NCERT Solutions for Class-XII Maths

Chapter-1
Exercise- 1.4

1. Determine whether or not each of the definition of given below gives a binary operation. In the event that * is not a binary operation, give justification for this.

(i) On Z+, define * by a * b = a − b

(ii) On Z+, define * by a * b = ab

(iii) On R, define * by a * b = ab2

(iv) On Z+, define * by a * b = |a − b|

(v) On Z+, define * by a * b = a

1. (i) On Z+, * is defined by a * b = a − b.

It is not a binary operation

as the image of (1, 2) under * is $1 * 2 = 1 - 2 = -1 \notin Z^+$.

(ii) On Z^+, * is defined by a * b = ab.

It is seen that for each a, b $\in Z^+$, there is a unique element ab in Z^+.

This means that * carries each pair (a, b) to a unique element a * b = ab in Z^+. Therefore, * is a binary operation.

(iii) On R, * is defined by a * b = ab^2.

It is seen that for each a, b $\in$ R, there is a unique element ab^2 in R.

This means that * carries each pair (a, b) to a unique element a * b = ab^2 in R.

Therefore, * is a binary operation.

(iv) On Z^+, * is defined by a * b = |a − b|.

It is seen that for each a, b $\in Z^+$, there is a unique element |a − b| in Z^+.

This means that * carries each pair (a, b) to a unique element a * b = |a − b| in Z^+.

Therefore, * is a binary operation.

(v) On Z^+, * is defined by a * b = a.

It is seen that for each a, b $\in Z^+$, there is a unique element a in Z^+.

This means that * carries each pair (a, b) to a unique element a * b = a in Z^+. Therefore, * is a binary operation.

2. For each binary operation * defined below, determine whether * is commutative or associative.

(i) On Z, define a * b = a − b

(ii) On Q, define a * b = ab + 1

(iii) On Q, define a * b $= \frac{ab}{2}$

(iv) On Z^+, define a * b = 2^{ab}

(v) On Z^+, define a * b = a^b

(vi) On R − {−1}, define $a * b = \frac{a}{b+1}$

2. **(i)** It is given that On Z, define a ∗ b = a – b

a –b ϵ Z. so the operation * is binary.

We can see that 1 * 2 = 1-2 = -1 and 2 * 1 = 2-1 = 1.

⇒ 1 *2 ≠ 2 * 1, where 1,2 ϵ Z.

⇒ the operation * is not commutative.

Also, we get,

(1 * 2) * 3 = (1 – 2) * 3 = -1 * 3 = -1 -3 = -4

1 * (2 * 3) = 1 * (2 – 3) = 1 * -1 = 1 – (-1) = 2

⇒ the operation * is not associative.

(ii) It is given that On Q, define a ∗ b = ab + 1

ab + 1 ϵ Q, so operation * is binary

We know that ab = ba for a,b ϵ Q

⇒ ab + 1 = ba + 1for a,b ϵ Q

⇒ a * b = a * b for a,b ϵ Q

⇒ 1 *2 ≠ 2 * 1, where 1,2 ϵ Z.

⇒ The operation * is commutative.

Also, we get,

(1 * 2) * 3 = (1 × 2) * 3 = 3 * 3 = 3 × 3 + 1 = 10

1 * (2 * 3) = 1 * (2 × 3) = 1 * 7 = 1 × 7 + 1 = 8

⇒ (1 * 2) * 3 ≠ 1 * (2 * 3)

⇒ the operation * is not associative.

(iii) It is given that On Q, define a ∗ b = $\frac{ab}{2}$

Now, $\frac{ab}{2}$ ϵ Q, so the operation * is binary.

We know that ab = ba for a,b ϵ Q

⇒ $\frac{ab}{2} = \frac{ba}{2}$ for a,b ϵ Q

⇒ a * b = a * b for a,b ϵ Q

⇒ The operation * is commutative.

Also for all a,b,c ϵ Q, we get,

$(a * b) * c = (\frac{ab}{2}) * c = \frac{\left(\frac{ab}{2}\right)c}{2} = \frac{abc}{4}$

$a * (b * c) = a * (\frac{bc}{2}) = \frac{a\left(\frac{bc}{2}\right)}{2} = \frac{abc}{4}$

$\Rightarrow$ (a * b) * c = a * (b * c)

$\Rightarrow$ the operation * is associative.

(iv) It is given that on Z^+, define $a * b = 2^{ab}$

$2^{ab} \in Z^+$, so operation * is binary

We know that ab = ba for a,b $\in Z^+$

$\Rightarrow$ $2^{ab} = 2^{ba}$ for a,b $\in Z^+$

$\Rightarrow$ a * b = a * b for a,b $\in Z^+$

$\Rightarrow$ The operation * is commutative.

Also, we get,

$(1 * 2) * 3 = 2^{(1\times2)} * 3 = 4 * 3 = 2^{(4\times3)} = 2^{12}$

$1 * (2 * 3) = 1 * 2^{(2\times3)} = 1 * 2^6 = 1 \times 64 = 2^{64}$

$\Rightarrow$ $(1 * 2) * 3 \neq 1 * (2 * 3)$

$\Rightarrow$ The operation * is not associative.

(v) It is given that On Z^+, define $a * b = a^b$

$a^b \in Z^+$, so operation * is binary

We know that ab = ba for a,b $\in Z^+$

$\Rightarrow$ $1 * 2 = 1^2$ and $2 * 1 = 2^1 = 2$

$\Rightarrow$ $1 * 2 \neq 2 * 1$, where 1,2 $\in Z^+$

$\Rightarrow$ The operation * is not commutative.

Also, we get,

$(1 * 2) * 3 = 2^3 * 3 = 8 * 3 = 24$

$1 * (2 * 3) = 1 * 3^2 = 1 * 9 = 9$

$\Rightarrow$ $(1 * 2) * 3 \neq 1 * (2 * 3)$, where 1,2,3 $\in Z^+$

$\Rightarrow$ The operation * is not associative.

(vi) It is given that On R, $* - \{-1\}$, define $a * b = \frac{a}{b+1}$

$\frac{a}{b+1} \in R$ for $b \neq -1$, so the operation * is binary.

We can see that $1 * 2 = \frac{1}{2+1} = \frac{1}{3}$ and $2 * 1 = \frac{2}{1+1} = \frac{2}{2} = 1$

$\Rightarrow$ $1 * 2 \neq 2 * 1$; where $1,2 \in R - \{-1\}$

$\Rightarrow$ the operation * is not commutative.

Now, we can observed that

$(1 * 2) * 3 = \frac{1}{3} * 3 = \frac{\frac{1}{3}}{3+1} = \frac{1}{12}$

$1 * (2 * 3) = 1 * \frac{2}{3+1} = 1 * \frac{2}{4} = 1 * \frac{1}{2} = \frac{1}{\frac{1}{2}+1} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

$\Rightarrow$ $(1 * 2) * 3 \neq 1 * (2 * 3)$, where $1,2,3 \in R * - \{-1\}$

$\Rightarrow$ The operation * is not associative.

3. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b = \min \{a, b\}$. Write the operation table of the operation $\wedge$.

3. The binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ is defined as $a \wedge b = \min \{a, b\}$ for all $a, b \in \{1, 2, 3, 4, 5\}$.

Thus, the operation table for the given operation $\wedge$ can be given as:

$\wedge$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

4. Consider a binary operation * on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table.

(i) Compute $(2 * 3) * 4$ and $2 * (3 * 4)$

(ii) Is * commutative?

(iii) Compute $(2 * 3) * (4 * 5)$.

(Hint: use the following table)

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

4. (i) $(2 * 3) * 4 = 1 * 4 = 1$

$2 * (3 * 4) = 2 * 1 = 1$

(ii) For every a,b ϵ {1, 2, 3, 4, 5},
We have, a * b = b * a
$\Rightarrow$ the operation * is commutative.

(iii) $(2 * 3) = 1$
$\Rightarrow (2 * 3) * (4 * 5) = 1 * 1 = 1$

5. Let *′ be the binary operation on the set {1, 2, 3, 4, 5} defined by a *′ b = H.C.F. of a and b. Is the operation *′ same as the operation * defined in Exercise 4 above? Justify your answer.

5. The binary operation *′ on the set {1, 2, 3 4, 5} is defined as a *′ b = H.C.F of a and b. The operation table for the operation *′ can be given as:

*′	**1**	**2**	**3**	**4**	**5**
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

We observe that the operation tables for the operations * and *′ are the same. Thus, the operation *′ is same as the operation*.

6. Let * be the binary operation on N given by $a * b$ = L.C.M. of a and b. Find
(i) 5 * 7, 20 * 16
(ii) Is * commutative?
(iii) Is * associative?
(iv) Find the identity of * in N
(v) Which elements of N are invertible for the operation *?

6. It is given that the binary operation on N given by a $*$ b = L.C.M. of a and b.
Then, 5 * 7 = LCM of 5 and 7 = 35
20 * 16 = LCM of 20 and 16 = 80.

(ii) It is given that the binary operation on N given by a $*$ b = L.C.M. of a and b.
We know that LCM of a and b = LCM of b and a, a,b ϵ N.
$\Rightarrow$ a * b = b * a
Therefore, the operation * is commutative.

(iii) It is given that the binary operation on N given by a ∗ b = L.C.M. of a and b.
For a, b, c ∈ N
(a * b) * c = (LCM of a and b) * c = LCM of a, b and c
a * (b * c) = a * (LCM of b and c) =LCM of a, b and c
⇒ (a * b)* c = a * (b * c)
Therefore, the operation * is associative.

(iv) It is given that the binary operation on N given by a ∗ b = L.C.M. of a and b.
We know that LCM of a and 1 = a = LCM of 1 and 1, a ∈ N
⇒ a * 1 = a = 1 * a, a ∈ N
Therefore, 1 is the identity of * in N.

(v) It is given that the binary operation on N given by a ∗ b = L.C.M. of a and b.
An element a in N is invertible w.r.t. the operation * if there exists an element b in N,
Such that a * b = e =b * a
Now, if e = 1
⇒ LCM of a and b = 1= LCM of b and a
⇒ This is only possible when a = b = 1
Therefore, 1 is the only invertible element of N w.r.t. the operation *.

7. Is * defined on the set {1, 2, 3, 4, 5} by $a * b$ = L.C.M. of a and b a binary operation? Justify your answer.

7. The operation * on the set A = {1, 2, 3, 4, 5} is defined as $a * b$ = L.C.M. of a and b.
Then, the operation table for the given operation * can be given as:

*	1	2	3	4	5
1	1	2	3	4	5
2	2	2	6	4	10
3	3	6	3	12	15
4	4	4	12	4	20
5	5	10	15	20	5

It can be observed from the obtained table that 3 * 2 = 2 * 3 = 6 ∉ A,
5 * 2 = 2 * 5 = 10 ∉ A,
3 * 4 = 4 * 3 = 12 ∉ A,
3 * 5 = 5 * 3 = 15 ∉ A,
4 * 5 = 5 * 4 = 20 ∉ A
Hence, the given operation * is not a binary operation.

8. Let * be the binary operation on N defined by $a * b$ = H.C.F. of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on N?

8. It is given that the binary operation on N defined by a ∗ b = H.C.F. of a and b.

We know that HCF of a and b = HCF of b and a, a,b ϵ N.

⇒ a * b = b * a

⇒ **The operation * is commutative.**

For a, b c ϵ N, we get,

(a * b) * c = (HCF of a and b) * c = HCF of a, b and c

a * (b * c) = a * (HCF of b and c) = HCF of a, b and c

⇒ (a * b) * c = a * (b * c)

⇒ **The operation * is associative.**

Now, an element e ϵ N will be the identity for the operation.

Now, if a * e = a = e * a, ∀ a ϵ N.

But, this is not true for any a ϵ N.

Therefore, the operation * does not have any identity in N.

9. Let * be a binary operation on the set Q of rational numbers as follows:

(i) $a * b = a - b$

(ii) $a * b = a^2 + b^2$

(iii) $a * b = a + ab$

(iv) $a * b = (a - b)2$

(v) $a * b = \frac{ab}{4}$

(vi) a * b = ab2

Find which of the binary operations are commutative and which are associative.

9. (i) On Q, the operation * is defined as $a * b = a - b$. It can be observed that:

$$\frac{1}{2} * \frac{1}{3} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

And

$$\frac{1}{3} * \frac{1}{2} = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$$

$$\therefore \frac{1}{2} * \frac{1}{3} \neq \frac{1}{3} * \frac{1}{2}, \text{ where } \frac{1}{2}, \frac{1}{3} \in Q$$

Thus, the operation * is not commutative.

It can also be observed that

$$\left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{3} = \left(\frac{1}{2} - \frac{1}{3}\right) * \frac{1}{4}\left(\frac{3-2}{6}\right) * \frac{1}{4} = \frac{1}{6} * \frac{1}{4} = \frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = \frac{-1}{12}$$

and

$\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)=\frac{1}{2}*\left(\frac{1}{3}-\frac{1}{4}\right)=\frac{1}{2}*\left(\frac{4-3}{12}\right)=\frac{1}{2}*\frac{1}{12}=\frac{1}{2}-\frac{1}{12}=\frac{6-1}{12}=\frac{5}{12}$

$\therefore\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}\neq\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)$, where $\frac{1}{2},\frac{1}{3},\frac{1}{4}\in Q$

Thus, the operation * is not associative.

(ii) On Q, the operation * is defined as $a * b = a^2 + b^2$.

For $a, b \in$ Q, we have

$a * b = a^2 + b^2 = b^2 + a^2 = b * a$

$\therefore a * b = b * a$

Thus, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = (1^2 + 2^2) * 3 = (1 + 4) * 3 = 5 * 3 = 5^2 + 3^2 = 34$ and

$1 * (2 * 3) = 1 * (2^2 + 3^2) = 1 * (4 + 9) = 1 * 13 = 1^2 + 13^2 = 170$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in$ Q

Thus, the operation * is not associative.

(iii) On Q, the operation * is defined as $a * b = a + ab$.

It can be observed that

$1 * 2 = 1 + 1\times2 = 1 + 2 = 3$

$2 * 1 = 2 + 2\times1 = 2 + 2 = 4$

$\therefore 1 * 2 \neq 2 * 1$, where $1, 2 \in$ Q

Thus, the operation * is not commutative.

It can also be observed that

$(1 * 2) * 3 = (1+ 1\times2) * 3 = (1 + 2) * 3 = 3 * 3 = 3 + 3\times3 = 3 + 9 = 12$ and

$1 * (2 * 3) = 1 * (2 + 2 \times 3) = 1 * (2 + 6) = 1 * 8 = 1 + 1 \times 8 = 1 + 8 = 9$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in$ Q

Thus, the operation * is not associative.

(iv) On Q, the operation * is defined by $a * b = (a - b)2$.

For $a, b \in$ Q, we have

$a * b = (a - b)^2$

$b * a = (b - a)^2 = [-(a - b)]^2 = (a - b)^2$

$\therefore a * b = b * a$

Thus, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = (1 - 2)^2 * 3 = (-1)^2 * 3 = 1 * 3 = (1 - 3)^2 = (-2)^2 = 4$

and

$1 * (2 * 3) = 1 * (2 - 3)^2 = 1 * (-1)^2 = 1 * 1 = (1 - 1)^2 = 0$

∴ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1, 2, 3 ∈ Q

Thus, the operation * is not associative.

(v) On Q, the operation * is defined as a*b $= \frac{ab}{4}$

For a, b ∈ Q, we have

$$a*b = \frac{ab}{4} = \frac{ba}{4} = b*a$$

∴ a * b = b * a

Thus, the operation * is commutative.

For a, b, c ∈ Q, we have

$$(a*b)*c = \left(\frac{ab}{4}\right)*c = \frac{\left(\frac{ab}{4}\right).c}{4} = \frac{abc}{16}$$

And

$$a*(b*c) = a*\left(\frac{bc}{4}\right) = \frac{a.\left(\frac{bc}{4}\right)}{4} = \frac{abc}{16}$$

∴ $(a * b) * c = a * (b * c)$, where $a, b, c \in Q$

Thus, the operation * is associative.

(vi) On Q, the operation * is defined as $a * b = ab^2$

It can be observed that

$$\frac{1}{2}*\frac{1}{3} = \frac{1}{2}.\left(\frac{1}{3}\right)^2 = \frac{1}{2}.\frac{1}{9} = \frac{1}{18}$$

and

$$\frac{1}{3}*\frac{1}{2} = \frac{1}{3}.\left(\frac{1}{2}\right)^2 = \frac{1}{3}.\frac{1}{4} = \frac{1}{12}$$

$\therefore \frac{1}{2}*\frac{1}{3} \neq \frac{1}{3}*\frac{1}{2}$, where $\frac{1}{2}$ and $\frac{1}{3} \in Q$

Thus, the operation * is not commutative. It can also be observed that

$$\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4} = \left[\frac{1}{2}\left(\frac{1}{3}\right)^2\right]*\frac{1}{4} = \frac{1}{18}*\frac{1}{4} = \frac{1}{18}.\left(\frac{1}{4}\right)^2 = \frac{1}{18\times16} = \frac{1}{288}$$

and

$$\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right) = \frac{1}{2}*\left[\frac{1}{3}\left(\frac{1}{4}\right)^2\right] = \frac{1}{2}*\frac{1}{18} = \frac{1}{2}\left(\frac{1}{48}\right)^2 = \frac{1}{2\times2304} = \frac{1}{4608}$$

$\therefore \left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4} \neq \frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)$, where $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \in Q$

Thus, the operation * is not associative.

Hence, the operations defined in (ii), (iv), (v) are commutative and the operation defined in (v) is associative.

10. Find which of the operations given above has identity.

10. (i) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \ \forall \ a \in Q.$

$a * b = a - b$

This operation is not commutative,

Therefore, it does not have identity element.

(ii) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \ \forall \ a \in Q.$

$a * b = a^2 + b^2$

If $a * e = a$, then $a^2 + e^2 = a$.

For $a = -2$, $(-2)^4 + e^2 \neq -2$.

Therefore, there is no identity element.

(iii) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \ \forall \ a \in Q.$

Now, $a * b = a + ab$

This is not commutative.

Therefore, there is no identity element.

(iv) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \ \forall \ a \in Q.$

$a * b = (a - b)^2$

If $a * e = a$, then $(a - e)^2 = a$.

A square is always positive, thus for $a = -2$, $(-2 - e)^2 \neq -2$.

Therefore, there is no identity element.

(v) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \ \forall \ a \in Q.$

$a * b = \frac{ab}{4}$

If $a * e = a$, then $\frac{ae}{4} = a.$

Therefore, $e = 4$ is the identity element.

$a * 4 = 4 * a = \frac{4a}{4} = a.$

(vi) An element $e \in Q$ will be the identity element for the operation * if

a * e = a = e * a, $\forall$ a ϵ Q.
Now, a * b = ab2
This operation is not commutative,
Therefore, there is not have identity element.

11. Let A = N × N and * be the binary operation on A defined by
$(a, b) * (c, d) = (a + c, b + d)$
Show that * is commutative and associative. Find the identity element for * on A, if any.

11. A = N × N and * is a binary operation on A and is defined by
$(a, b) * (c, d) = (a + c, b + d)$
Let $(a, b), (c, d) \in A$
Then, $a, b, c, d \in N$
We have:
$(a, b) * (c, d) = (a + c, b + d)$
$(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d)$
[Addition is commutative in the set of natural numbers]
$\therefore (a, b) * (c, d) = (c, d) * (a, b)$
Therefore, the operation * is commutative.
Now, let $(a, b), (c, d), (e, f) \in A$
Then, $a, b, c, d, e, f \in N$
We have
$[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)$ and
$(a, b) * [(c, d) * (e, f)] = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)$
$\therefore [(a, b) * (c, d)] * (e, f) = (a, b) * [(c, d) * (e, f)]$
Therefore, the operation * is associative.
Let an element $e = (e_1, e_2) \in A$ will be an identity element for the operation *
if $a * e = a = e * a$ for all $a = (a_1, a_2) \in A$
i.e., $(a_1 + e_1, a_2 + e_2) = (a_1, a_2) = (e_1 + a_1, e_2 + a_2)$
Which is not true for any element in A.
Therefore, the operation * does not have any identity element.

12. State whether the following statements are true or false. Justify.
(i) For an arbitrary binary operation * on a set N, $a * a = a \forall a \in N$.
(ii) If * is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$

12. (i) For an arbitrary binary operation $*$ on a set N, $a * a = a \forall a \in N$.
The above statement is false.
Explanation: It is given that an operation $*$ on a set N, $a * a = a \forall a \in N$
Then, in particular, for b = a = 3, we get,

$3 * 3 = 3 + 3 = 6 \neq 3$

13. Consider a binary operation * on N defined as $a * b = a^3 + b^3$. Choose the correct answer.

(A) Is * both associative and commutative?

(B) Is * commutative but not associative?

(C) Is * associative but not commutative?

(D) Is * neither commutative nor associative?

13. On N, the operation * is defined as $a * b = a^3 + b^3$.

For, $a, b, \in$ N, we have

$a * b = a^3 + b^3 = b^3 + a^3 = b * a$ [Addition is commutative in N]

Therefore, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = (1^3 + 2^3) * 3 = (1 + 8) * 3 = 9 * 3 = 9^3 + 3^3 = 729 + 27 = 756$

and

$1* (2 * 3) = 1 * (2^3 + 3^3) = 1* (8 + 27) = 1*35 = 1^3 + 35^3 = 1 + 42875 = 42876$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in$ N

Therefore, the operation * is not associative.

Hence, the operation * is commutative, but not associative.

Thus, the correct answer is B.