of order 2 × 1

$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$

$\begin{bmatrix} 3x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$

Equating similar terms,

3x - 4y = 3 equation 1

x + 2y = 11 equation 2

equation 1 + 2(equation 2) and solving the above equations,

3x - 4y = 3
+
2x + 4y = 22

5x = 3 + 22 = 25

5x = 25

$x = \frac{25}{5} = 5$

x = 5 , substituting x = 2 in equation 2,

5 + 2y = 11

2y =11 - 5 = 6

2y = 6

$y = \frac{6}{2} = 3$

x = 5 and y = 3

21. Question

If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$.

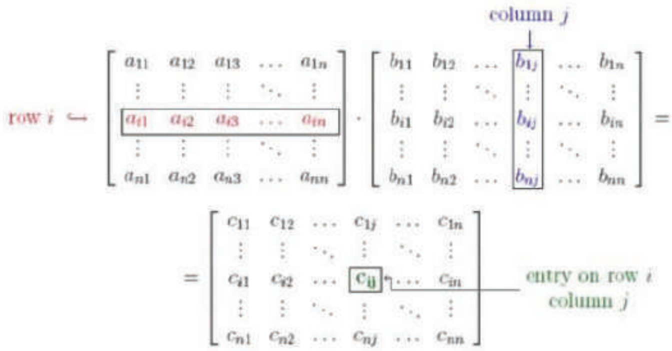
Answer

Given : $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, $A^2 + xI = yA$.

A is a matrix of order 2 x 2

To find : x and y

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

A^2 is a matrix of order 2 x 2

$A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3(3)+1(7) & 3(1)+1(5) \\ 7(3)+5(7) & 7(1)+5(5) \end{bmatrix} = \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$

$A^2 = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$

$xI = x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

$xI = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

$A^2 + xI = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 16+x & 8+0 \\ 56+0 & 32+x \end{bmatrix} = \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix}$

$A^2 + xI = \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix}$

$yA = y \times \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

$yA = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

It is given that $A^2 + xI = yA$,

$\begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

Equating similar terms in the given matrices,

16 + x = 3y and 8 = y,

hence y = 8

Substituting y = 8 in equation 16 + x = 3y

16 + x = 3 × 8 = 24

16 + x = 24

x = 24 - 16 = 8

x = 8

x = 8, y = 8

22. Question

If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the value of a and b such that $A^2 + aA + bI = O$.

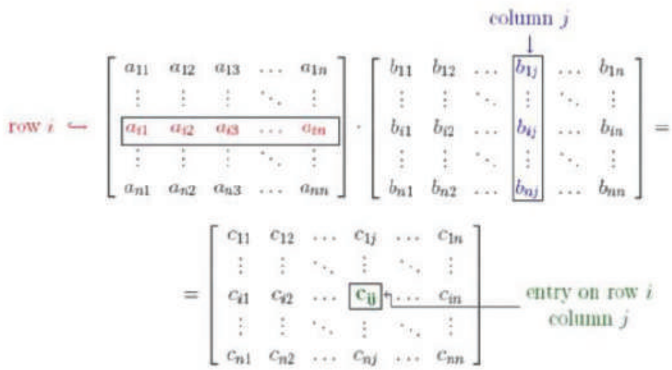
Answer

Given : $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, $A^2 + aA + bI = O$

A is a matrix of order 2 x 2

To find : a and b

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

A^2 is a matrix of order 2 x 2

$A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3(3)+2(1) & 3(2)+2(1) \\ 1(3)+1(1) & 1(2)+1(1) \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

$aA = a \times \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3a & 2a \\ 1a & 1a \end{bmatrix}$

$bI = b \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$

$bI = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$

$A^2 + aA + bI = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ 1a & 1a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix}$

$A^2 + aA + bI = \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix}$

It is given that $A^2 + aA + bI = 0$

$\begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equating similar terms in the matrices,we get

4 + a = 0 and 3 + a + b = 0

a = 0 - 4 = -4

a = -4

substituting a = -4 in 3 + a + b = 0

3 - 4 + b = 0

-1 + b = 0

b = 0 + 1 = 1

b = 1

a = -4 and b = 1

23. Question

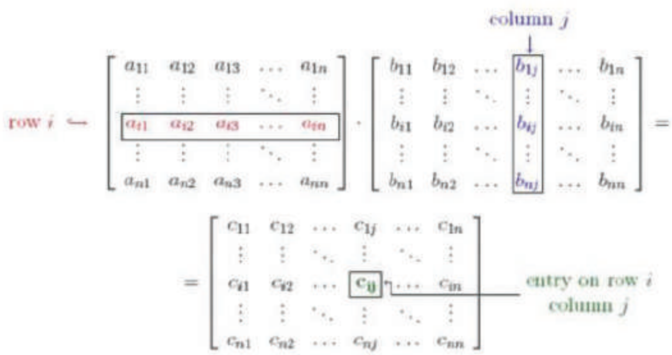
Find the matrix A such that $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}. A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

Answer

Given : $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}. A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

To find : matrix A

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

IF XA = B, then $A = X^{-1}B$

$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} . A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

To find $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1}$

Determinant of given matrix = $\begin{vmatrix} 5 & -7 \\ -2 & 3 \end{vmatrix}$ = 5(3) - (-7)(-2) = 15 - 14 = 1

Adjoint of matrix $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} = \frac{1}{1} \times \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 3(-16) + 7(7) & 3(-6) + 7(2) \\ 2(-16) + 5(7) & 2(-6) + 5(2) \end{bmatrix} = \begin{bmatrix} -48 + 49 & -18 + 14 \\ -32 + 35 & -12 + 10 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$

24. Question

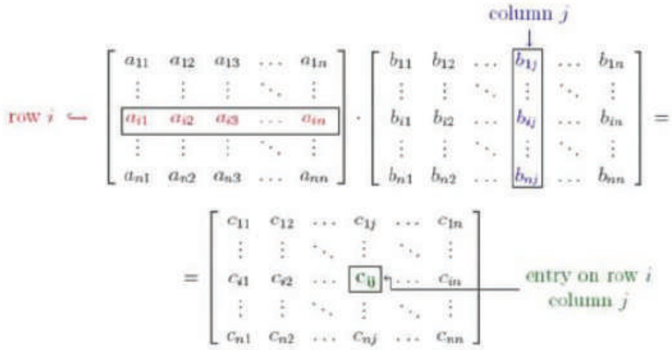
Find the matrix A such that $A. \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$.

Answer

Given : $A. \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$.

To find : matrix A

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

IF AX = B, then $A = BX^{-1}$

A. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$

To find $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$

Determinant of given matrix = $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$ = 5(2) - (4)(3) = 10 - 12 = -2

Adjoint of matrix $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{-2} \times \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$A = \frac{1}{-2} \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 0(5) - 4(-4) & 0(-3) - 4(2) \\ 10(5) + 3(-4) & 10(-3) + 3(2) \end{bmatrix}$

$A = \frac{1}{-2} . \begin{bmatrix} 0 + 16 & 0 - 8 \\ 50 - 12 & -30 + 6 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 16 & -8 \\ 38 & -24 \end{bmatrix} = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

$A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

$A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

25. Question

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = (A^2 + B^2)$ then find the values of a and b.

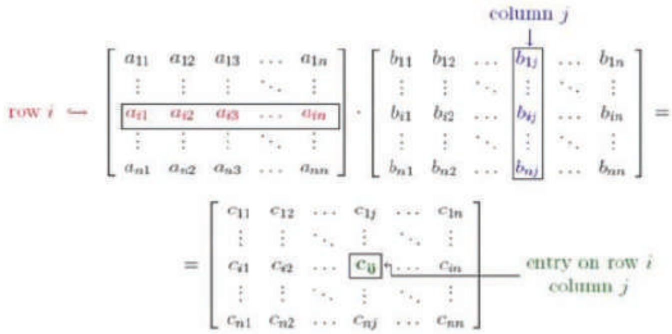
Answer

Given : $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$

$(A + B)^2 = (A^2 + B^2)$

To find : a and b

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 1+a & -1-1 \\ 2+b & -1-1 \end{bmatrix} = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix}$

$A + B = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix} \times \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix} = \begin{bmatrix} (1+a)(1+a) - 2(2+b) & (1+a)(-2) - 2(-2) \\ (2+b)(1+a) - 2(2+b) & (2+b)(-2) - 2(-2) \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} 1 + a^2 + 2a - 4 - 2b & -2 - 2a + 4 \\ 2 + 2a + b + ab - 4 - 2b & -4 - 2b + 4 \end{bmatrix} = \begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1(1) - 1(2) & 1(-1) - 1(-1) \\ 2(1) - 1(2) & 2(-1) - 1(-1) \end{bmatrix} = \begin{bmatrix} 1-2 & -1+1 \\ 2-2 & -2+2 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

$B^2 = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} \times \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a(a) - 1(b) & a(-1) - 1(-1) \\ b(a) - 1(b) & b(-1) - 1(-1) \end{bmatrix} = \begin{bmatrix} a^2 - b & -a+1 \\ ab - b & -b+1 \end{bmatrix}$

$B^2 = \begin{bmatrix} a^2 - b & -a+1 \\ ab - b & -b+1 \end{bmatrix}$

$(A^2 + B^2) = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a^2 - b & -a+1 \\ ab - b & -b+1 \end{bmatrix} = \begin{bmatrix} -1 + a^2 - b & -a+1 \\ ab - b & -b+1 \end{bmatrix}$

$(A^2 + B^2) = \begin{bmatrix} -1 + a^2 - b & -a+1 \\ ab - b & -b+1 \end{bmatrix}$

It is given that $(A + B)^2 = (A^2 + B^2)$

$\begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix} = \begin{bmatrix} -1 + a^2 - b & -a+1 \\ ab - b & -b+1 \end{bmatrix}$

Equating similar terms in the given matrices we get,

2 - 2a = -a + 1 and -2b = -b + 1

2 - 1 = -a + 2a and -2b + b = 1

1 = a and -b = 1

a = 1 and b = -1

26. Question

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that F(x) . F(y) = F(x + y).

Answer

Given : $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$,

To show : F(x) . F(y) = F(x + y).

Formula used :

$$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\times\begin{bmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{bmatrix}$$

$$=\begin{bmatrix}a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33}\\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33}\\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33}\end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

F(x) = $\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

F(y) = $\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x + y) = $\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x) . F(y) = $\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}\cos x(\cos y)-\sin x(\sin y)+0(0) & \cos x(-\sin y)-\sin x(\cos y)+0(0) & \cos x(0)-\sin x(0)+0(1)\\ \sin x(\cos y)+\cos x(\sin y)+0(0) & \sin x(-\sin y)+\cos x(\cos y)+0(0) & \sin x(0)+\cos x(0)+0(1)\\ 0(\cos y)+0(\sin y)+1(0) & 0(-\sin y)+0(\cos y)+1(0) & 0(0)+0(0)+1(1)\end{bmatrix}$$

F(x) . F(y) = $\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\cos x\sin y+\sin x\cos y & 0\\ \sin x\cos y+\cos x\sin y & -\sin x\sin y+\cos x\cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

We know that,

cosx(cosy) - sinx (siny) = cos(x+y) and -cosx(siny) - sinx(cosy) = -sin(x+y)

F(x) . F(y) = $\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x + y) = F(x) . F(y) = $\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x + y) = F(x) . F(y)

27. Question

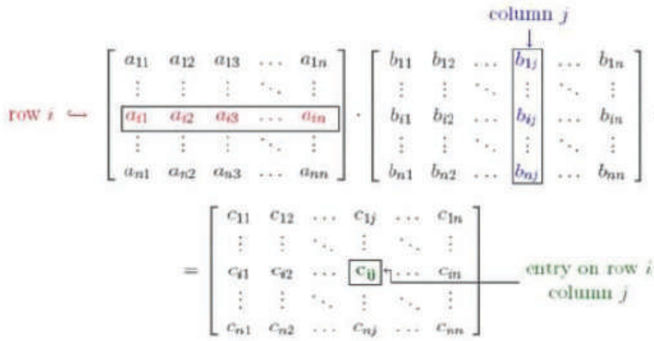
If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, show that $A^2=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix}$

Answer

Given : $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$,

To show : $A^2=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix}$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \times \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha(\cos\alpha) + \sin\alpha(-\sin\alpha) & \cos\alpha(\sin\alpha) + \sin\alpha(\cos\alpha) \\ -\sin\alpha(\cos\alpha) + \cos\alpha(-\sin\alpha) & -\sin\alpha(\cos\alpha) + \cos\alpha(\cos\alpha) \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & -2\sin\alpha\cos\alpha \\ -2\sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

We know that $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ and $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

28. Question

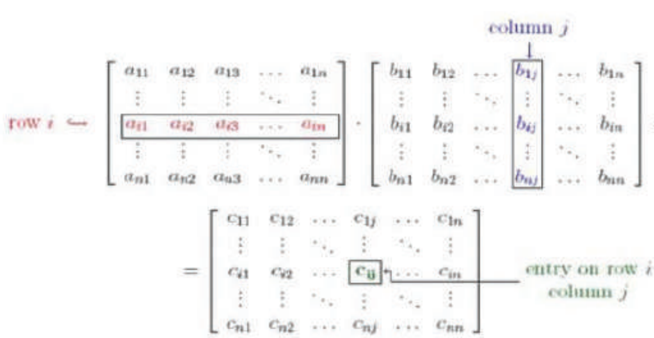
If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = O$, find x.

Answer

Given : $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = O$,

To find : x

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$[1 \quad x \quad 1]\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\-2\\3\end{bmatrix} = 0$$

$$[1 \quad x \quad 1] \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = [1(1) + x(4) + 1(3) \quad 1(2) + x(5) + 1(2) \quad 1(3) + x(6) + 1(5)]$$

$$[1 \quad x \quad 1] \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = [1 + 4x + 3 \quad 2 + 5x + 2 \quad 6x + 8]$$

$$[1 \quad x \quad 1] \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = [4x + 4 \quad 5x + 4 \quad 6x + 8]$$

$$[1 \quad x \quad 1]\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\-2\\3\end{bmatrix} = [4x + 4 \quad 5x + 4 \quad 6x + 8] \times \begin{bmatrix}1\\-2\\3\end{bmatrix}$$

$$[4x + 4 \quad 5x + 4 \quad 6x + 8] \times \begin{bmatrix}1\\-2\\3\end{bmatrix} = [(4x + 4)(1) + (5x + 4)(-2) + (6x + 8)(3)]$$

$$[4x + 4 \quad 5x + 4 \quad 6x + 8] \times \begin{bmatrix}1\\-2\\3\end{bmatrix} = [4x + 4 - 10x - 8 + 18x + 24] = [12x + 20]$$

$$[1 \quad x \quad 1]\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\-2\\3\end{bmatrix} = [12x + 20] = 0$$

$12x + 20 = 0$

$12x = -20$

$x = \frac{-20}{12} = \frac{-5}{3}$

$x = \frac{-5}{3}$

29. Question

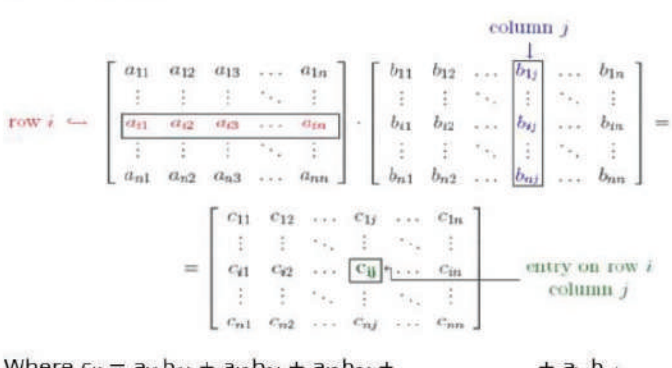
If $[x \ 4 \ 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = O$, find x.

Answer

Given : $[x \ 4 \ 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = O$,

To find : x

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$[x \quad 4 \quad 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = 0$

$[x \quad 4 \quad 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix} = [x(2)+4(1)+1(0) \quad x(1)+4(0)+1(2) \quad x(2)+4(2)+1(-4)]$

$[x \quad 4 \quad 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix} = [2x+4 \quad x+2 \quad 2x+4]$

$[2x+4 \quad x+2 \quad 2x+4]\begin{bmatrix}x\\4\\-1\end{bmatrix} = [(2x+4)(x)+4(x+2)+(2x+4)(-1)]$

$[x \quad 4 \quad 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = [2x^2+4x+4x+8-2x-4] = [2x^2+6x+4] = 0$

$2x^2 + 6x + 4 = 0$

$x^2 + 3x + 2 = 0$

$(x + 1)(x + 2) = 0$

$x + 1 = 0$ or $x + 2 = 0$

$x = -1$ or $x = -2$

$x = -1$ or $x = -2$

30. Question

Find the values of a and b for which

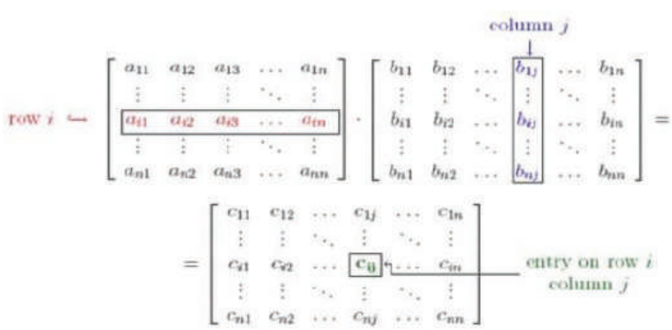
$\begin{bmatrix}a & b\\-a & 2b\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$.

Answer

Given : $\begin{bmatrix}a & b\\-a & 2b\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$.

To find : a and b

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$,then matrix AB exists and is of order $a \times d$,if and only if b = c

$\begin{bmatrix}a & b\\-a & 2b\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$

$\begin{bmatrix}a & b\\-a & 2b\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}a(2)+b(-1)\\-a(2)+2b(-1)\end{bmatrix} = \begin{bmatrix}2a-b\\-2a-2b\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$

$\begin{bmatrix}2a-b\\-2a-2b\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$

Equating similar terms,

2a - b = 5

-2a - 2b = 4

Adding the above two equations,we get

-3b = 9

b = $\frac{9}{-3}$ = -3

b = -3

substituting b = -3 in 2a - b = 5,we get

2a + 3 = 5

2a = 5 - 3 = 2

a = 1

a = 1 and b = -3

31. Question

If $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, find f(A), where f(x) = x2 - 5x + 7.

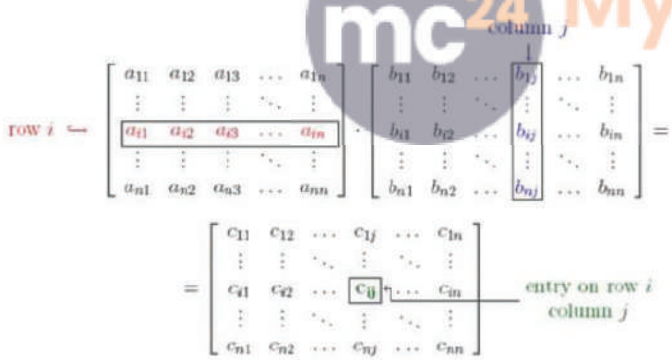
Answer

Given : $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, and f(x) = x^2 - 5x + 7.

Matrix A is of order 2 $\times$ 2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

f(x) = x^2 - 5x + 7

f(A) = A^2 - 5A + 7I

$A^2 = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 3(3)+4(-4) & 3(4)+4(-3) \\ -4(3)-3(-4) & -4(4)-3(-3) \end{bmatrix} = \begin{bmatrix} 9-16 & 12-12 \\ -12+12 & -16+9 \end{bmatrix}$

$A^2 = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$

$5A = 5 \times \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$

$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$f(A) = A^2 - 5A + 7I = \begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix} - \begin{bmatrix}15 & 20\\ -20 & -15\end{bmatrix} + \begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix} = \begin{bmatrix}-7-15+7 & 0-20+0\\ 0+20+0 & -7+15+7\end{bmatrix}$

$f(A) = A^2 - 5A + 7I = \begin{bmatrix}-15 & -20\\ 20 & 15\end{bmatrix}$

$f(A) = A^2 - 5A + 7I = \begin{bmatrix}-15 & -20\\ 20 & 15\end{bmatrix}$

32. Question

If $A = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, prove that $A^n = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$ for all $n \in N$.

Answer

Given : $A = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$,

Matrix A is of order 2×2.

To prove : $A^n = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$

Proof :

$A = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

Let us assume that the result holds for A^{n-1}

$A^{n-1} = \begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix}$

We need to prove that the result holds for A^n by mathematical induction .

$A^n = A^{n-1} \times A = \begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix} \times \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1(1)+(n-1)(0) & 1(1)+(n-1)(1)\\ 0(1)+1(0) & 0(1)+1(1)\end{bmatrix}$

$A^n = \begin{bmatrix}1+0 & 1+n-1\\ 0+0 & 0+1\end{bmatrix} = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$

$A^n = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$

33. Question

Given an example of two matrices A and B such that

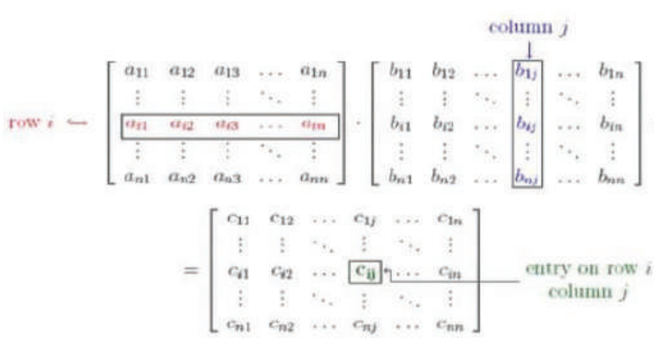
$A \neq O$, $B \neq O$, $AB = O$ and $BA \neq O$.

Answer

Given : $A \neq 0, B \neq 0, AB = 0, BA \neq 0$

To Find : matrix A and B

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

Let $A = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ and $B = \begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$

$A \neq 0, B \neq 0$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(0)+0(1) & 1(0)+0(0) \\ 0(0)+0(1) & 0(0)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0(1)+0(0) & 0(0)+0(0) \\ 1(1)+0(0) & 1(0)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

34. Question

Give an example of three matrices A, B, C such that

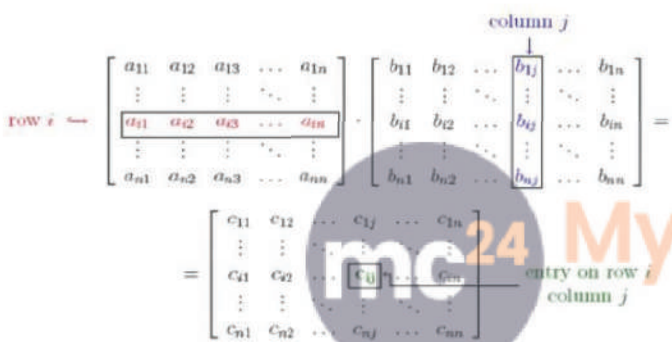
AB = AC but B ≠ C.

Answer

Given : AB = AC and B ≠ C.

To Find : matrix A and B

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots + a_{in}b_{nj}$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

B ≠ C

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(0)+0(1) & 1(0)+0(0) \\ 0(0)+0(1) & 0(0)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(0)+0(0) & 1(0)+0(1) \\ 0(0)+0(0) & 0(0)+0(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AC = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

AB = AC = 0

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

35. Question

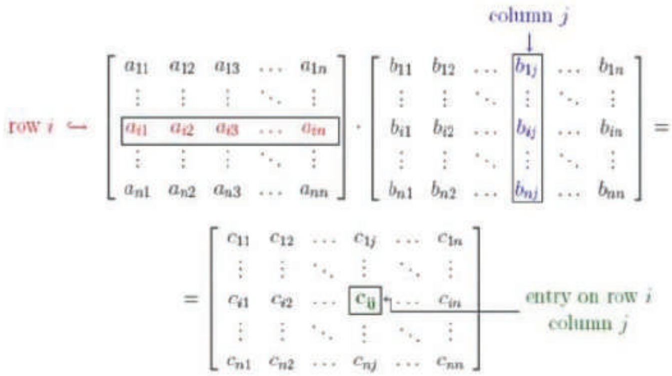
If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $(3A^2 - 2B + I)$.

Answer

Given : $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, Matrices A and B are of order 2 $\times$ 2.

To find : $(3A^2 - 2B + I)$.

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1(1)+0(-1) & 1(0)+0(7) \\ -1(1)+7(-1) & -1(0)+7(7) \end{bmatrix}$

$A^2 = \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$

$3A^2 = 3 \times \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix}$

$3A^2 = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix}$

$2B = 2 \times \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$2B = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-0+1 & 0-8+0 \\ -24+2+0 & 147-14+1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -8 \\ -22 & 134 \end{bmatrix}$

36. Question

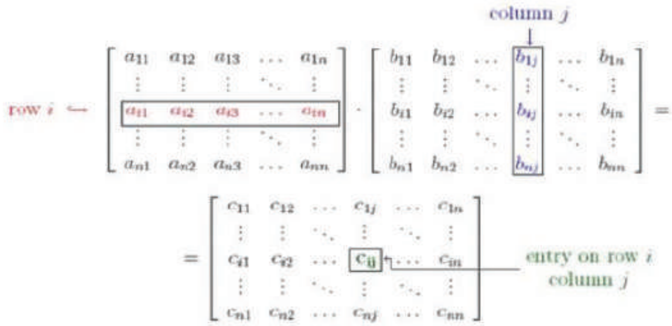
If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find the value of x.

Answer

Given : $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$,

To find : x

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

$$\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix} \times \begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix} \times \begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix} = \begin{bmatrix}2(1)+3(-2) & 2(-3)+3(4)\\5(1)+7(-2) & 5(-3)+7(4)\end{bmatrix} = \begin{bmatrix}2-6 & -6+12\\5-14 & -15+28\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & 13\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix} \times \begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & 13\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\begin{bmatrix}-4 & 6\\-9 & 13\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

Equating similar terms in the two matrices, we get

$x = 13$

$x = 13$

Exercise 5D

1. Question

If $A = \begin{bmatrix}2 & -3 & 5\\0 & 7 & -4\end{bmatrix}$, verify that $(A')' = A$.

Answer

Transpose of a matrix is obtained by interchanging the rows and the columns of matrix A. It is denoted by A'.

e.g. $A_{12} = A_{21}$

$$A = \begin{bmatrix}2 & -3 & 5\\0 & 7 & -4\end{bmatrix}$$

Hence transpose of matrix A is,

$$A' = \begin{bmatrix}2 & 0\\-3 & 7\\5 & -4\end{bmatrix}$$

$(A')' = \begin{bmatrix}2 & -3 & 5\\0 & 7 & -4\end{bmatrix}$ (A')' = AHence, Proved.

2. Question

If $A = \begin{bmatrix}3 & 5\\-2 & 0\\4 & -6\end{bmatrix}$, verify that $(2A)' = 2A'$.

Answer

Given $A = \begin{bmatrix}3 & 5\\-2 & 0\\4 & -6\end{bmatrix}$

To Prove: (2A)' = 2A'

Proof: Let us consider, B = 2A

Now, $B = 2\begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 10 \\ -4 & 0 \\ 8 & -12 \end{bmatrix}$

LHS $\Rightarrow B' = \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$

Again to find RHS, we will find the transpose of matrix A

$A' = \begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix}$

RHS = 2A'

$\Rightarrow 2\begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$

LHS = RHS

Hence proved.

3. Question

If $A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$, verify that (A + B)' = (A' + B').

Answer

Given $A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$

To Prove: (A + B)' = A' + B'

Proof: Let us consider C = A + B

$C = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix} + \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -1 & -3 & -3 \\ -2 & 1 & 2 \end{bmatrix}$

Now LHS = C'

$\Rightarrow \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$

To find RHS, we will find transpose of matrix A and B

$$A' = \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} \text{ And } B' = \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$$

RHS = A' + B'

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} + \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$$

LHS = RHS

Hence proved.

4. Question

If $P = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix}$ and $P = \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$, verify that (P + Q)′ = (P′ + Q′).

Answer

Given $P = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix}$ and $Q = \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$

To Prove: (P + Q)′ = P′ + Q′

Proof: Let us consider R = P + Q,

$$R = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -1 \\ -2 & -1 \\ 2 & 11 \end{bmatrix}$$

LHS = R $\Rightarrow$(P + Q)′

$$LHS = \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

To find RHS, we will first find the transpose of matrix P and Q

$$P' = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} \text{ And } Q' = \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$$

RHS = P′ + Q′

$$\Rightarrow \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

LHS = RHS

Hence proved.

5. Question

If $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that (A + A') is symmetric.

Answer

Given $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

To Prove: A + A' is symmetric.(Note:A matrix P is symmetric if P' = P)

Proof: We will find A',

$$A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

Now let us take P = A + A'

$$P = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

Now $P' = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

$$\Rightarrow P' = P$$

Hence A + A' is a symmetric matrix.

6. Question

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that (A + A') is skew-symmetric.

Answer

Given $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To prove: A-A' is a skew-symmetric matrix.(Note: A matrix P is skew-symmetric if P' = -P)

Proof: First we will find the transpose of matrix A

$$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

Let us take P = A-A'

$$P = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$\Rightarrow$P' = P

Hence A-A' is a skew symmetric matrix.

7. Question

Show that the matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is skew-symmetric.

HINT: Show that A' = -A.

Answer

Given $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

To Prove: A is a skew symmetric matrix.

Proof: As for a matrix to be skew symmetric A' = -A

We will find A'.

$$A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$\Rightarrow$A' = -A

So A is A skew symmetric matrix.

8. Question

Express the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

Answer

Given $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, As for a symmetric matrix A' = A hence

A + A' = 2A

A = $\frac{1}{2}(A + A') \Rightarrow P$ (Symmetric Matrix)

Similarly for a skew symmetric matrix since A' = -A hence

A-A' = 2A

A = $\frac{1}{2}(A - A') \Rightarrow Q$ (Skew Symmetric Matrix)

So a matrix can be represented as a sum of a symmetric matrix P and skew symmetric matrix Q.

First, we will find the transpose of matrix A,

$$A' = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

Now using the above formulas,

$P = \frac{1}{2}(A + A')$

$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\right)$

$\Rightarrow \frac{1}{2}\begin{bmatrix} 4 & 2 \\ 2 & 8 \end{bmatrix}$

$P = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$

$Q = \frac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\right)$

$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Hence A = P + Q

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ [Matrix A as the sum of P and Q]

$\Rightarrow \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

9. Question

Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

Answer

Given $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$,to express as sthe um of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,we will find transpose of matrix A

$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

Now using the above formulas

$P = \frac{1}{2}(A + A')$

$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$

$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

Hence A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$ [Matrix A as the sum of P and Q]

$$\Rightarrow \begin{bmatrix} 3 & \frac{-8}{2} \\ \frac{2}{2} & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

10. Question

Express the matrix $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Answer

Given $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$.

First, we find A'

$$A' = \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}$$

Now using the above mentioned formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} -2 & 7 & 8 \\ 7 & 6 & 4 \\ 8 & 4 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & 3 & -6 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{3}{2} & -3 \\ \frac{-3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix} + \begin{bmatrix} 0 & \frac{3}{2} & -3 \\ \frac{-3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$ [Matrix A as sum of P and Q]

$$\Rightarrow \begin{bmatrix} -1 & \frac{10}{2} & 1 \\ \frac{4}{2} & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$$