NCERT Solutions for Class 9 Maths Chapter 8 Quadrilaterals Ex 8.2

Question 1.

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal. Show that :

(i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

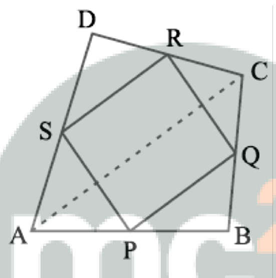

Fig. 8.29

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Solution.

(i) In ΔADC, S and R are the mid-points of sides AD and CD respectively

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it

SR || AC and SR = $\frac{1}{2}$ AC (1)

(ii) In ΔABC, P and Q are mid-points of sides AB and BC respectively.

Therefore, by using mid-point theorem

PQ || AC and PQ = $\frac{1}{2}$ AC (2)

Using equations (1) and (2), we obtain

PQ || SR and PQ = SR (3)

PQ = SR

(iii) From equation (3), we obtained

PQ || SR and

PQ = SR

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Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal

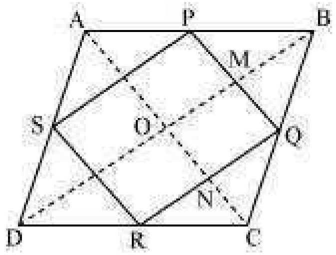
Hence, PQRS is a parallelogram

Question 2.

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Solution.

In ΔABC, P and Q are the mid-points of sides AB and BC respectively



AC (Using mid-point theorem) (1)

R and S are the mid-points of CD and AD respectively

RS || AC and RS = $\frac{1}{2}$ AC (Using mid-point theorem)

∠ PQ || AC and PQ = $\frac{1}{2}$ AC (2)

In ΔADC,

From equations (1) and (2), we obtain

PQ || RS and PQ = RS

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other it is a parallelogram

Let the diagonals of rhombus ABCD intersect each other at point O

In quadrilateral OMQN,

MQ || ON (PQ || AC)

QN|| OM (QR || BD)

Therefore,

OMQN is a parallelogram

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∠MQN = ∠NOM

∠PQR = ∠NOM

However, NOM = 90^0 (Diagonals of a rhombus are perpendicular to each other)

PQR = 90^0

Clearly, PQRS is a parallelogram having one of its interior angles as 90^0

Hence, PQRS is a rectangle

Question 3.

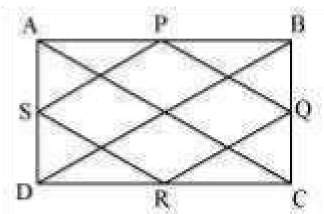
ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Solution.

Let us join AC and BD.

In ΔABC,

P and Q are the mid-points of AB and BC respectively



PQ || AC and PQ = $\frac{1}{2}$ AC (Mid-point theorem) (1)

Similarly in ΔADC,

SR || AC and SR = $\frac{1}{2}$ AC (Mid-point theorem) (2)

Clearly,

PQ || SR and PQ = SR

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram

PS || QR and PS = QR (Opposite sides of parallelogram) (3)

In ΔBCD, Q and R are the mid-points of side BC and CD respectively.

QR || BD and QR = $\frac{1}{2}$ BD (Mid-point theorem) (4)

However, the diagonals of a rectangle are equal.

AC = BD (5)

By using equation (1), (2), (3), (4), and (5), we obtain

PQ = QR = SR = PS

Therefore, PQRS is a rhombus

Question 4.

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.30). Show that F is the mid-point of BC.

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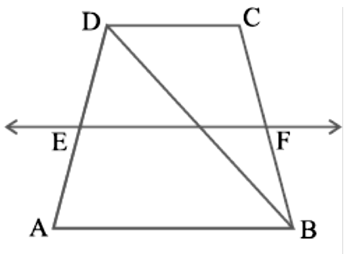
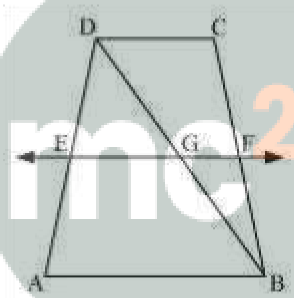


Fig. 8.30

Solution.

Let EF intersect DB at G

By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side



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In ΔABD,

EF || AB and E is the mid-point of AD

Therefore,

G will be the mid-point of DB

EF || AB and

AB || CD

EF || CD (Two lines parallel to the same line are parallel to each other)

In ΔBCD,

GF || CD and

G is the mid-point of line BD. Therefore, by using converse of mid-point theorem, F is the mid-point of BC

Question 5.

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.31). Show that the line segments AF and EC trisect the diagonal BD.

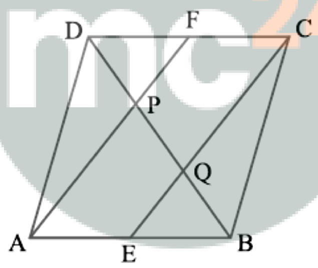


Fig. 8.31

Solution.

ABCD is a parallelogram.

AB || CD

And

AE || FC

Again,

AB = CD (Opposite sides of parallelogram ABCD)

$\frac{1}{2}$ AB = $\frac{1}{2}$ CD

AE = FC (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides is parallel and equal to each other

Therefore, AECF is a parallelogram

AF || EC (Opposite sides of a parallelogram)

In ΔDQC,

F is the mid-point of side DC and FP || CQ

Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ

DP = PQ (1)

AB = CD

Similarly,

In ΔAPB,

E is the mid-point of side AB and EQ || AP

Therefore, by using the converse of mid-point theorem, it can be said that Q is the mid-point of PB

PQ = QB (2)

From equations (1) and (2),

DP = PQ = BQ

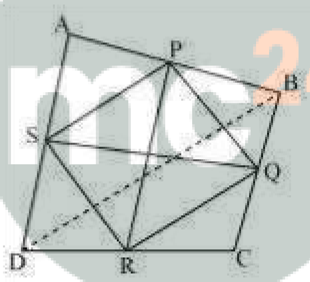
Hence, the line segments AF and EC trisect the diagonal BD

Question 6.

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution.

Let ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively. Join PQ, QR, RS, SP, and BD



In ΔABD, S and P are the mid-points of AD and AB respectively. Therefore, by using mid-point theorem:

SP || BD and SP = $\frac{1}{2}$ BD (1)

Similarly in ΔBCD,

QR || BD and QR = $\frac{1}{2}$ BD (2)

From equations (1) and (2), we obtain

SP || QR and SP = QR

In quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other

Therefore, SPQR is a parallelogram.

We know that diagonals of a parallelogram bisect each other

Hence, PR and QS bisect each other

Question 7.

ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

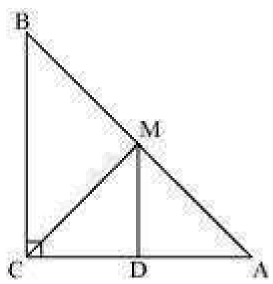
(i) D is the mid-point of AC

(ii) MD ⊥ AC

(iii) CM = MA = $\frac{1}{2}$AB

Solution.

(i) In ΔABC,



Given that: M is the mid-point of AB and

MD || BC

Therefore,

D is the mid-point of AC (By converse of mid-point theorem)

(ii) As DM || CB and AC is a transversal line for them

Therefore,

$\angle MDC + \angle DCB = 180^0$ (Co-interior angles)

$\angle MDC + 90^0 = 180^0$

$\angle MDC = 90^0$

MD is perpendicular to AC

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(iii) Join MC

In ΔAMD and ΔCMD,

AD = CD (D is the mid-point of side AC)

$\angle ADM = \angle CDM$ (Each 90^0)

DM = DM (Common)

ΔAMD ≅ ΔCMD (By SAS congruence rule)

Therefore,

AM = CM (By CPCT)

However,

AM = $\frac{1}{2}$ AB (M is the mid-point of AB)

Therefore,

CM = AM = $\frac{1}{2}$ AB

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