NCERT Solutions for Class-XI Maths

Chapter-6 Exercise-6.1

NCERT Math Class 11

1. Solve $24x < 100$, when

(i) x is a natural number.

(ii) x is an integer.

1. Given that: $24x < 100$

$\Rightarrow x < \frac{100}{24} \Rightarrow x < \frac{25}{6}$

(i) x is a natural number.

We know that $1, 2, 3$, and 4 are the only natural numbers less than $\frac{25}{6}$.

Thus, when x is a natural number, the solutions of the given inequality are $1, 2, 3$, and 4.

Hence, in this case, the solution set is $\{1, 2, 3, 4\}$.

(ii) x is an integer.

The integers less than $\frac{25}{6}$ are $\ldots -3, -2, -1, 0, 1, 2, 3, 4$.

Thus, when x is an integer, the solutions of the given inequality are ...-3, $-2, -1, 0, 1, 2, 3, 4$.

Hence, in this case, the solution set is $\{\ldots -3, -2, -1, 0, 1, 2, 3, 4\}$.

2. Solve – 12x > 30, when

(i) x is a natural number. (ii) x is an integer.

2. It is given in the question that,

– 12x > 30

Dividing the inequality by -12 on both sides we get,

$x < \frac{-5}{2}$

i. When x is a natural integer.

It can be clearly observed that there is no natural number less than $\frac{-5}{2}$ because- $\frac{5}{2}$ is a negative number and natural numbers are positive numbers.

∴there would be no solution of the given inequality when x is a natural number.

ii. When x is an integer

It can be clearly observed that the integer number less than $\frac{-5}{2}$ are…, -5, -4, -3 .

Thus, solution of– 12x > 30is …,-5, -4, -3, when x is an integer.

∴ {…, -5, -4, -3} is the solution set.

3. Solve $5x-3<7$, when

(i) x is an integer.

(ii) x is a real number.

3. Given that: $5x-3<7$

$\Rightarrow 5x<10 \Rightarrow x<\frac{10}{5} \Rightarrow x<2$

(i) x is an integer.

The integers less than 2 are $\ldots,-4,-3,-2,-1,0,1$.

Thus, when x is an integer, the solutions of the given inequality are ...,

$-4,-3,-2,-1,0,1$.

Hence, in this case, the solution set is $\{\ldots,-4,-3,-2,-1,0,1\}$.

(ii) x is a real number.

When x is a real number, the solutions of the given inequality are given by $x<2$, that is, all real numbers x which are less than 2.

Thus, the solution set of the given inequality is $x\in(-\infty,2)$.

4. Solve 3x + 8 >2, when:

(i) x is an integer. (ii) x is a real number.

4. It is given in the question that,

3x + 8 >2

Subtracting 8 from both sides we get,

3x + 8 – 8 >2 – 8

3x >- 6

Dividing both sides by 3 we get,

$\frac{3x}{3}>\frac{-6}{3}$

x > -2

i. When x is an integer

It can be clearly observed that the integer number greater than -2 are -1, 0, 1, 2,...

Thus, solution of3x + 8 >2is -1, 0, 1, 2,… when x is an integer.

∴ {-1, 0, 1, 2,…} is the solution set.

ii. When x is a real number.

It can be clearly observed that the solutions of 3x + 8 >2 will be given by x >-2 which states that all the real numbers that are greater than -2.

∴ x ∈ (-2, ∞) is the solution set.

5. Solve the following inequality for real $x: 4x + 3 < 6x + 7$

5. $4x + 3 < 5x + 7$

$\Rightarrow 4x + 3 - 7 < 5x + 7 - 7$

$\Rightarrow 4x - 4 < 5x$

$\Rightarrow 4x - 4 - 4x < 5x - 4x$

$\Rightarrow -4 < x$

Thus, all real numbers x, which are greater than -4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-4, \infty)$.

6. $3x - 7 > 5x - 1$

6. It is given in the question that,

$3x - 7 > 5x - 1$

Adding 7 to both the sides,

$3x - 7 + 7 > 5x - 1 + 7$

$3x > 5x + 6$

Subtracting 5x from both the sides,

$3x - 5x > 5x + 6 - 5x$

$-2x > 6$

Dividing both sides by -2

$-\frac{2x}{-2} < \frac{6}{-2}$

$x < -3$

∴ The solutions of the given inequality are defined by all the real numbers less than -3.

Thus, (-∞, -3) is the required solution set.

7. Solve the following inequality for real $x: 3(x-1) \le 2(x-3)$

7. $3(x-1) \le 2(x-3)$

$\Rightarrow 3x - 3 \le 2x - 6$

$\Rightarrow 3x - 3 + 3 \le 2x - 6 + 3$

$\Rightarrow 3x \le 2x - 3$

$\Rightarrow 3x - 2x \le 2x - 3 - 2x$

$\Rightarrow x \leq -3$

Thus, all real numbers x, which are less than or equal to -3, are the solutions of the given inequality. Hence, the solution set of the given inequality is $(-\infty, -3]$.

8. Solve the following inequality for real x : $3(2-x) \geq 2(1-x)$

8. It is given in the question that,

$3(2-x) \geq 2(1-x)$

$6 - 3x \geq 2 - 2x$

Adding 2x to both the sides,

$6 - 3x + 2x \geq 2 - 2x + 2x$

$6 - x \geq 2$

Subtracting 6 from both the sides,

$6 - x - 6 \geq 2 - 6$

$-x \geq -4$

$x \leq 4$

∴ The solutions of the given inequality are defined by all the real numbers greater than or equal to 4.

Thus, [4, ∞) is the required solution set.

9. Solve the following inequality for real x : $x + \frac{x}{2} + \frac{x}{3} < 11$

9.

$x + \frac{x}{2} + \frac{x}{3} < 11$

$\Rightarrow x\left(1 + \frac{1}{2} + \frac{1}{3}\right) < 11 \Rightarrow x\left(\frac{6+3+2}{6}\right) < 11 \Rightarrow \frac{11}{6}x < 11 \Rightarrow x < 11 \times \frac{6}{11} \Rightarrow x < 6$

Thus, all real numbers x, which are less than 6, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 6)$.

10. Solve the following inequality for real x : $\frac{x}{3} > \frac{x}{2} + 1$

10. It is given in the question that,

$\frac{x}{3} > \frac{x}{2} + 1$

$\left(\frac{2x - 3x}{6}\right) > 1$

$\frac{-x}{6} > 1$

$-x > 6$

$x < -6$

∴ The solutions of the given inequality are defined by all the real numbers less than - 6.

Thus, $(-\infty, -6)$ is the required solution set.

11. Solve the following inequality for real x :

$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

11.

$\Rightarrow \frac{3x}{5} - \frac{6}{5} \leq \frac{10}{3} - \frac{5x}{3} \Rightarrow \frac{3x}{5} + \frac{5x}{3} \leq \frac{10}{3} + \frac{6}{5} \Rightarrow \frac{9x+25x}{15} x \leq \frac{50+18}{15}$

$\Rightarrow \frac{34x}{15} \leq \frac{68}{15} \Rightarrow 34x \leq 68 \Rightarrow x \leq \frac{68}{34} \Rightarrow x \leq 2$

Thus, all real numbers x , which are less than or equal to 2 , are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

12. Solve the following inequality for real x : $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$

12. It is given in the question that,

$\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$

Cross - multiplying the denominators, we get:

$3\left(\frac{3x}{5} + 4\right) \geq 2(x-6)$

$\left(\frac{9x}{5} + 12\right) \geq 2x - 12$

$12 + 12 \geq 2x - \frac{9x}{5}$

$24 \geq \frac{10x - 9x}{5}$

$24 \geq \frac{x}{5}$

$120 \geq x$

∴ The solutions of the given inequality are defined by all the real numbers less than or equal to 120.
Thus, (-∞, 120] is the required solution set.

13. Solve the following inequality for real x :
$2(2x+3)-10<6(x-2)$

13. $2(2x+3)-10<6(x-2)$

$\Rightarrow 4x-4<6x-12$
$\Rightarrow 4x-6x<-12+4$
$\Rightarrow 4x+6-10<6x-12$

$\Rightarrow -2x<-8 \qquad \Rightarrow -x<-4 \qquad \Rightarrow x>4$

Thus, all real numbers x, which are greater than or equal to 4 , are the solutions of the given inequality. Hence, the solution set of the given inequality is $[4,\infty)$.

14. $37-(3x+5) \geq 9x-8(x-3)$

14. We have,
$37-(3x+5) \geq 9x-8(x-3)$
$= 37-3x-5 \geq 9x-8x+24$
$= 32-3x \geq x+24$
$= 32-24 \geq x+3x$
$= 8 \geq 4x$
$= 2 \geq x$
∴ All the real numbers of x which are less than or equal to 2 are the solutions of the given inequality
Hence, $(-\infty,2]$ will be the solution for the given inequality

15. Solve the following inequality for real x :
$\frac{x}{4}<\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$

15. $\frac{x}{4}<\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$

$\Rightarrow \frac{x}{4}<\frac{5x}{3}-\frac{2}{3}-\frac{7x}{5}+\frac{3}{5} \qquad \Rightarrow \frac{x}{4}-\frac{5x}{3}+\frac{7x}{5}<-\frac{2}{3}+\frac{3}{5} \qquad \Rightarrow \frac{15x-100x+84x}{60}<\frac{-10+9}{15}$

$\Rightarrow -\frac{x}{60}<-\frac{1}{15} \qquad \Rightarrow -x<-\frac{60}{15} \qquad \Rightarrow x>4$

Thus, all real numbers x, which are greater than 4 , are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(4,\infty)$.

16. Solve the following inequality for real x : $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

16. We have,

$\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5} = \frac{(2x-1)}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$

$= \frac{(2x-1)}{3} \geq \frac{15x-10-8+4x}{20}$

$= \frac{(2x-1)}{3} \geq \frac{19x-18}{20}$

$=20\ (2x\text{-}1) \geq 3\ (19x\text{-}18)$

$=40x\text{-}20 \geq 57x\text{-}54$

$= -\ 20+54 \geq 57x\text{-}40x$

$= 34 \geq 17x$

$= 2 \geq x$

∴ All the real numbers of x which are less than or equal to 2 are the solutions of the given inequality

Hence, $(-\infty, 2]$ will be the solution for the given inequality

17. Solve the following inequality and show the graph of the solution on number line:

$3x-2<2x+1$

17. $3x-2<2x+1$

$\Rightarrow 3x-2x<1+2$

3

The graphical representation of the solutions of the given inequality is as follows:

18. $5x - 3 \geq 3x - 5$

18. We have,

$5x-3 \geq 3x-5$

Solving the given inequality, we get:

$5x - 3 \geq 3x - 5$

$= 5x - 3x \geq -5 + 3$

$= 2x \geq -2 = \frac{2x}{2} \geq \frac{-2}{2}$

$= x \geq -1$

Now, the graphical representation of the solution is as follows:

19. Solve the following inequality and show the graph of the solution on number line:

$3(1-x)<2(x+4)$

19. $3(1-x)<2(x+4)$

$$\Rightarrow 3-3x<2x+8$$

$\Rightarrow -3x-2x\langle 8-3\Rightarrow -5x\rangle 5\Rightarrow -x>1\Rightarrow x<-1$

The graphical representation of the solutions of the given inequality is as follows:

20. $\frac{x}{4}\geq\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$

20. We have,

$\frac{x}{4}\geq\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$

Solving the given inequality, we get:

$\frac{x}{4}\geq\frac{(5x-2)}{3}-\frac{(7x-3)}{5}=\frac{x}{2}\geq\frac{5(5x-2)-3(7x-3)}{15}$

$=\frac{x}{2}\geq\frac{25x-10-21x+9}{15}$

$=\frac{x}{2}\geq\frac{4x-1}{15}$

$= 15x \geq 2\ (4x - 1)$

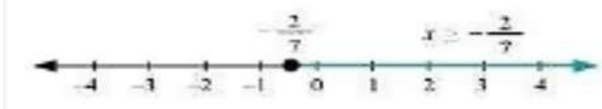
$= 15x \geq 8x - 2$

$= 15x - 8x \geq 8x - 2 - 8x$

$= 7x \geq -2$

$=x\geq-\frac{2}{7}$

Now, the graphical representation of the solution is as follows:



21. Ravi obtained 70 and 75 marks in first two unit test. Find the number if minimum marks he should get in the third test to have an average of at least 60 marks.

21. Let x be the marks obtained by Ravi in the third unit test.

Since the student should have an average of at least 60 marks, so

$\frac{70+75+x}{3} \geq 60 \quad \Rightarrow 145+x \geq 180 \qquad \Rightarrow x \geq 35$

Hence, the student must obtain a minimum of 35 marks to have an average of at least 60 marks.

22. To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.

22. Let us assume Sunita scored x marks in her fifth examination

Now, according to the question in order to receive A grade in the course she must have to obtain average 90 marks or more in her five examinations

$\therefore \frac{87+92+94+95+x}{5} \geq 90$

$= \frac{368+x}{5} \geq 90$

$= 368 + x \geq 450$

$= x \geq 450 - 368$

$= x \geq 82$

Hence, she must have to obtain 82 or more marks in her fifth examination

23. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

23. Let x be the smaller of the two consecutive odd positive integers. Then, the other integer is $x+2$.

Since both the integers are smaller than 10 , therefore

$x+2<10 \qquad \Rightarrow x<10-2 \qquad \Rightarrow x<8$

Also, the sum of the two integers is more than 11.

$\therefore x+(x+2)>11 \quad \Rightarrow 2x+2>11 \quad \Rightarrow 2x>11-2$

$\Rightarrow x>\frac{9}{2} \qquad \Rightarrow x>4.5 \qquad \text{... (ii)}$

From (i) and (ii), we obtain that the value of x can be $4,5,6$ or 7..

Since x is an odd number, x can take the values, 5 and 7 .

Hence, the required possible pairs of numbers are $(5,7)$ and $(7,9)$.

24. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

24. Let us assume x be the smaller of the two consecutive even positive integers

$\therefore$ Other integer = x + 2

It is also given in the question that, both the integers are larger than 5

$\therefore x > 5$ (i)

Also, it is given in the question that sum off two integers is less than 23

$\therefore x + (x + 2) < 23$

$2x + 2 < 23$

$x < \frac{21}{2}$

$x > 10.5$ (ii)

Thus, from (i) and (ii) we have x is an even number and it can take values 6, 8 and 10

Hence, possible pairs are (6, 8), (8, 10) and (10, 12)

25. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

25. Let the length of the shortest side of the triangle be $x$cm.

Then, length of the longest side $= 3x$ cm and the length of the third side $= (3x - 2)$cm

Since the perimeter of the triangle is at least 61 cm,

$\Rightarrow x + 3x + (3x - 2) \geq 61 \Rightarrow 7x - 2 \geq 61$

Hence, the minimum length of the shortest side is 9 cm.

26. A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second?

[Hint: If x is the length of the shortest board, then x, (x + 3) and 2x are the lengths of the second and third piece, respectively. Thus, $x + (x + 3) + 2x \leq 91$ and $2x \geq (x + 3) + 5$].

26. Let the length of the shortest piece be $x$cm. Then, length of the second piece and the third piece are $(x + 3)$cm and $2x$ cm respectively.

Since the three lengths are to be cut from a single piece of board of length 91 cm,

$x + (x + 3) + 2x \leq 91 \Rightarrow 4x + 3 \leq 91 \Rightarrow 4x \leq 91 - 3 \Rightarrow 4x \leq 88$

$\Rightarrow x \leq 22$ …(1)

Also, the third piece is at least 5 cm longer than the second piece.

$\therefore 2x \geq (x+3)+5 \qquad \Rightarrow x \geq 8$

$\Rightarrow 2x \geq x+8 \qquad \ldots(2)$

From (1) and (2), we obtain, $8 \leq x \leq 22$

Thus, the possible length of the shortest board is greater than or equal to 8 cm but less than or equal to 22 cm.