

NCERT Solutions for Class-XI Maths

Chapter-10 Exercise-10.3

NCERT Math Class 11

1. Reduce the following equations into slope-intercept form and find their slopes and the y intercepts.

(i) $x + 7y = 0$

(ii) $6x + 3y - 5 = 0$

(iii) $y = 0$

1. (i) The given equation is $x + 7y = 0$. It can be written as

$$y = -\frac{1}{7}x + 0$$

This equation is of the form $y = mx + c$, where $m = -\frac{1}{7}$ and $c = 0$.

Therefore, equation (1) is in the slope-intercept form, where the slope and the y intercept are $-\frac{1}{7}$ and 0 respectively.

- (ii) The given equation is $6x + 3y - 5 = 0$.

It can be written as

$$y = \frac{1}{3}(-6x + 5)$$

$$y = -2x + \frac{5}{3}$$

This equation is of the form $y = mx + c$, where $m = -2$ and $c = \frac{5}{3}$.

Therefore, equation (2) is in the slope-intercept form, where the slope and the y intercept are -2 and $\frac{5}{3}$ respectively.

- (iii) The given equation is $y = 0$.

It can be written as

$$y = 0.x + 0 \dots (3)$$

This equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$.

Therefore, equation (3) is in the slope-intercept form, where the slope and the y - intercept are 0 and 0 respectively.

2. Reduce the following equations into intercept form and find their intercepts on the axes.

(i) $3x + 2y - 12 = 0$

2. (i) $3x + 2y - 12 = 0$

Equation of line in intercept form is given by $\frac{x}{a} + \frac{y}{b} = 1$, where 'a' and 'b' are intercepts

on x axis and y - axis respectively.

Given equation is $3x + 2y - 12 = 0$

$$\Rightarrow 3x + 2y = 12$$

Dividing both sides by 12, we get

$$\Rightarrow \frac{3x}{12} + \frac{2y}{12} = \frac{12}{12}$$

$$\Rightarrow \frac{x}{4} + \frac{y}{6} = 1$$

The above equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 4$, $b = 6$

Intercept on x - axis = 4

Intercept on y - axis = 6

(ii) $4x - 3y = 6$

Ans. Equation of line in intercept form is given by $\frac{x}{a} + \frac{y}{b} = 1$, where 'a' and 'b' are

intercepts on x axis and y - axis respectively.

Given equation is $4x - 3y = 6$

$$\Rightarrow 4x - 3y = 6$$

Dividing both sides by 6, we get

$$\Rightarrow \frac{4x}{6} - \frac{3y}{6} = \frac{6}{6}$$

$$\Rightarrow \frac{2x}{3} - \frac{y}{2} = 1$$

$$\Rightarrow \frac{x}{3/2} + \frac{y}{-2} = 1$$

The above equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 3/2$, $b = -2$

Intercept on x - axis = $3/2$

Intercept on y - axis = -2

(iii) $3y + 2 = 0$

Ans. Equation of line in intercept form is given by $\frac{x}{a} + \frac{y}{b} = 1$, where 'a' and 'b' are intercepts on x axis and y – axis respectively.

Given equation is $3y + 2 = 0$

$$\Rightarrow 3y = -2$$

Dividing both sides by -2, we get

$$\Rightarrow \frac{3y}{-2} = \frac{-2}{-2}$$

$$\Rightarrow \frac{3y}{-2} = 1$$

$$\Rightarrow \frac{y}{-2/3} = 1$$

The above equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 0$, $b = -2/3$

Intercept on x – axis = 0

Intercept on y – axis = $-2/3$

3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

(i) $x - \sqrt{3}y + 8 = 0$

(ii) $y - 2 = 0$

(iii) $x - y = 4$

3. (i) The given equation is $x - \sqrt{3}y + 8 = 0$.

It can be reduced as:

$$x - \sqrt{3}y = -8$$

$$\Rightarrow -x + \sqrt{3}y = 8$$

On dividing both sides by $\sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$, we obtain

$$-\frac{x}{2} + \frac{\sqrt{3}}{2}y = \frac{8}{2}$$

$$\Rightarrow \left(-\frac{1}{2}\right)x + \left(\frac{\sqrt{3}}{2}\right)y = 4$$

$$\Rightarrow x \cos 120^\circ + y \sin 120^\circ = 4$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line $x \cos \omega + y \sin \omega = p$, we obtain $\omega = 120^\circ$ and $p = 4$.

Thus, the perpendicular distance of the line from the origin is 4 , while the angle between the perpendicular and the positive x -axis is 120° .

(ii) The given equation is $y - 2 = 0$. .

It can be reduced as $0 \cdot x + 1 \cdot y = 2$

On dividing both sides by $\sqrt{0^2 + 1^2} = 1$, we obtain $0 \cdot x + 1 \cdot y = 2$

$$\Rightarrow x \cos 90^\circ + y \sin 90^\circ = 2 .$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line $x \cos \omega + y \sin \omega = p$, we obtain $\omega = 90^\circ$ and $p = 2$.

Thus, the perpendicular distance of the line from the origin is 2 , while the angle between the perpendicular and the positive x -axis is 90° (iii) The given equation is $x - y = 4$.

It can be reduced as $1 \cdot x + (-1) \cdot y = 4$

On dividing both sides by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}} x + \left(-\frac{1}{\sqrt{2}} \right) y = \frac{4}{\sqrt{2}}$$

$$\Rightarrow x \cos \left(2\pi - \frac{\pi}{4} \right) + y \sin \left(2\pi - \frac{\pi}{4} \right) = 2\sqrt{2}$$

$$\Rightarrow x \cos 315^\circ + y \sin 315^\circ = 2\sqrt{2}$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line $x \cos \omega + y \sin \omega = p$, we obtain $\omega = 315^\circ$ and $p = 2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive x -axis is 315° .

4. Find the distance of the point $(-1, 1)$ from the line $12(x + 6) = 5(y - 2)$.

4. Given equation of the line $12(x + 6) = 5(y - 2)$.

$$\Rightarrow 12x + 72 = 5y - 10$$

$$\Rightarrow 12x - 5y + 82 = 0 \dots (1)$$

Comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 12$, $B = -5$, and $C = 82$

Perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Given point $(x_1, y_1) = (-1, 1)$

\therefore distance of point $(-1, 1)$ from the given line

$$d = \frac{|12 \times (-1) + (-5) \times 1 + 82|}{\sqrt{12^2 + (-5)^2}} = \frac{|-12 - 5 + 82|}{\sqrt{144 + 25}} = \frac{|65|}{\sqrt{169}} = \frac{65}{13} \text{ units}$$

$$d = 5 \text{ units}$$

5. Find the points on the x -axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

5. The given equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$

$$\text{or, } 4x + 3y - 12 = 0$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$,

we obtain $A = 4, B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the x -axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point

$$(x_1, y_1) \text{ is given by } d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

Therefore,

$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$\Rightarrow 4 = \frac{|4a - 12|}{5}$$

$$\Rightarrow |4a - 12| = 20$$

$$\Rightarrow \pm(4a - 12) = 20$$

$$\Rightarrow (4a - 12) = 20 \text{ or } -(4a - 12) = 20$$

$$\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12$$

$$\Rightarrow a = 8 \text{ or } -2$$

Thus, the required points on the x -axis are $(-2, 0)$ and $(8, 0)$.

6. Find the distance between parallel lines

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

6. (i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

Given parallel lines are $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Here $A = 15$, $B = 8$, $C_1 = -34$, $C_2 = 31$

Distance between parallel lines is

$$d = \frac{|-34 - 31|}{\sqrt{15^2 + 8^2}} = \frac{|-65|}{\sqrt{225 + 64}} = \frac{65}{\sqrt{289}}$$

$$\therefore d = 65/17$$

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$.

Ans. Given parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$.

$$\Rightarrow lx + ly + p = 0 \text{ and } lx + ly - r = 0$$

Distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Here $A = l$, $B = l$, $C_1 = p$, $C_2 = -r$

Distance between parallel lines is

$$d = \frac{|p - (-r)|}{\sqrt{l^2 + l^2}} = \frac{|p + r|}{\sqrt{2}l}$$

$$\therefore d = \frac{|p + r|}{l\sqrt{2}}$$

7. Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.

7. The equation of the given line is

$$3x - 4y + 2 = 0$$

$$\text{or } y = \frac{3x}{4} + \frac{2}{4}$$

$$\text{or } y = \frac{3}{4}x + \frac{1}{2}$$

which is of the form $y = mx + c$

$$\therefore \text{Slope of the given line} = 3/4$$

It is known that parallel lines have the same slope.

$$\therefore \text{Slope of the other line} = m = \frac{3}{4} \text{ Now, the equation of the line that has a slope of } \frac{3}{4}$$

and passes through the point $(-2, 3)$ is

$$(y-3) = \frac{3}{4}\{x - (-2)\}$$

$$4y - 12 = 3x + 6$$

$$\text{i.e., } 3x - 4y + 18 = 0$$

8. Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

8. Given equation of line is $x - 7y + 5 = 0$

$$\Rightarrow y = \frac{1}{7}x + \frac{5}{7}$$

Which is of the form $y = mx + c$, where m is the slope of the given line.

\therefore slope of the given line is $1/7$

Slope of the line perpendicular to the line having slope m is $-1/m$

Slope of the line perpendicular to the line having a slope of $1/7$ is $-\frac{1}{\frac{1}{7}} = -7$

Equation of line with slope -7 and x intercept 3 is given by $y = m(x - d)$

$$\Rightarrow y = -7(x - 3)$$

$$\Rightarrow y = -7x + 21$$

$$\Rightarrow 7x + y = 21$$

9. Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

9. The given lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

$$y = -\sqrt{3}x + 1 \dots (1) \quad \text{and} \quad y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}$$

The slope of line (1) is $m_1 = -\sqrt{3}$, while the slope of line (2) is $m_2 = -\frac{1}{\sqrt{3}}$

The acute angle i.e., θ between the two lines is given by

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan\theta = \left| \frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + (-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)} \right|$$

$$\tan\theta = \left| \frac{-3 + 1}{\sqrt{3}} \right| = \left| \frac{-2}{1 + 1} \right| = 2 \times \sqrt{3}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Thus, the angle between the given lines is either 30° or $180^\circ - 30^\circ = 150^\circ$.

10. The line through the points $(h, 3)$ and $(4, 1)$ intersects the line $7x - 9y - 19 = 0$ at right angle. Find the value of h .

10. Let the slope of the line passing through $(h, 3)$ and $(4, 1)$ be m_1

$$\text{Then, } m_1 = \frac{1-3}{4-h} = -\frac{2}{4-h}$$

Let the slope of line $7x - 9y - 19 = 0$ be m_2

$$\Rightarrow 7x - 9y - 19 = 0$$

$$\Rightarrow y = \frac{7}{9}x - \frac{19}{9}$$

$$\therefore m_2 = -\frac{19}{9}$$

Since, the given lines are perpendicular

Therefore, $m_1 \times m_2 = -1$

$$\Rightarrow \left(-\frac{2}{4-h}\right) \times \frac{7}{9} = -1$$

$$\Rightarrow -\frac{14}{36-9h} = -1$$

$$\Rightarrow -14 = -1 \times (36 - 9h)$$

$$\Rightarrow 36 - 9h = 14$$

$$\Rightarrow 9h = 36 - 14$$

$$\Rightarrow h = 22/9$$

$$\therefore h = 22/9$$

11. Prove that the line through the point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is A

$$(x - x_1) + B(y - y_1) = 0.$$

11. The slope of line $Ax + By + C = 0$ or $y = \left(\frac{-A}{B}\right)x + \left(\frac{-C}{B}\right)$ is $m = -\frac{A}{B}$

It is known that parallel lines have the same slope.

$$\therefore \text{Slope of the other line} = m = -\frac{A}{B}$$

The equation of the line passing through point (x_1, y_1) and having a slope $m = -\frac{A}{B}$ is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = -\frac{A}{B}(x - x_1)$$

$$B(y - y_1) = -A(x - x_1)$$

$$A(x - x_1) + B(y - y_1) = 0$$

Hence, the line through point (x_1, y_1) and parallel to line $Ax + By + C = 0$ is A

$$(x - x_1) + B(y - y_1) = 0$$

12. Two lines passing through the point $(2, 3)$ intersects each other at an angle of 60° . If slope of one line is 2, find equation of the other line.

12. Let the slope of the first line be m_1

Given, $m_1 = 2$

Let the slope of the other line be m_2 .

Angle between the two lines is 60° .

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\Rightarrow \tan 60^\circ = \left| \frac{2 - m_2}{1 + 2m_2} \right|$$

$$\Rightarrow \sqrt{3} = \pm \left(\frac{2 - m_2}{1 + 2m_2} \right)$$

$$\Rightarrow \sqrt{3} = \frac{2 - m_2}{1 + 2m_2}$$

or

$$\sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2} \right)$$

$$\Rightarrow \sqrt{3}(1 + 2m_2) = 2 - m_2$$

or

$$\sqrt{3}(1 + 2m_2) = -(2 - m_2)$$

$$\Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2$$

or

$$\sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$

$$\Rightarrow m_2(2\sqrt{3} + 1) = 2 - \sqrt{3}$$

or

$$m_2(2\sqrt{3} - 1) = -(2 + \sqrt{3})$$

$$\Rightarrow m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)}$$

or

$$m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$$

Case 1: When $m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)}$

The equation of the line passing through point $(2, 3)$ and having a slope m_2 is

$$y - 3 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1} \right) (x - 2)$$

$$\Rightarrow (2\sqrt{3} + 1)y - 3(2\sqrt{3} + 1) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$$

$$\Rightarrow (\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3$$

$$\Rightarrow (\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = 8\sqrt{3} - 1$$

∴ Equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = 8\sqrt{3} - 1$

Case 2: When $m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$

The equation of the line passing through point (2, 3) and having a slope m_2 is

$$y - 3 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)} (x - 2)$$

$$\Rightarrow (2\sqrt{3} - 1)y - 3(2\sqrt{3} - 1) = -(2 + \sqrt{3})x + 2(2 + \sqrt{3})$$

$$\Rightarrow (2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 4 + 2\sqrt{3} + 6\sqrt{3} - 3$$

$$\Rightarrow (2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 8\sqrt{3} + 1$$

Equation of the other line is $(2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 8\sqrt{3} + 1$

- 13.** Find the equation of the right bisector of the line segment joining the points (3,4) and (1,2)
- 13.** The right bisector of a line segment bisects the line segment at 90° . The end-points of the line segment are given as A(3,4) and B(-1,2).

Accordingly, mid-point of AB = $\left(\frac{3-1}{2}, \frac{4+2}{2} \right) = (1,3)$

Slope of AB = $\frac{2-4}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$

∴ Slope of the line perpendicular to AB = $-\frac{1}{\left(\frac{1}{2}\right)} = -2$

The equation of the line passing through (1,3) and having a slope of -2 is

$$(y - 3) = -2(x - 1)$$

$$-3 = -2x + 2$$

$$2x + y = 5$$

Thus, the required equation of the line is $2x + y = 5$.

14. Find the coordinates of the foot of perpendicular from the point $(-1, 3)$ to the line $3x - 4y - 16 = 0$.

14.

Let the co-ordinates of the foot of the perpendicular from $(-1, 3)$ to the line $3x - 4y - 16 = 0$ be (a, b)

Let the slope of the line joining $(-1, 3)$ and (a, b) be m_1

$$m_1 = \frac{b-3}{a+1}$$

Let the slope of the line $3x - 4y - 16 = 0$ be m_2

$$\Rightarrow y = \frac{3}{4}x - 4$$

$$\therefore m_2 = \frac{3}{4}$$

Since these two lines are perpendicular $m_1 \times m_2 = -1$

$$\Rightarrow \left(\frac{b-3}{a+1}\right) \times \left(\frac{3}{4}\right) = -1$$

$$\Rightarrow \frac{3b-9}{4a+4} = -1$$

$$\Rightarrow 3b - 9 = -4a - 4$$

$$\Rightarrow 4a + 3b = 5 \quad \dots\dots(1)$$

Point (a, b) lies on the line $3x - 4y = 16$

$$\therefore 3a - 4b = 16 \quad \dots\dots(2)$$

Solving equations (1) and (2), we obtain

$$a = 68/25 \quad \text{and} \quad b = -49/25$$

Co-ordinates of the foot of perpendicular is $(68/25, -49/25)$

15. The perpendicular from the origin to the line $y = mx + c$ meets it at the point $(-1, 2)$. Find the values of m and c .

15. The given equation of line is $y = mx + c$.

It is given that the perpendicular from the origin meets the given line at $(-1, 2)$.

Therefore, the line joining the points $(0, 0)$ and $(-1, 2)$ is perpendicular to the given line.

$$\therefore \text{Slope of the line joining } (0,0) \text{ and } (-1,2) = \frac{2}{-1} = -2$$

The slope of the given line is m .

$$\therefore m \times -2 = -1 \quad [\text{The two lines are perpendicular}]$$

Since point $(-1,2)$ lies on the given line, it satisfies the equation $y = mx + c$.

$$\Rightarrow m = \frac{1}{2}$$

$$\therefore 2 = m(-1) + c$$

$$\Rightarrow 2 = \frac{1}{2}(-1) + c$$

$$\Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$.

16. If p and q are the lengths of perpendiculars from the origin to the lines $x \cos \theta - y \sin \theta = k \cos 2\theta$ and $x \sec \theta + y \operatorname{cosec} \theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

16. The equations of given lines are

$$x \cos \theta - y \sin \theta = k \cos 2\theta \quad \dots\dots\dots (1)$$

$$x \sec \theta + y \operatorname{cosec} \theta = k \quad \dots\dots\dots (2)$$

Perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Comparing equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \cos \theta$, $B = -\sin \theta$, and $C = -k \cos 2\theta$

Given that p is the length of the perpendicular from $(0, 0)$ to line (1).

$$\therefore p = \frac{|A \times 0 + B \times 0 + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k \cos 2\theta|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = k \cos 2\theta$$

$$p = k \cos 2\theta$$

Squaring both sides

$$p^2 = k^2 \cos^2 2\theta \quad \dots\dots\dots (3)$$

Comparing equation (2) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \sec \theta$, $B = \operatorname{cosec} \theta$, and $C = -k$

Given that q is the length of the perpendicular from $(0, 0)$ to line (2)

$$q = \frac{|A \times 0 + B \times 0 + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k|}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} = \frac{k}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}}$$

$$= \frac{k}{\sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}} = \frac{k \cos \theta \sin \theta}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = k \cos \theta \sin \theta$$

$$q = k \cos \theta \sin \theta$$

Multiplying both sides by 2

$$2q = 2k \cos \theta \sin \theta = k \times 2 \sin \theta \cos \theta$$

$$2q = k \sin 2\theta$$

Squaring both sides

$$4q^2 = k^2 \sin^2 2\theta \quad \dots\dots\dots(4)$$

Adding (3) and (4) we get

$$p^2 + 4q^2 = k^2 \cos^2 2\theta + k^2 \sin^2 2\theta$$

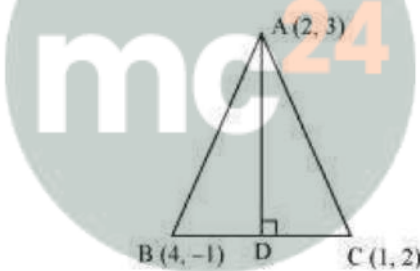
$$\Rightarrow p^2 + 4q^2 = k^2 (\cos^2 2\theta + \sin^2 2\theta) \quad [\because \cos^2 2\theta + \sin^2 2\theta = 1]$$

$$\therefore p^2 + 4q^2 = k^2$$

17. In the triangle ABC with vertices A(2,3), B(4,-1) and C(1,2), find the equation and length of altitude from the vertex A .

17. Let AD be the altitude of triangle ABC from vertex A .

Accordingly, AD ⊥ BC



The equation of the line passing through point (2,3) and having a slope of 1 is

$$(y-3) = 1(x-2)$$

$$\Rightarrow x - y + 1 = 0$$

$$\Rightarrow y - x = 1$$

Therefore, equation of the altitude from vertex A = y - x = 1.

Length of AD = Length of the perpendicular from A(2,3) to BC The equation of BC is

$$(y+1) = \frac{2+1}{1-4}(x-4)$$

$$\Rightarrow (y+1) = -1(x-4)$$

$$\Rightarrow y + 1 = -x + 4$$

$$\Rightarrow x + y - 3 = 0$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = 1, B = 1$, and $C = -3$.

$$\therefore \text{Length of AD} = \frac{|1 \times 2 + 1 \times 3 - 3|}{\sqrt{1^2 + 1^2}} \text{ units} = \frac{|2|}{\sqrt{2}} \text{ units} = \frac{2}{\sqrt{2}} \text{ units} = \sqrt{2} \text{ units}$$

Thus, the equation and the length of the altitude from vertex A are $y - x = 1$ and $\sqrt{2}$ units respectively.

18. If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b , then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

18. Equation of a line whose intercepts on the axes are a and b is $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow bx + ay = ab$$

$$\Rightarrow bx + ay - ab = 0 \quad \dots\dots\dots(1)$$

Perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Comparing equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = b, B = a$ and $C = -ab$

If p is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (1), we obtain

$$p = \frac{|A \times 0 + B \times 0 - ab|}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow p = \frac{|-ab|}{\sqrt{a^2 + b^2}}$$

Squaring both sides we get

$$p^2 = \frac{(-ab)^2}{a^2 + b^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$$

$$\therefore \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$



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