Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

Therefore,

$^{62}P_{3} = \frac{62!}{(62-3)!}$

$^{62}P_{3} = 62 \times 61 \times 60 \times 59 = 226920$

Thus, the value of $^{62}P_{3}$ is 226920.

Q. 1. C. Evaluate:

$^{6}P_{6}$

Answer : To find: the value of $^{6}P_{6}$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

Therefore,

$^{6}P_{6} = \frac{6!}{(6-6)!}$

$^{62}P_{3} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Thus, the value of $^{6}P_{6}$ is 720.

Q. 1. D. Evaluate:

$^{9}P_{0}$

Answer : To find: the value of $^{9}P_{0}$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

Therefore,

$^9P_0 = \frac{9!}{(9-0)!}$

$^9P_0 = 1$

Thus, the value of $^9P_0$ is 1.

Q. 2. Prove that $^9P_3 + 3 \times {}^9P_2 = {}^{10}P_3$.

Answer : To Prove: $^9P_3 + 3 \times {}^9P_2 = {}^{10}P_3$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

The equation given below needs to be proved i.e

$^9P_3 + 3 \times {}^9P_2 = {}^{10}P_3$.

$\frac{9!}{(9-3)!} + \left(3 \times \frac{9!}{(9-2)!}\right) = \frac{10!}{(10-3)!}$

$(9 \times 8 \times 7) + (3 \times 9 \times 8) = 10 \times 9 \times 8$

$10 \times 9 \times 8 = 10 \times 9 \times 8$

Hence, proved.

$^9P_3 + 3 \times {}^9P_2 = {}^{10}P_3$.

Q. 3. (i) If $^nP_5 = 20 \times {}^nP_3$, find n.

(ii) If 16 × $^nP_3$ = 13 × $^{n+1}P_3$, find n.
(iii) If $^{2n}P_3$ = 100 × $^nP_2$, find n.

Answer : (i) To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$$^nP_r = \frac{n!}{(n-r)!}$$

$^nP_5$ = 20 × $^nP_3$.

$$\frac{n!}{(n-5)!} = \left(20 \times \frac{n!}{(n-3)!}\right)$$

$$\frac{1}{(n-5)!} = \left(20 \times \frac{1}{(n-3)(n-4)(n-5)!}\right)$$

$$1 = \left(20 \times \frac{1}{(n-3)(n-4)}\right)$$

20 = (n - 3)(n - 4)

$$n^2 - 7n + 12 = 20$$

$$n^2 - 7n - 8 = 0$$

(n - 8)(n + 1) = 0

n = 8, -1

We know, that n cannot be a negative number.

Hence, value of n is 8

(ii) To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$${}^nP_r = \frac{n!}{(n-r)!}$$

$16 \times {}^nP_3 = 13 \times {}^{n+1}P_3$.

$$16 \times \frac{n!}{(n-3)!} = \left(13 \times \frac{(n+1)!}{(n-2)!}\right)$$

$$16 \times \frac{n!}{(n-3)!} = \left(13 \times \frac{(n+1)n!}{(n-2)(n-3)!}\right)$$

$$16 = 13 \times \frac{(n+1)}{(n-2)}$$

16n – 32 = 13n + 13

3n = 45

n = 15

Hence, value of n is 15.

(iii) To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$${}^nP_r = \frac{n!}{(n-r)!}$$

${}^{2n}P_3 = 100 \times {}^nP_2$

$$\frac{2n!}{(2n-3)!} = \left(100 \times \frac{n!}{(n-2)!}\right)$$

$$\frac{2n(2n-1)(2n-2)(2n-3)!}{(2n-3)!} = \left(100 \times \frac{n(n-1)(n-2)!}{(n-2)!}\right)$$

$$\frac{2n(2n-1)(2n-2)(2n-3)!}{(2n-3)!} = \left(100 \times \frac{n(n-1)(n-2)!}{(n-2)!}\right)$$

2n(2n - 1)(2n - 2) = 100 × n(n - 1)

4n(2n - 1)(n - 1) = 100 × n(n-1)

$8n^2 - 4n - 100n = 0$

$8n^2 - 104n = 0$

8n(n - 13) = 0

n = 0, 13

We know that n should be greater than zero.

Hence, value of n is 13

Q. 4. (i) If If $^5P_r = 2 \times {}^6P_{r-1}$, find r.

(ii) If $^{20}P_r = 13 \times {}^{20}P_{r-1}$, find r.
(iii) If $^{11}P_r = {}^{12}P_{r-1}$, find r.

Answer : (i) To find: the value of r

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

$^5P_r = 2 \times {}^6P_{r-1}$

$$\frac{5!}{(5-r)!} = \left(2 \times \frac{6!}{(7-r)!}\right)$$

$$\frac{5!}{(5-r)!} = \left(2 \times \frac{6 \times 5!}{(7-r)(6-r)(5-r)!}\right)$$

$$1 = \left(\frac{12}{(7-r)(6-r)}\right)$$

$$r^2 - 13r + 30 = 0$$

r = 10, 3

Hence, value of r is 3, 10

(ii) To find: the value of r

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

$^{20}P_r = 13 \times {}^{20}P_{r-1}$

$$\frac{20!}{(20-r)!} = \left(13 \times \frac{20!}{(21-r)!}\right)$$

$$\frac{1}{(20-r)!} = \left(13 \times \frac{1}{(21-r)(20-r)!}\right)$$

21 – r = 13

r = 8

Hence, value of r is 8.

(iii) To find: the value of r

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

$^{11}P_{r} = {}^{12}P_{r-1}$

$$\frac{11!}{(11-r)!} = \left(\frac{12!}{(13-r)!}\right)$$

$$\frac{11!}{(11-r)!} = \left(\frac{12 \times 11!}{(13-r)(12-r)(11-r)!}\right)$$

$$1 = \frac{12}{(13-r)(12-r)}$$

$$r^2 - 25r + 144 = 0$$

(r - 16)(r - 9) = 0

r = 16, 9

Since r cannot be 16 as it creates a negative factorial in denominator. Therefore, r = 16 is not possible.

Hence, value of r is 9.

Q. 5. (i) If $^{n}P_{4}$: $^{n}P_{5}$, = 1 : 2, find n.

(ii) If $^{n-1}P_{3}$: $^{n+1}P_{3}$, = 5 : 12, find n.

Answer : To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

$^{n}P_{4} : {}^{n}P_{5} = 1:2$

$$\frac{n!}{(n-4)!} : \frac{n!}{(n-5)!} = \frac{1}{2}$$

$$\frac{n!}{(n-4)(n-5)!} : \frac{n!}{(n-5)!} = \frac{1}{2}$$

$$\frac{n!}{(n-4)(n-5)!} \times \frac{(n-5)!}{n!} = \frac{1}{2}$$

$$\frac{1}{(n-4)} = \frac{1}{2}$$

n – 4 = 2

n = 6

Hence, value of n is 6.

(ii) To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

$^{n-1}P_3 : {}^{n+1}P_3$, = 5 : 12

$$\frac{(n-1)!}{(n-4)!} : \frac{(n+1)!}{(n-2)!} = \frac{5}{12}$$

$$\frac{(n-1)!}{(n-4)!} : \frac{(n+1)n(n-1)!}{(n-2)(n-3)(n-4)!} = \frac{5}{12}$$

$$\frac{(n-1)!}{(n-4)!} \times \frac{(n-2)(n-3)(n-4)!}{(n+1)n(n-1)!} = \frac{5}{12}$$

$$\frac{(n-2)(n-3)}{(n+1)n} = \frac{5}{12}$$

$$\frac{n^2 - 5n + 6}{n^2 + n} = \frac{5}{12}$$

$12n^2 - 60n + 72 = 5n^2 + 5n$

$7n^2 - 65n + 72 = 0$

n = 8, 2.25

Since n cannot be 2.25 as it creates a negative factorial in denominator. Therefore, n = 2.25 is not possible.

Hence, value of n is 8.

Q. 6. If $^{15}P_{r-1} : {}^{16}P_{r-2}$, = 3 : 4, find r.

Answer : To find: the value of r

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

$^{15}P_{r-1} : {}^{16}P_{r-2}$, = 3 : 4

$$\frac{15!}{(16-r)!} : \frac{16!}{(18-r)!} = \frac{3}{4}$$

$$\frac{15!}{(16-r)!} : \frac{16 \times 15!}{(18-r)(17-r)(16-r)!} = \frac{3}{4}$$

$$\frac{15!}{(16-r)!} \times \frac{(18-r)(17-r)(16-r)!}{16 \times 15!} = \frac{3}{4}$$

$$\frac{(18-r)(17-r)}{4} = 3$$

$$\frac{(18-r)(17-r)}{16} = \frac{3}{4}$$

$$r^2 - 35r + 306 = 12$$

$r^2 - 35r + 294 = 0$

$(r - 21)(r - 14) = 0$

$r = 21,14$

Since r cannot be 21 as it creates a negative factorial in denominator. Therefore, $r = 14$ is not possible.

Hence, value of r is 14

Q. 7. If $^{2n-1}P_n : {}^{2n+1}P_{n-1}$, = 22 : 7, find n.

Answer : To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

$^{2n-1}P_n : {}^{2n+1}P_{n-1}$, = 22 : 7

$$\frac{(2n-1)!}{(n-1)!} : \frac{(2n+1)!}{(n+2)!} = \frac{22}{7}$$

$$\frac{(2n-1)!}{(n-1)!} : \frac{(2n+1)(2n)(2n-1)!}{(n+2)(n+1)n(n-1)!} = \frac{22}{7}$$

$$\frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)(n+1)n(n-1)!}{(2n+1)(2n)(2n-1)!} = \frac{22}{7}$$

$$\frac{(n+2)(n+1)}{(2n+1)2} = \frac{22}{7}$$

$$\frac{n^2 + 3n + 2}{2n + 1} = \frac{44}{7}$$

$7n^2 + 21n + 14 = 88n + 44$

$7n^2 - 67n - 30 = 0$

n = 10, -0.42

Since n cannot be -0.42

Hence, value of n is 10.

Q. 8. Find n, If $^{n+5}P_{n+1} = \frac{11}{2}(n-1).\ ^{n+3}P_n$, find n.

Answer : To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_r = \frac{n!}{(n-r)!}$

$^{n+5}P_{n+1} = \frac{11}{2}(n-1).\ ^{n+3}P_n$

$$\frac{(n+5)!}{4!} = \frac{11}{2}(n-1)\frac{(n+3)!}{3!}$$

$$\frac{(n+5)(n+4)(n+3)!}{4 \times 3!} = \frac{11}{2}(n-1)\frac{(n+3)!}{3!}$$

$$\frac{(n+5)(n+4)}{2} = 11(n-1)$$

$$n^2 + 9n + 20 = 22n - 22$$

$$n^2 - 13n + 42 = 0$$

(n - 7)(n - 6) = 0

n = 7, 6

Hence, values of n are 7 & 6

Q. 9. Prove that $1 + 1.\ {}^{1}P_{1} + 2.\ {}^{2}P_{2} + 3.\ {}^{3}P_{3} + \ldots.\ n.\ {}^{n}P_{n} = {}^{n+1}P_{n+1}$.

Answer : To Prove: $1 + 1.\ {}^{1}P_{1} + 2.\ {}^{2}P_{2} + 3.\ {}^{3}P_{3} + \ldots.\ n.\ {}^{n}P_{n} = {}^{n+1}P_{n+1}$.

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$1 + 1.\ {}^{1}P_{1} + 2.\ {}^{2}P_{2} + 3.\ {}^{3}P_{3} + \ldots.\ n.\ {}^{n}P_{n} = {}^{n+1}P_{n+1}$.

$1 + (2! - 1!) + (3! - 2!) + (4! - 3!) + \ldots\ldots.((n + 1)! - n!) = (n + 1)!$

$1 + ((n + 1)! - 1!) = (n + 1)!$

$(n + 1)! = (n + 1)!$

Hence proved.

Q. 10. Find the number of permutations of 10 objects, taken 4 at a time.

Answer : To find: the number of permutations of 10 objects, taken 4 at a time.

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$${}^{10}P_{4} = \frac{10!}{6!}$$

$${}^{10}P_{4} = 10 \times 9 \times 8 \times 7$$

$${}^{10}P_{4} = 5040$$

Hence, the number of permutations of 10 objects, taken 4 at a time is 5040.

Exercise 8D

Q. 1. In how many ways can 5 persons occupy 3 vacant seats?

Answer : To find: number of arrangements of 5 people in 3 seats.

Consider three seats A B C

Now, place A can be occupied by any 1 person out of 5.

Then place B can be occupied by any 1 person from remaining 4 and for C there are 3 people to occupy the seat.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 5 different objects in 3 places is

P(5,3) = $\frac{5!}{(5-3)!}$

= $\frac{5!}{2!}$ = $\frac{120}{2}$ = 60.

Therefore, the number of possible solutions is 60.

Q. 2. In how many ways can 7 people line up at a ticket window of a cinema hall?

Answer : To find: number of arrangements of 7 people in a queue.

Here there are 7 spaces to be occupied by 7 people.

Therefore 7 people can occupy first place.

Similarly, 6 people can occupy second place and so on.

Lastly, there will be a single person to occupy the 7 positions.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 7 different objects in 7 places is

P(7,7) = $\frac{7!}{(7-7)!}$

= $\frac{7!}{0!}$ = $\frac{5040}{1}$ = 5040.

Therefore, the number of possible ways is 5040

Q. 3. In how many ways can 5 children stand in a queue?

Answer : To find: number of arrangements of 5 children in a queue.

Here, 5 places are needed to be occupied by 5 children.

Therefore any one of the 5 children can occupy first place.

Similarly, any 4 children can occupy second place and so on.

Lastly, there will be a single person to occupy the 5 position

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 5 different objects in 5 places is

P(5,5) = $\frac{5!}{(5-5)!}$

= $\frac{5!}{0!}$ = $\frac{120}{1}$ = 120.

Hence, this can be done in 120 ways.

Q. 4. In how many ways can 6 women draw water from 6 wells if no well remains unused?

Answer : To find: number of arrangements of 6 women drawing water from 6 wells

Here, 6 wells are needed to be used by 6 women.

Therefore any one of the 6 women can draw water from the 1 well.

Similarly, any 5 women can draw water from the 2nd well and so on.

Lastly, there will be single women left to draw water from the 6th well.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 6 different objects in 6 places is

P(6,6) = $\frac{6!}{(6-6)!}$

= $\frac{6!}{0!}$ = $\frac{720}{1}$ = 720.

Hence, this can be done in 720 ways.

Q. 5. In how many ways can 4 different books, one each in chemistry, physics, biology and mathematics, be arranged on a shelf?

Answer : To find: number of arrangements of 4 different books in a shelf.

There are 4 different books.

Any one of the four different books can be placed on the shelf first.

Similarly, in the next position, 1 book out of 3 can be placed.

Finally, the last book will have a single place to fit.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 4 different objects in 4 places is