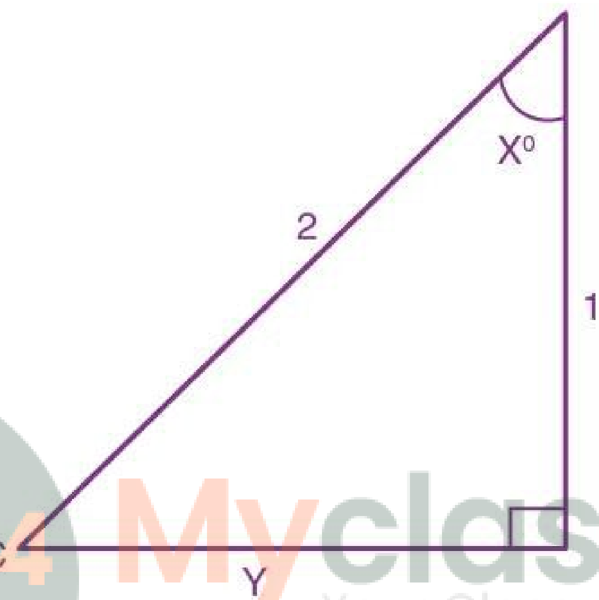
Exercise 22(B)

1. From the following figure, find:
(i) y
(ii) sin x°
(iii)
(iv) (sec x° - tan x°) (sec x° + tan x°)



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Solution:

In the given figure,
(i) As it's a right-angled triangle, so using Pythagorean Theorem
$2^2 = y^2 + 1^2$
$y^2 = 2^2 - 1^2$
$= 4 – 1$
$= 3$
Taking square root on both sides, we get
$y = \sqrt{3}$

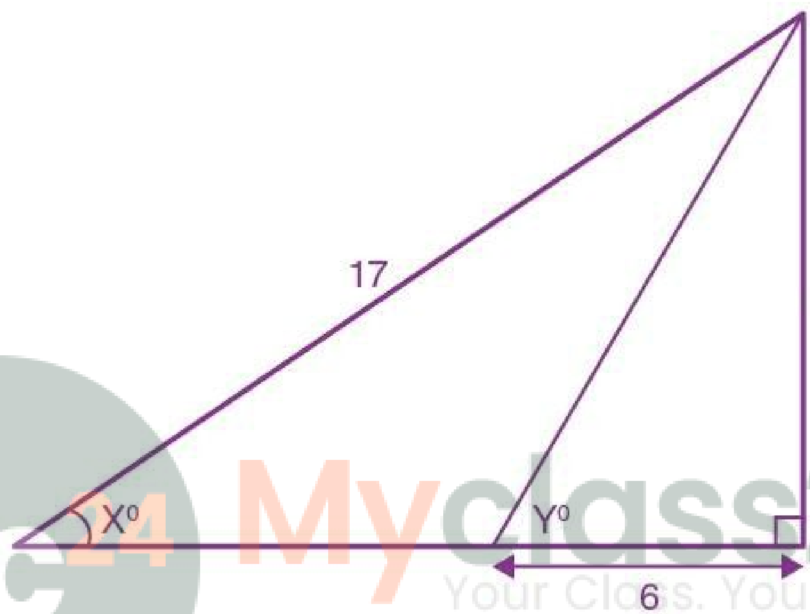
(ii) sin x° = perpendicular/hypotenuse
$= \sqrt{3}/2$

(iii) tan x° = perpendicular/base
$= \sqrt{3}$
sec x° = hypotenuse/base
$= 2$

Therefore,
(sec x° – tan x°) (sec x° + tan x°) = $(2 - \sqrt{3})(2 + \sqrt{3})$
$= 4 - \sqrt{3}$

= 1

2. Use the given figure to find:
(i) sin x°
(ii) cos y°
(iii) 3 tan x° - 2 sin y° + 4 cos y°



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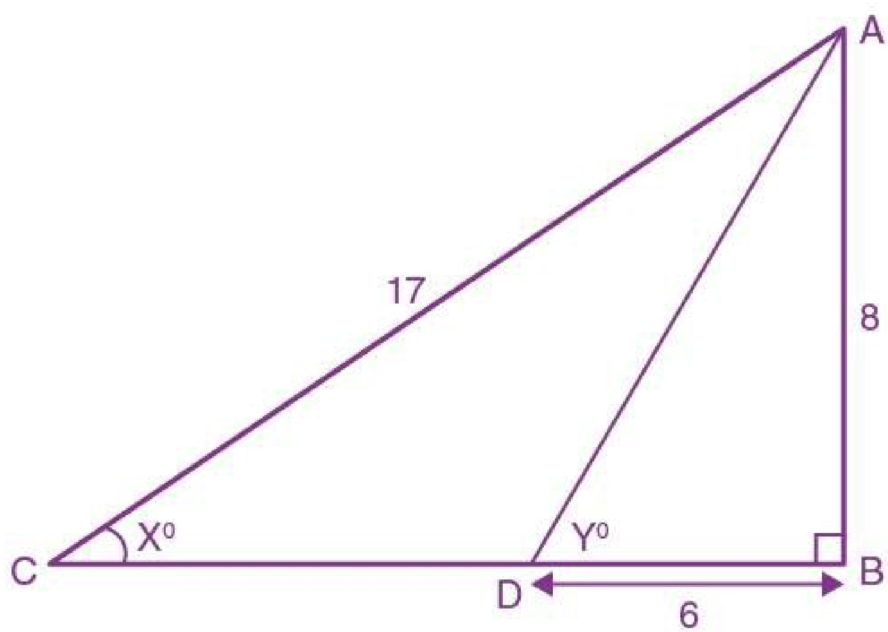
Solution:

Let's consider the given figure,



As the triangle is a right-angled triangle, so using Pythagorean Theorem

$AD^2 = 8^2 + 6^2$

$= 64 + 36$

$= 100$

Taking square root on both sides, we get

AD = 10

Also, by Pythagorean Theorem

$BC^2 = AC^2 – AB^2$

$= 17^2 - 8^2$

$= 289 – 64$

$= 225$

Taking square root on both sides, we get

BC = 15

Now,

(i) sin x^o = perpendicular/hypotenuse

= 8/17

(ii) cos y^o = base/hypotenuse

= 6/10

= 3/5

(iii) sin y^o = perpendicular/base

= AB/AD

= 8/10

= 4/5

And,

cos y^o = 6/10

= 3/5

So,

tan x^o = perpendicular/base

= AB/BC

= 8/15

Therefore,

3 tan x^o – 2 sin y^o + 4 cos y^o

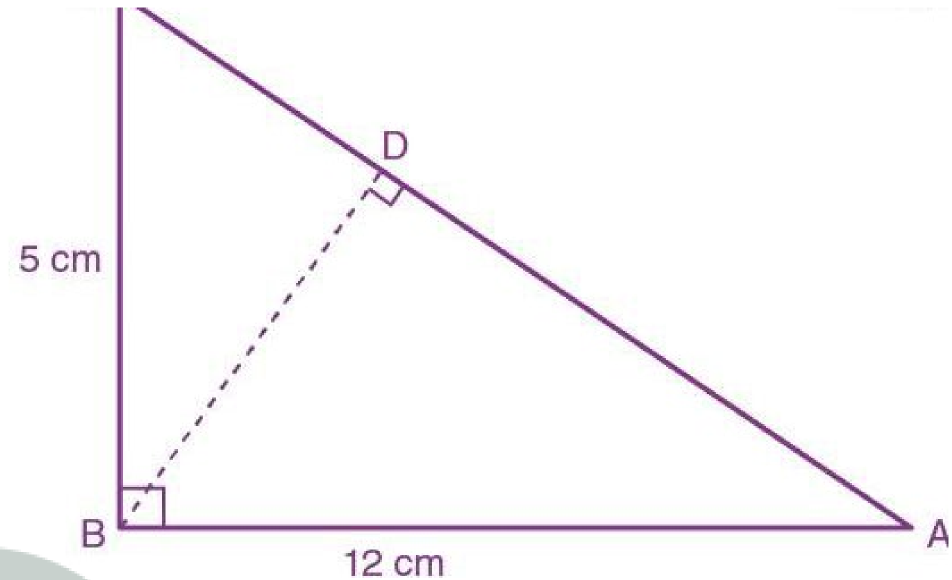
= 3(8/15) – 2(4/5) + 4(3/5)

= 8/5 – 8/5 + 12/5

= 12/5

3. In the diagram, given below, triangle ABC is right-angled at B and BD is perpendicular to AC. Find:

(i) cos ∠DBC (ii) cot ∠DBA



Solution:

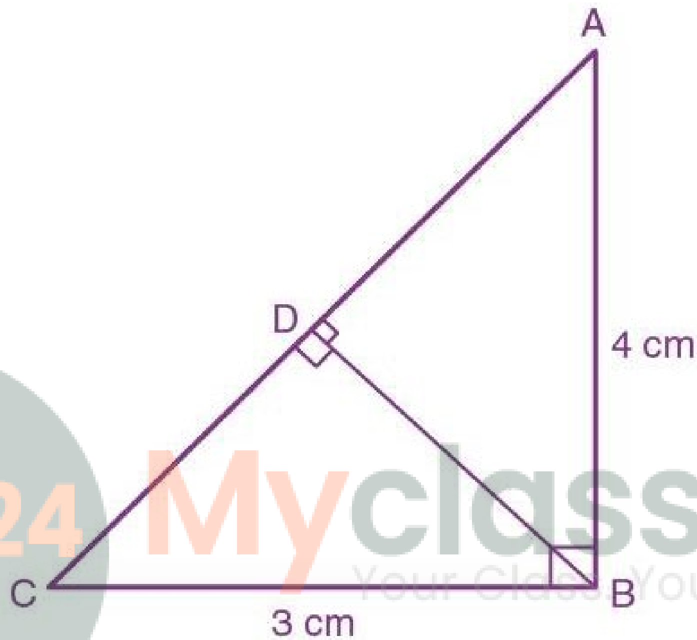
Let's consider the given figure,
As the triangle is a right-angled triangle, so using Pythagorean Theorem
$AC^2 = 5^2 + 12^2$
$= 25 + 144$
$= 169$
Taking square root on both sides, we get
AC = 13
In ΔCBD and ΔCBA,
$\angle$C is common to both the triangles
$\angle CDB = \angle CBA = 90^o$
Hence, $\angle CBD = \angle CAB$
Thus, ΔCBD and ΔCBA are similar triangles according to AAA criterion
So, we have
AC/BC = AB/BD
13/5 = 12/BD
BD = 60/13

Now,
(i) cos $\angle$DBC = base/hypotenuse
= BD/BC
= (60/13)/5
= 12/13

(ii) cot $\angle$DBA = base/perpendicular
= BD/AB

= (60/13)/12
= 5/13

4. In the given figure, triangle ABC is right-angled at B. D is the foot of the perpendicular from B to AC. Given that BC = 3 cm and AB = 4 cm. Find:
(i) tan ∠DBC
(ii) sin ∠DBA



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Solution:

Considering the given figure, we have
A right-angled triangle ABC, so by using Pythagorean Theorem we have
$AC^2 = BC^2 + AB^2$
$= 4^2 + 3^2$
= 16 + 9
= 25
Taking square root on both sides, we get
AC = 5

In ΔCBD and ΔCAB, we have
∠BCD = ∠ACB (Common)
∠CDB = ∠CBA = 90°
Hence, ΔCBD ~ ΔCAB by AA similarity criterion
So,
AC/BC = AB/BD
5/3 = 4/BD
BD = 12/5

Now, using Pythagorean Theorem in ∆BDC
$DC^2 = BC^2 – BD^2$
$= 3^2 – (12/5)^2$
$= 9 – 144/25$
$= (225 – 144)/25$
$= 81/25$
Taking square root on both sides, we get
DC = 9/5
Therefore,
AD = AC – DC
= 5 – 9/5
= 16/5

Now,
(i) tan ∠DBC = perpendicular/ base
= DC/BD
= (9/5)/(12/5)
= 3/4

(ii) sin ∠DBA = AD/AB
= (16/5)/4
= 4/5

5. In triangle ABC, AB = AC = 15 cm and BC = 18 cm, find cos ∠ABC.
Solution:

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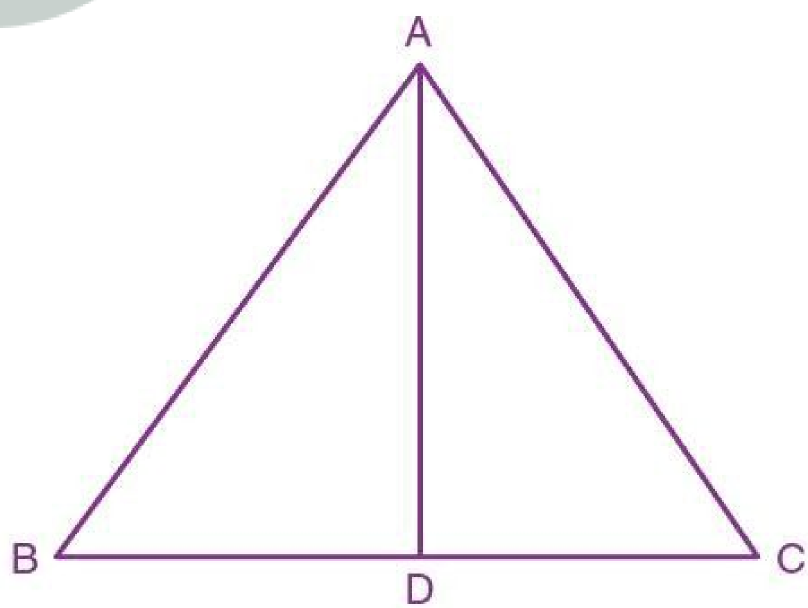
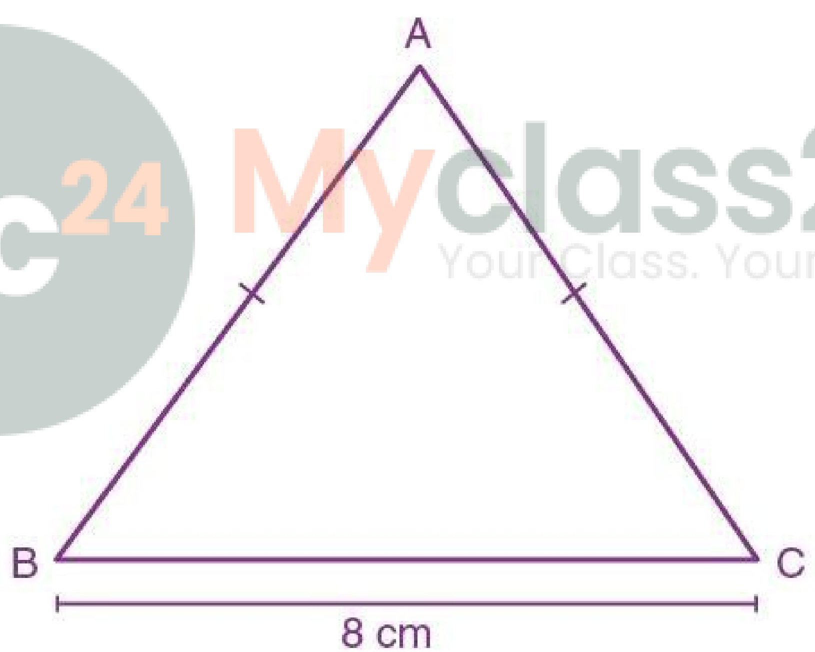
Let's consider the figure below:



In the isosceles ∆ABC, we have

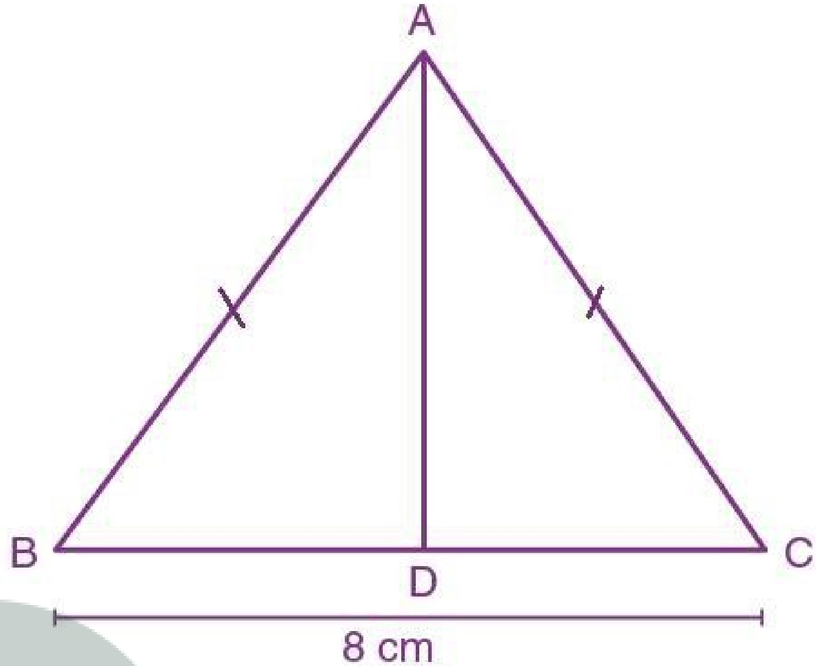
AB = AC = 15 cm
BC = 18 cm
Now, the perpendicular drawn from angle A to its opposite BC divides its into two equal parts
i.e., BD = DC = 9cm
Hence,
cos ∠ABC = base/hypotenuse
= BD/AB
= 9/15
= 3/5

6. In the figure given below, ABC is an isosceles triangle with BC = 8 cm and AB = AC = 5 cm. Find:
(i) sin B
(ii) tan C
(iii) $\sin^2 B + \cos^2 B$
(iv)
(v) tan C - cot B



Solution:

Let's consider the figure below:



In the isosceles ΔABC, we have
AB = AC = 5 cm
BC = 8 cm
Now, the perpendicular drawn from angle A to its opposite BC divides its into two equal parts
i.e., BD = DC = 4cm

As, $\angle ADB = 90^o$ in ΔABD, we have
$AB^2 = AD^2 + BD^2$
$AD^2 = AB^2 - BD^2$
$= 5^2 - 4^2$
$= 25 - 16$
$= 9$
Taking square root on both sides, we get
AD = 3

Now,
(i) sin B = AD/AB
= 3/5

(ii) tan C = AD/DC
= 3/4

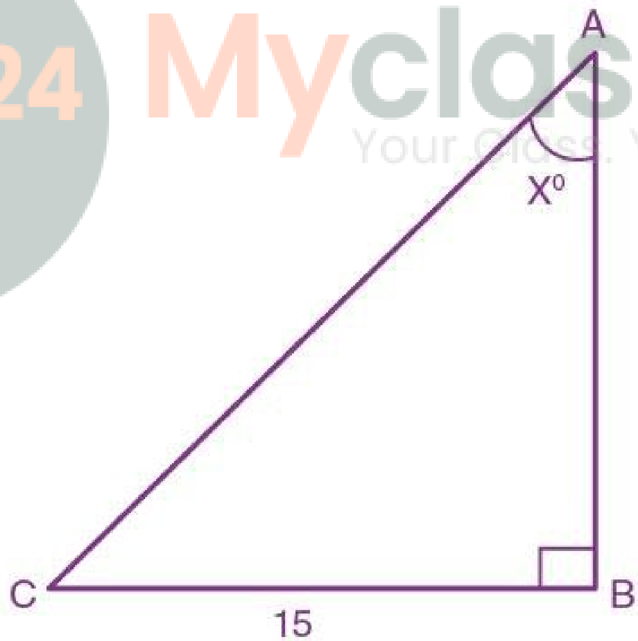
(iii) sin B = AD/AB
= 3/5
cos B = BD/AB
= 4/5
Hence,

$\sin^2 B + \cos^2 B = (3/5)^2 + (4/5)^2$
$= 9/25 + 16/25$
$= 25/25$
$= 1$

(iv) tan C = AD/DC
= ¾
cot B = BD/AD
= 4/3
Hence,
tan C – cot B = ¾ - 4/3
= (9 - 16)/12
= -7/12

7. In triangle ABC; ∠ABC = 90°, ∠CAB = x°, tan x° = ¾ and BC = 15 cm. Find the measures of AB and AC.
Solution:

Let's consider the figure below:



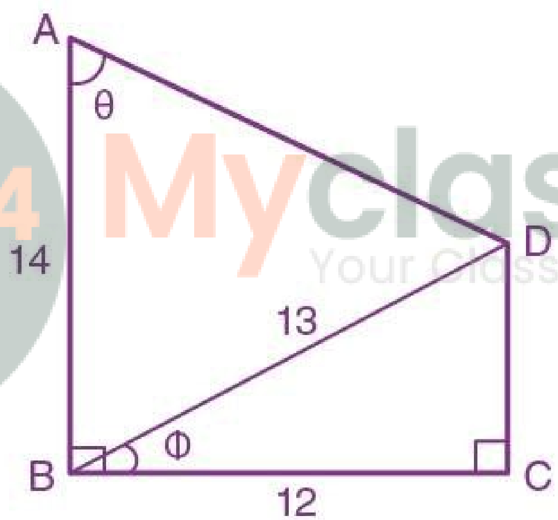
Given, tan x° = ¾
⇒ perpendicular/base = ¾
BC/AB = ¾
Hence,
If length of base AB = 4x, the length of perpendicular BC = 3x
So, by Pythagoras Theorem
$BC^2 + AB^2 = AC^2$
$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2$
$= 25x^2$
Taking square root on both sides, we get
AC = 5x, which is the hypotenuse

Now, we have
BC = 15
⇒ 3x = 15
x = 5

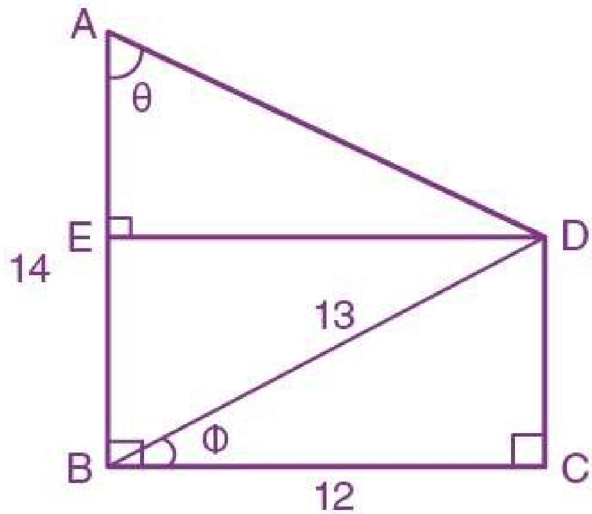
Therefore, AB = 4x = 4(5) = 20 cm
And, AC = 5x = 5 × 5 = 25 cm

8. Using the measurements given in the following figure:
(i) Find the value of sin Ø and tan θ
(ii)
(iii)Write an expression for AD in terms of θ



Solution:

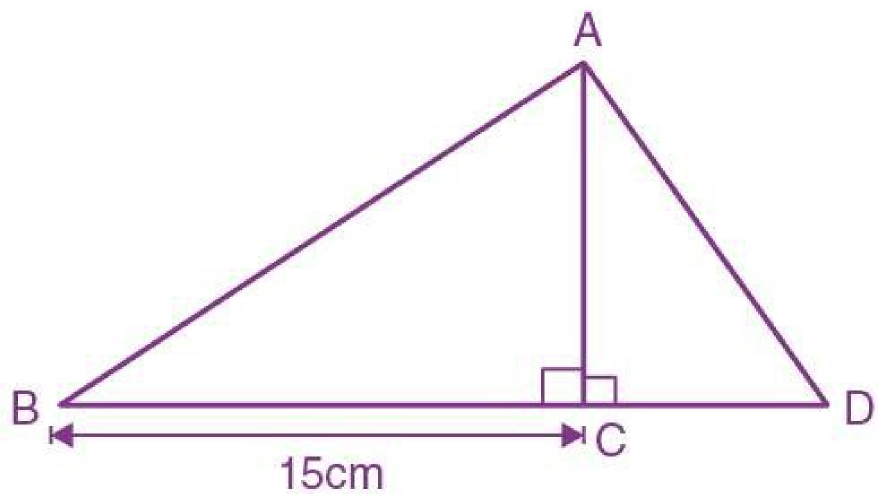
Let's consider the figure below:



Constructing a perpendicular from D to the side AB at point E which makes BCDE a rectangle.
Now, in right angled ΔBCD using Pythagorean Theorem, we have
$BD^2 = BC^2 + CD^2$ [As AB is the hypotenuse]
$CD^2 = BD^2 - BC^2$
$CD^2 = 13^2 - 12^2$
$= 169 – 144$
$= 25$
Taking square root on both sides, we get
CD = 5
As BCDE is a rectangle,
ED = 12 cm, EB = 5 cm and AE = (14 – 5) cm = 9 cm
Now,
(i) sin Ø = CD/BD
= 5/13
tan θ = ED/AE
= 12/9
= 4/3

(ii) sec θ = AD/AE
= AD/9
AD = 9 sec θ
Or
cosec θ = AD/ED
= AD/12
AD = 12cosec θ

9. In the given figure:
BC = 15 cm and sin B = 4/5



(i) Calculate the measure of AB and AC.
(ii) Now, if tan ∠ADC = 1; calculate the measures of CD and AD.
Also, show that: $\tan^2 B - 1/\cos^2 B = -1$
Solution:

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Given, BC = 15 cm and sin B = 4/5
⇒ Perpendicular/hypotenuse = AC/AB
= 4/5
Hence, if the length of perpendicular is 4x, the length of hypotenuse will be 5x
In right triangle ABC, we have
$BC^2 + AC^2 = AB^2$ [By Pythagoras Theorem]
$BC^2 = AB^2 - AC^2$
$= (5x)^2 - (4x)^2$
$= 25x^2 - 16x^2$
$= 9x^2$
Taking square root on both sides, we get
BC = 3x
Now, as BC = 15 (given)
3x = 15
x = 15/3
x = 5

(i) AC = 4x
= 4(5)
= 20 cm
And,
AB = 5x
= 5(5)
= 25 cm

(ii) Given,

tan ∠ADC = 1
perpendicular/base = AC/CD
= 1/1
Hence,
If length of perpendicular is x, then the length of hypotenuse will be x
And, we have
$AC^2 + CD^2 = AD^2$ [Using Pythagoras Theorem]
$x^2 + x^2 = AD^2$
$AD^2 = 2x^2$
Taking square root on both sides, we get
$AD = \sqrt{2}x$

Now,
AC = 20 ⇒x = 20
So,
$AD = \sqrt{2}x = \sqrt{2}(20) = 20\sqrt{2}$ cm
And,
CD = 20 cm
Hence,
tan B = AC/BC
= 20/15
= 4/3
cos B = BC/AB
= 15/25
= 3/5
Thus,
$\tan^2 B – 1/\cos^2 B = (4/3)^2 – 1/(3/5)^2$
$= 16/9 – 1/(9/25)$
$= 16/9 – 25/9$
$= - 9/9$
$= -1$

10. If sin A + cosec A = 2;
Find the value of $\sin^2 A + \text{cosec}^2 A$.
Solution:

Given, sin A + cosec A = 2
On squaring on both sides, we have
$(\sin A + \text{cosec } A)^2 = 2^2$
$\sin^2 A + \text{cosec}^2 A + 2\sin A. \text{cosec } A = 4$
$\sin^2 A + \text{cosec}^2 A + 2 = 4$ [As sin A. cosec A = sin A x 1/sin A = 1]
$\sin^2 A + \text{cosec}^2 A = 4 – 2 = 2$
Hence, the value of $(\sin^2 A + \text{cosec}^2 A)$ is 2