

Activity 6

OBJECTIVE

To find the sum of first n natural numbers.

MATERIAL REQUIRED

Cardboard, coloured papers, white paper, cutter, adhesive.

METHOD OF CONSTRUCTION

1. Take a rectangular cardboard of a convenient size and paste a coloured paper on it. Draw a rectangle ABCD of length 11 units and breadth 10 units.
2. Divide this rectangle into unit squares as shown in Fig. 1.
3. Starting from upper left-most corner, colour one square, 2 squares and so on as shown in the figure.

DEMONSTRATION

1. The pink colour region looks like a stair case.
2. Length of 1st stair is 1 unit, length of 2nd stair is 2 units, length of 3rd stair is 3 units, and so on, length of 10th stair is 10 units.

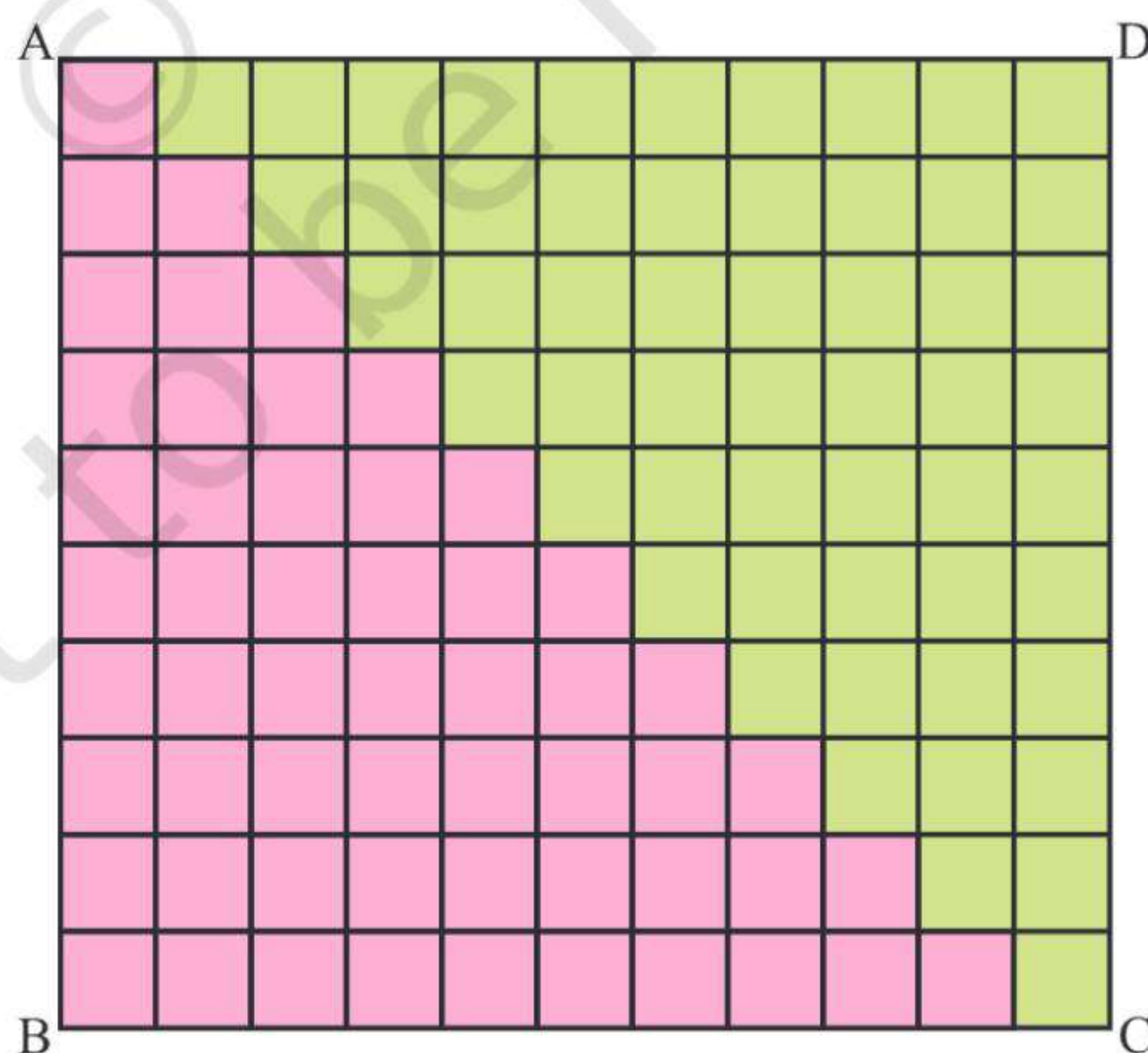


Fig. 1

3. These lengths give a pattern

1, 2, 3, 4, ..., 10,

which is an AP with first term 1 and common difference 1.

4. Sum of first ten terms

$$= 1 + 2 + 3 + \dots + 10 = 55 \quad (1)$$

Area of the shaded region $= \frac{1}{2}$ (area of rectangle ABCD)

$= \frac{1}{2} \times 10 \times 11$, which is same as obtained in (1) above. This shows that the

sum of the first 10 natural numbers is $\frac{1}{2} \times 10 \times 11 = \frac{1}{2} \times 10(10+1)$.

This can be generalised to find the sum of first n natural numbers as

$$S_n = \frac{1}{2} n(n+1) \quad (2)$$

OBSERVATION

For $n = 4$, $S_n = \dots$

For $n = 12$, $S_n = \dots$

For $n = 50$, $S_n = \dots$

For $n = 100$, $S_n = \dots$

APPLICATION

Result (2) may be used to find the sum of first n terms of the list of numbers:

1. $1^2, 2^2, 3^2, \dots$

2. $1^3, 2^3, 3^3, \dots$

to be studied in Class XI.

Activity 7

OBJECTIVE

To find the sum of the first n odd natural numbers.

MATERIAL REQUIRED

Cardboard, thermocol balls, pins, pencil, ruler, adhesive, white paper.

METHOD OF CONSTRUCTION

1. Take a piece of cardboard of a convenient size and paste a white paper on it.
2. Draw a square of suitable size on it (10 cm $\times$ 10 cm).
3. Divide this square into unit squares.
4. Fix a thermocol ball in each square with the help of a pin as shown in Fig. 1.
5. Enclose the balls as shown in the figure.

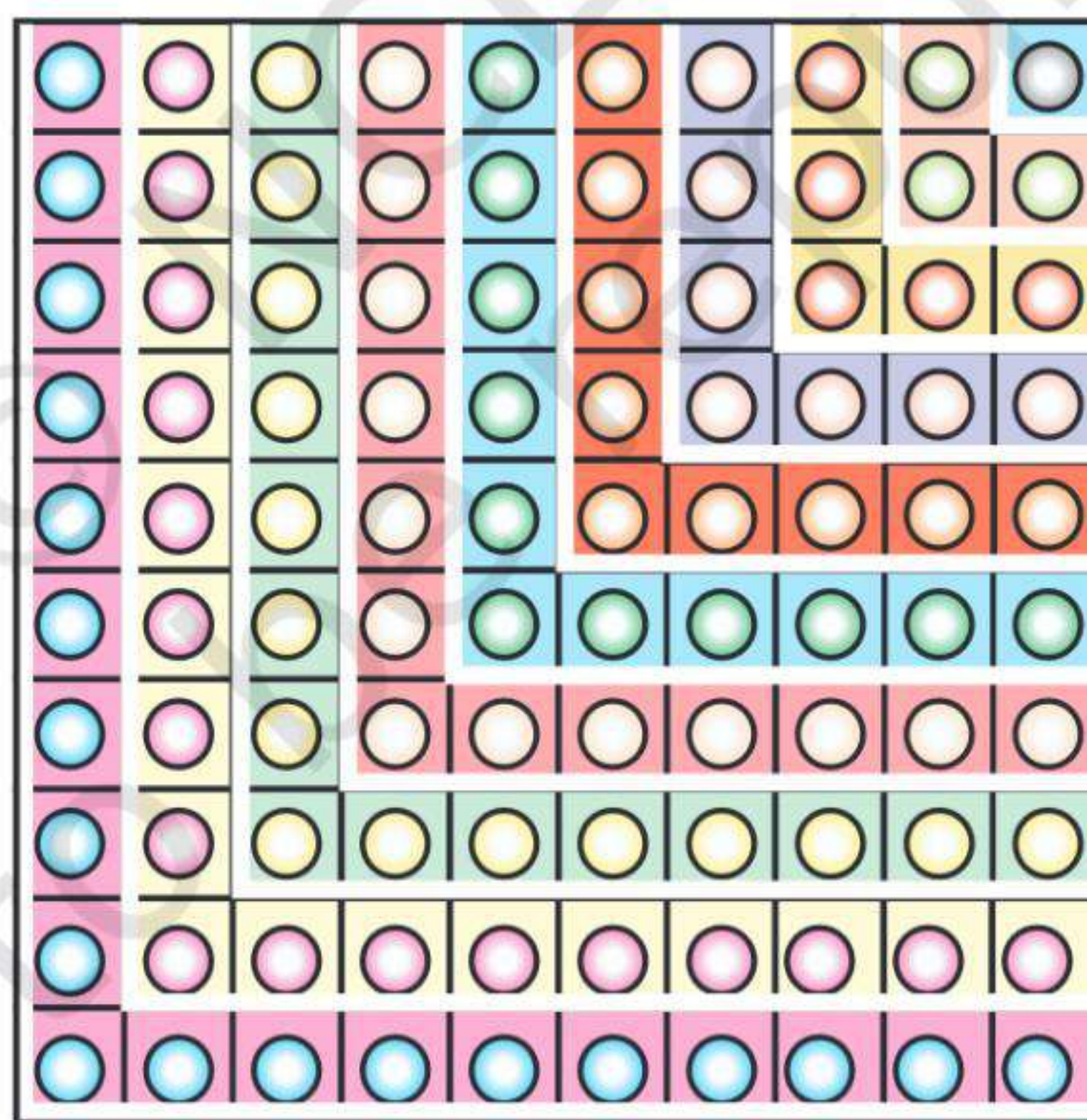


Fig. 1

DEMONSTRATION

Starting from the uppermost right corner, the number of balls in first enclosure (blue colour) = 1 ($=1^2$),

the number of balls in first 2 enclosures = $1 + 3 = 4 (=2^2)$,
 the number of balls in first 3 enclosures = $1 + 3 + 5 = 9 (=3^2)$,
 the number of balls in first 10 enclosures = $1 + 3 + 5 + \dots + 19 = 100 (=10^2)$.
 This gives the sum of first ten odd natural numbers. This result can be generalised for the sum of first n odd numbers as:

$$S_n = 1 + 3 + + (2n - 1) = n^2 \tag{1}$$

OBSERVATION

For $n = 4$ in (1), $S_n =$
 For $n = 5$ in (1), $S_n =$
 For $n = 50$ in (1), $S_n =$
 For $n = 100$ in (1), $S_n =$

APPLICATION

The activity is useful in determining formula for the sum of the first n odd natural numbers.

Activity 8

OBJECTIVE

To find the sum of the first n -even natural numbers.

MATERIAL REQUIRED

Cardboard, thermocol balls, pins, pencil, ruler, white paper, chart paper, adhesive.

METHOD OF CONSTRUCTION

1. Take a piece of cardboard of a convenient size and paste a white paper on it.
2. Draw a rectangle of suitable size on it (10 cm $\times$ 11 cm).
3. Divide this rectangle into unit squares.
4. Fix a thermocol ball in each square using a pin as shown in the Fig. 1.
5. Enclose the balls as shown in the figure.

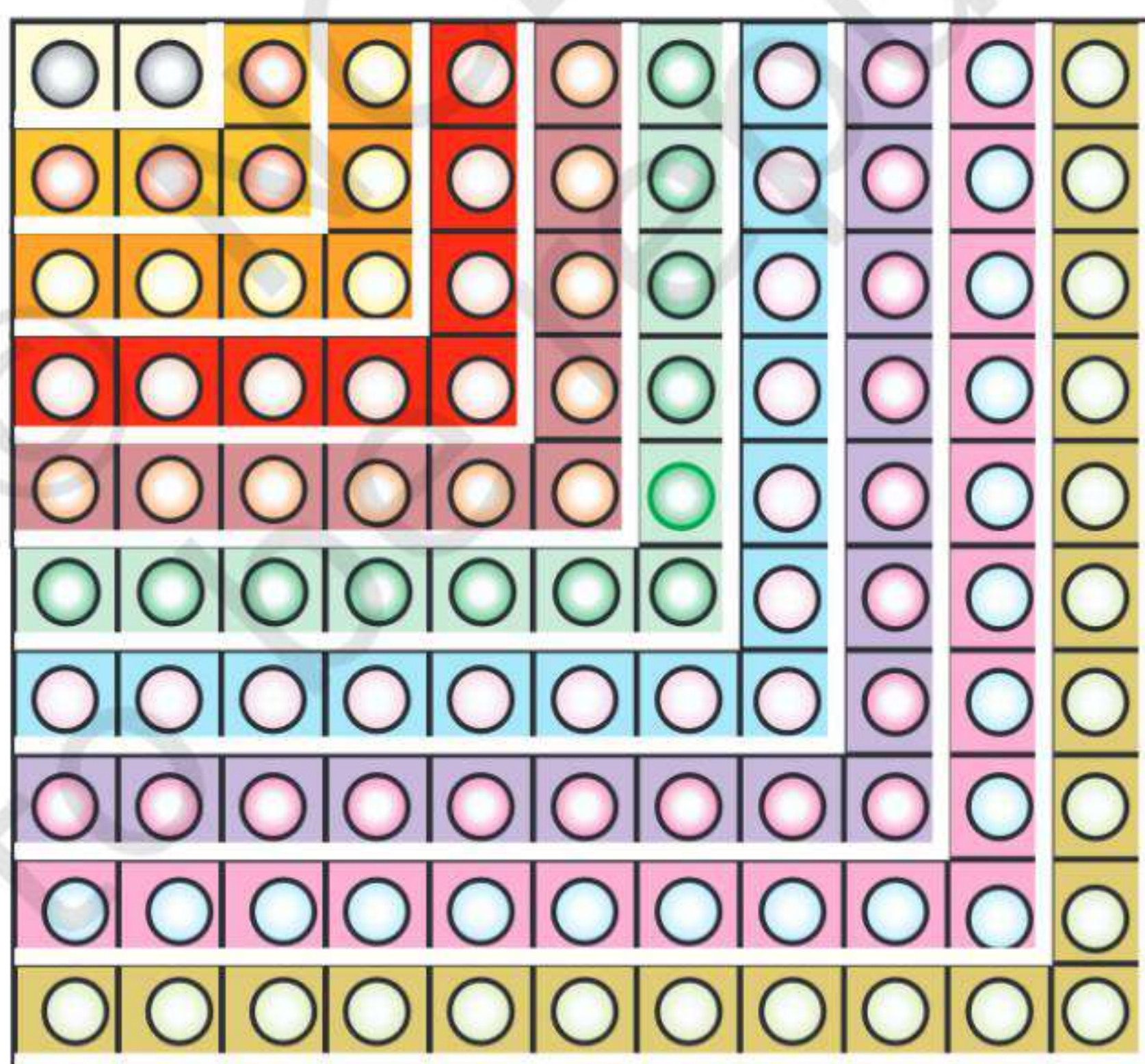


Fig. 1

DEMONSTRATION

Starting from the uppermost left corner,

the number of balls in first enclosure = $2 (= 1 \times 2)$,
the number of balls in first two enclosures = $2 + 4 = 6 (= 2 \times 3)$,
the number of balls in first three enclosures = $2 + 4 + 6 = 12 (= 3 \times 4)$,
:
the number of balls in first six enclosures = $2 + 4 + 6 + 8 + 10 + 12 = 42 (= 6 \times 7)$
the number of balls in first ten enclosures = $2 + 4 + 6 + 8 + \dots + 20 = 110 (= 10 \times 11)$
This gives the sum of first ten even natural numbers.
This result can be generalised for the sum of first n even natural numbers as

$$S_n = 2 + 4 + 6 + \dots + 2n = n \times (n + 1) \tag{1}$$

OBSERVATION

For $n = 4$ in (1), $S_n = \dots\dots\dots$
For $n = 7$ in (1), $S_n = \dots\dots\dots$
For $n = 40$ in (1), $S_n = \dots\dots\dots$
For $n = 70$ in (1), $S_n = \dots\dots\dots$
For $n = 100$ in (1), $S_n = \dots\dots\dots$

APPLICATION

The formula $S_n = n (n+1)$ is useful in finding out the sum of the first n even numbers.

Activity 9

OBJECTIVE

To establish a formula for the sum of first n terms of an Arithmetic Progression.

MATERIAL REQUIRED

Cardboard, coloured drawing sheets, white paper, cutter, adhesive.

METHOD OF CONSTRUCTION

1. Take a rectangular cardboard of a convenient size and paste a white paper on it. Draw a rectangle ABCD of length $(2a+9d)$ units and breadth 10 units.
2. Make some rectangular strips of equal length a units and breadth one unit and some strips of length d units and breadth 1 unit, using coloured drawing sheets.
3. Arrange/paste these strips on the rectangle ABCD as shown in Fig. 1.

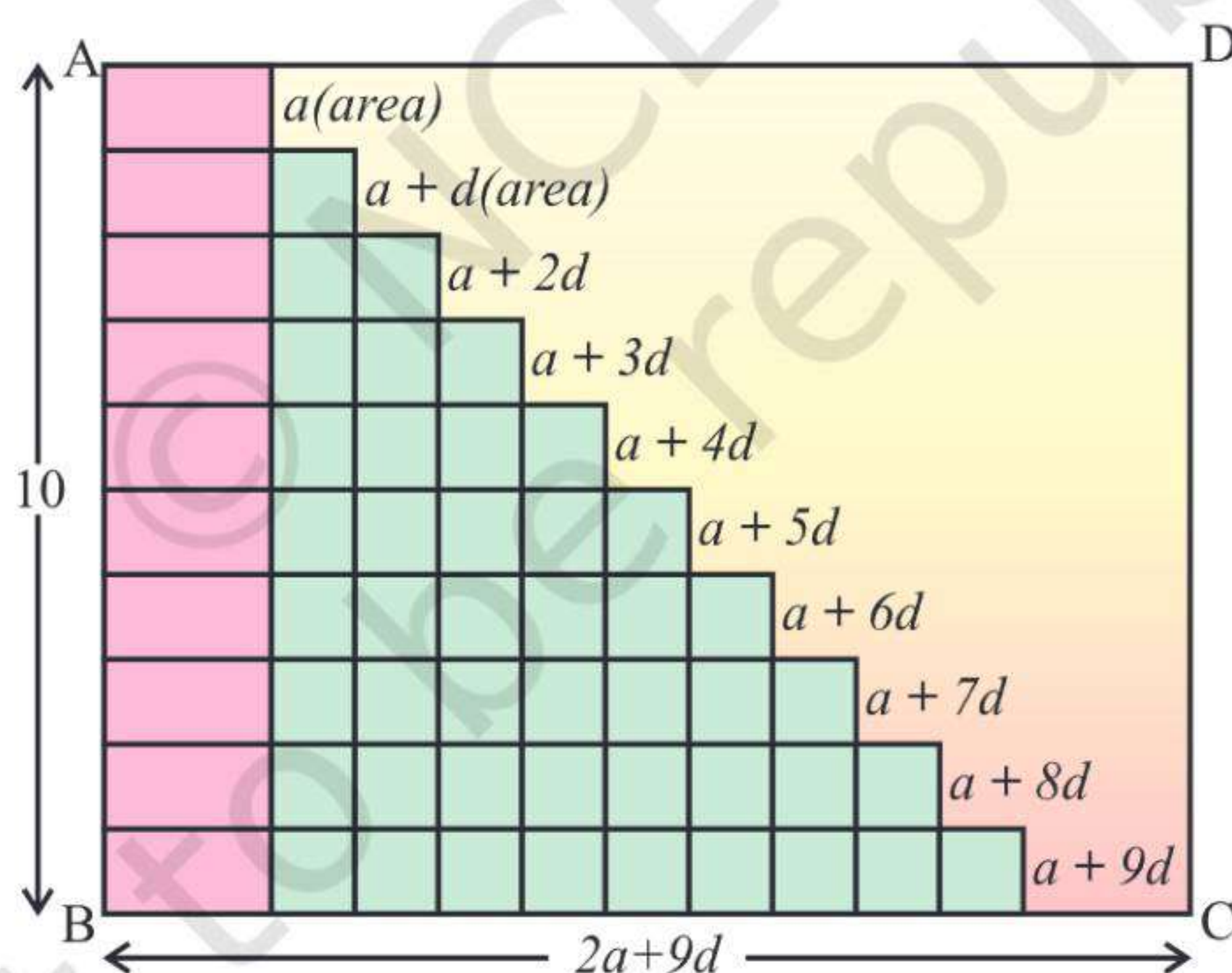


Fig. 1

DEMONSTRATION

1. The strips so arranged look like a stair case.
2. The first stair is of length a units, the second stair is of length $a+d$ (units), third of $a+2d$ units and so on and each is of breadth 1 unit. So, the areas (in sq. units) of these strips are $a, a + d, a + 2d, \dots, a+9d$, respectively.

3. This arrangement of strips gives a pattern $a, a + d, a + 2d, a + 3d, \dots$ which is an AP with first term a and the common difference d .

4. The sum of the areas (in square units) of these strips
 $= a + (a + d) + (a + 2d) + \dots + (a + 9d) = 10a + 45d \quad (1)$

5. Area of the designed formed by the stair case $= \frac{1}{2}$ (area of rectangle ABCD)
 $= \frac{1}{2}(10)(2a + 9d)$
 $= (10a + 45d)$, which is the same as obtained in (1) above.

This shows that the sum of first 10 terms of the AP $= \frac{1}{2}(10)(2a + 9d)$

$$= \frac{1}{2}(10) [2a + (10 - 1)d]$$

This can be further generalised to find the sum of first n terms of an AP as

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

OBSERVATION

On actual measurement:

$$a = \text{-----}, \quad d = \text{-----}, \quad n = \text{-----} \quad S_n = \text{-----}$$

$$\text{So, } S_n = \frac{n}{2} [- + (n - 1) -].$$

APPLICATION

This result may be used to find the sum of first n terms of the list of numbers :

1. $1^2, 2^2, 3^2, \dots$
2. $1^3, 2^3, 3^3, \dots$

to be studied in Class XI.

Activity 10

OBJECTIVE

To verify the distance formula by graphical method.

MATERIAL REQUIRED

Cardboard, chart paper, graph paper, glue, pen/pencil and ruler.

METHOD OF CONSTRUCTION

1. Paste a chart paper on a cardboard of a convenient size.
2. Paste the graph paper on the chart paper.
3. Draw the axes $X'OX$ and $Y'OY$ on the graph paper [see Fig. 1].
4. Take two points $A(a, b)$ and $B(c, d)$ on the graph paper and join them to get a line segment AB [see Fig. 2].

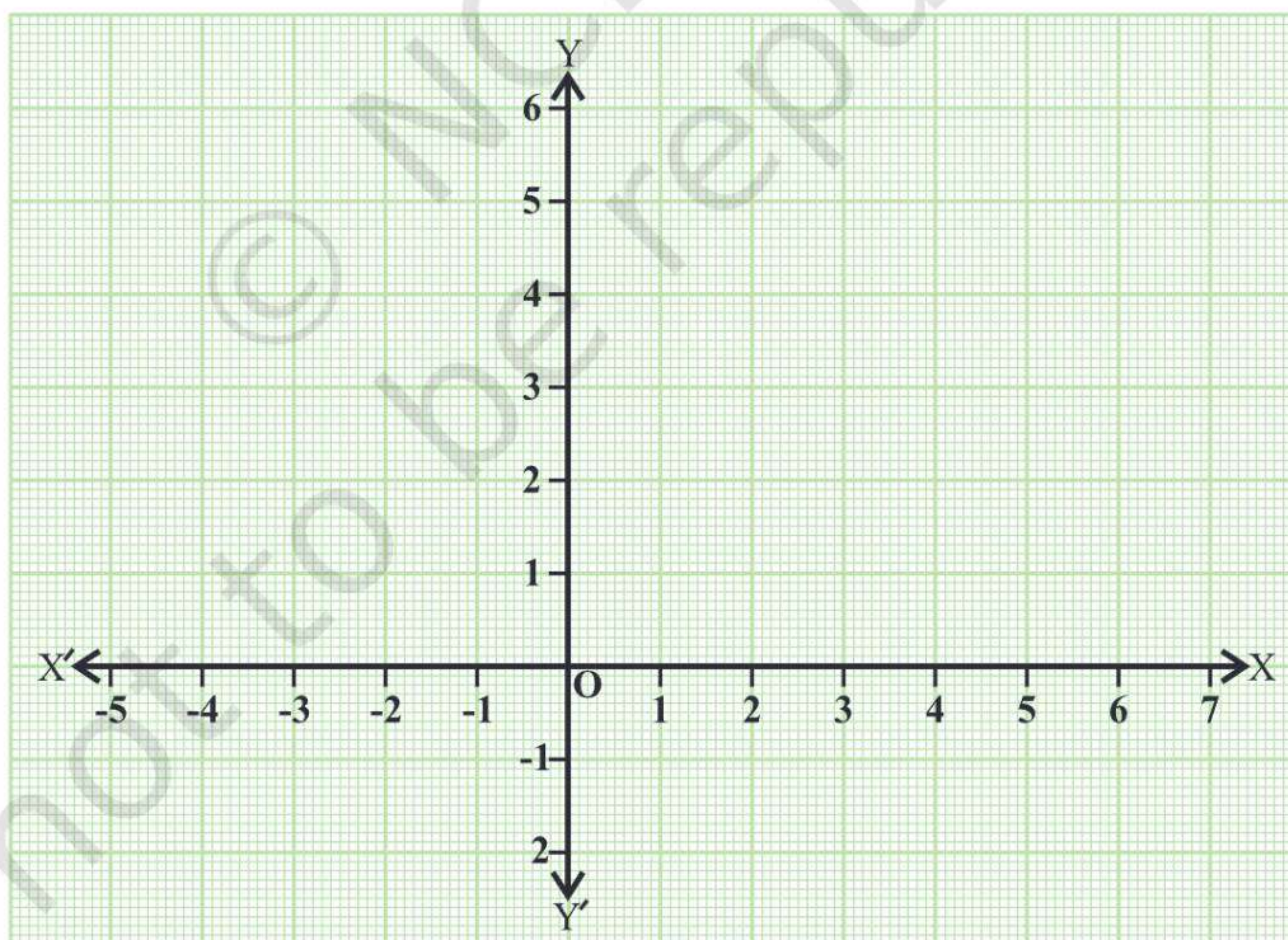


Fig. 1

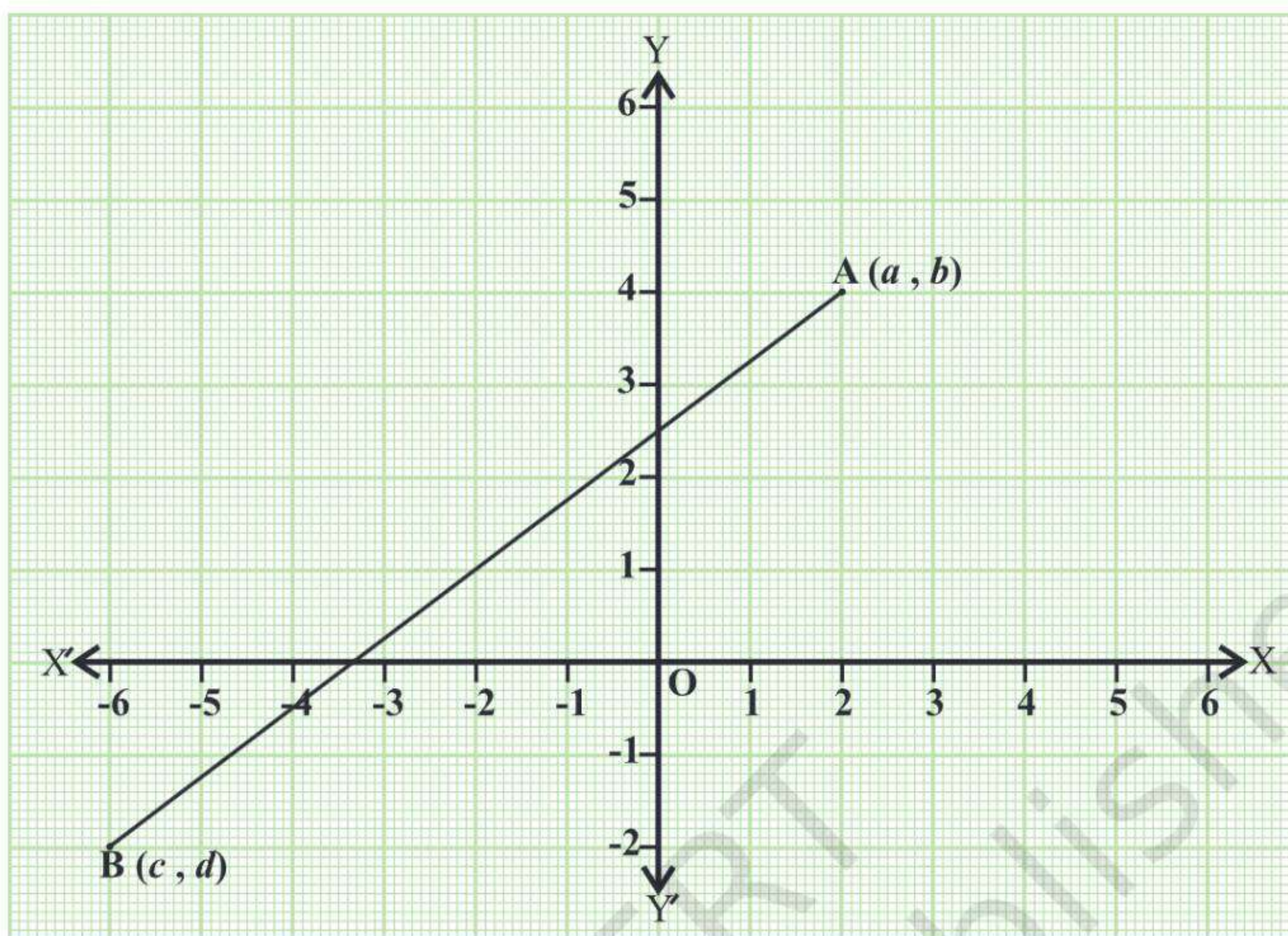


Fig. 2

DEMONSTRATION

1. Calculate the distance AB using distance formula.
2. Measure the distance between the two points A and B using a ruler.
3. The distance calculated by distance formula and distance measured by the ruler are the same.

OBSERVATION

1. Coordinates of the point A are _____.
Coordinates of the point B are _____.
2. Distance AB, using distance formula is _____.
3. Actual distance AB measured by ruler is _____.
4. The distance calculated in (2) and actual distance measured in (3) are _____.

APPLICATION

The distance formula is used in proving a number of results in geometry.