EXERCISE 12.1

1. If P (n) is the statement "n (n + 1) is even", then what is P (3)?

Solution:

Given:

P (n) = n (n + 1) is even.

So,

P (3) = 3 (3 + 1)

= 3 (4)

= 12

Hence, P (3) = 12, P (3) is also even.

2. If P (n) is the statement "$n^3 + n$ is divisible by 3", prove that P (3) is true but P (4) is not true.

Solution:

Given:

P (n) = $n^3 + n$ is divisible by 3

We have P (n) = $n^3 + n$

So,

P (3) = $3^3 + 3$

= 27 + 3

= 30

P (3) = 30, So it is divisible by 3

Now, let's check with P (4)

P (4) = $4^3 + 4$

= 64 + 4

= 68

P (4) = 68, so it is not divisible by 3

Hence, P (3) is true and P (4) is not true.

3. If P (n) is the statement "$2^n \geq 3n$", and if P (r) is true, prove that P (r + 1) is true.

Solution:

Given:

P (n) = "$2^n \geq 3n$" and p(r) is true.

We have, P (n) = $2^n \geq 3n$

Since, P (r) is true

So,

$2^r \geq 3r$

Now, let's multiply both sides by 2

$2 \times 2^r \geq 3r \times 2$

$2^{r+1} \geq 6r$

$2^{r+1} \geq 3r + 3r$ [since $3r > 3 = 3r + 3r \geq 3 + 3r$]

$\therefore 2^{r+1} \geq 3(r + 1)$

Hence, P (r + 1) is true.

4. If P (n) is the statement "$n^2 + n$" is even", and if P (r) is true, then P (r + 1) is true

Solution:

Given:

P (n) = $n^2 + n$ is even and P (r) is true, then $r^2 + r$ is even

Let us consider $r^2 + r = 2k$ … (i)

Now, $(r + 1)^2 + (r + 1)$

$r^2 + 1 + 2r + r + 1$

$(r^2 + r) + 2r + 2$

$2k + 2r + 2$ [from equation (i)]

$2(k + r + 1)$

2μ

$\therefore (r + 1)^2 + (r + 1)$ is Even.

Hence, P (r + 1) is true.

5. Given an example of a statement P (n) such that it is true for all n ϵ N.

Solution:

Let us consider

P (n) = 1 + 2 + 3 +--------- + n = n(n+1)/2

So,

P (n) is true for all natural numbers.

Hence, P (n) is true for all $n \in N$.

6. If P (n) is the statement "$n^2 – n + 41$ is prime", prove that P (1), P (2) and P (3) are true. Prove also that P (41) is not true.

Solution:

Given:

$P(n) = n^2 - n + 41$ is prime.

$P(n) = n^2 - n + 41$

$P(1) = 1 - 1 + 41$

$= 41$

P (1) is Prime.

Similarly,

$P(2) = 2^2 – 2 + 41$

$= 4 - 2 + 41$

$= 43$

P (2) is prime.

Similarly,

$P (3) = 3^2 – 3 + 41$

$= 9 – 3 + 41$

$= 47$

P (3) is prime

Now,

$P (41) = (41)^2 - 41 + 41$

$= 1681$

P (41) is not prime

Hence, P (1), P(2), P (3) are true but P (41) is not true.