$\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{4}$

Parallel vector to the line is

$\bar{b} = 6\hat{\imath} + \lambda\hat{\jmath} + 4\hat{k}$

For given equation of plane,

3x - y - 2z = 7

normal vector to the plane is

$\bar{n} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

As given line and plane are perpendicular to each other.

$\therefore \bar{b} \times \bar{n} = 0$

$\therefore \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & \lambda & 4 \\ 3 & -1 & -2 \end{vmatrix} = 0$

$\therefore \hat{\imath}(-2\lambda + 4) - \hat{\jmath}(-12 - 12) + \hat{k}(-6 - 3\lambda) = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

Comparing coefficients of $\hat{k}$ on both sides

$\therefore -6 - 3\lambda = 0$

$3\lambda = -6$

$\lambda = -2$

19. Question

Write the equation of the plane passing through the point (a, b, c) and parallel to the plane $\bar{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 2$.

Answer

Given :

A ≡ (a, b, c)

Equation of plane parallel to required plane

$\therefore \bar{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 2$

To Find : Equation of plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

For point A ≡ (a, b, c), position vector is

$\bar{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

As plane $\bar{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 2$ is parallel to the required plane, the vector normal to required plane is

$\bar{n} = \hat{\imath} + \hat{\jmath} + \hat{k}$

Now, $\bar{a}.\bar{n} = (a \times 1) + (b \times 1) + (c \times 1)$

= a + b + c

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) = a + b + c$

20. Question

Find the length of perpendicular drawn from the origin to the plane 2x - 3y + 6z + 21 = 0.

Answer

Given :

Equation of plane : 2x – 3y + 6z + 21 = 0

To Find :

Length of perpendicular drawn from origin to the plane = d

Formulae :

1) Distance of the plane from the origin :

Distance of the plane from the origin is given by,

$$d = \frac{p}{|\bar{n}|}$$

Answer :

For the given equation of plane

2x – 3y + 6z = -21

Direction ratios of normal vector are (2, -3, 6)

Therefore, equation of normal vector is

$\bar{n} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

$\therefore |\bar{n}| = \sqrt{2^2 + (-3)^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

From given equation of plane,

p = -21

Now, distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

$$\therefore d = \frac{-21}{7}$$

d = 3 units

21. Question

Find the direction cosines of the perpendicular from the origin to the plane $\bar{r}\cdot\left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) - 1 = 0$.

Answer

Given :

Equation of plane : $\bar{r}.\left(6\hat{\imath} - 3\hat{\jmath} - 2\hat{k}\right) + 1 = 0$

To Find :

Direction cosines of the normal i.e. $l, m \,\&\, n$

Formulae :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

$\bar{r}.\left(6\hat{\imath} - 3\hat{\jmath} - 2\hat{k}\right) + 1 = 0$

Equation of normal vector is

$\bar{n} = 6\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$

$\sqrt{a^2 + b^2 + c^2} = \sqrt{6^2 + (-3)^2 + (-2)^2}$

$= \sqrt{36 + 9 + 4}$

$= \sqrt{49}$

$= 7$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{6}{7}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{7}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{-2}{7}$$

$(l, m, n) = \left(\frac{6}{7}, \frac{-3}{7}, \frac{-2}{7}\right)$

22. Question

Show that the line $\vec{r} = \left(4\hat{i} - 7\hat{k}\right) + \lambda\left(4\hat{i} - 2\hat{j} + 3\hat{k}\right)$ is parallel to the plane $\vec{r} \cdot \left(5\hat{i} + 4\hat{j} - 4\hat{k}\right) = 7$.

Answer

Given :

Equation of plane : : $\bar{r}.\left(5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}\right) = 7$

Equation of line :

$\bar{r} = \left(4\hat{\imath} - 7\hat{k}\right) + \lambda\left(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right)$

To Prove : Given line is parallel to the given plane.

Answer :

Comparing given plane i.e.

$\bar{r}.\left(5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}\right) = 7$

with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$, we get,

$\bar{n} = 5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}$

This is the vector perpendicular to the given plane.

Now, comparing given equation of line i.e.

$\bar{r} = \left(4\hat{\imath} - 7\hat{k}\right) + \lambda\left(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right)$

with $\bar{r} = \bar{a} + \lambda\bar{b}$, we get,

$\bar{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

Now,

$\bar{n}.\bar{b} = \left(5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}\right).\left(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right)$

$= (5 \times 4) + (4 \times (-2)) + ((-4) \times 3)$

$= 20 - 8 - 12$

$= 0$

$\therefore \bar{n}.\bar{b} = 0$

Therefore, vector normal to the plane is perpendicular to the vector parallel to the line.

Hence, the given line is parallel to the given plane.

23. Question

Find the length of perpendicular from the origin to the plane $\bar{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k})+14=0$.

Answer

Given :

Equation of plane : $\bar{r}.\left(2\hat{\imath}-3\hat{\jmath}+6\hat{k}\right)+14=0$

To Find : Length of perpendicular = d

Formulae :

1) Unit Vector :

Let $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$$\hat{a}=\frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}|=\sqrt{{a_1}^2+{a_2}^2+{a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from the origin to the plane

$\bar{r}.\bar{n}=p$ is given by,

$$d=\frac{p}{|\bar{n}|}$$

Answer :

Given equation of the plane is

$$\bar{r}.\left(2\hat{\imath}-3\hat{\jmath}+6\hat{k}\right)+14=0$$

$$\therefore\bar{r}.\left(2\hat{\imath}-3\hat{\jmath}+6\hat{k}\right)=-14$$

$$\therefore\bar{r}.\left(-2\hat{\imath}+3\hat{\jmath}-6\hat{k}\right)=14$$

Comparing above equation with

$$\bar{r}.\bar{n}=p$$

We get,

$\bar{n}=-2\hat{\imath}+3\hat{\jmath}-6\hat{k}$ & p = 14

Therefore,

$$|\bar{n}|=\sqrt{(-2)^2+3^2+(-6)^2}$$

$$=\sqrt{4+9+36}$$

$$=\sqrt{49}$$

$$=7$$

The length of the perpendicular from the origin to the given plane is

$$d=\frac{p}{|\bar{n}|}$$

$$\therefore d=\frac{14}{7}$$

$$\therefore d=2\ units$$

24. Question

Find the value of λ for which the line

$\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{\lambda}$ is parallel to the plane $\bar{r}.\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=4$

Answer

Given :

Equation of line : $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{\lambda}$

Equation of plane : $\bar{r}.\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=4$

To Find : λ

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$\bar{b}=a_1\hat{\imath}+b_1\hat{\jmath}+c_1\hat{k}$

2) Angle between a line and a plane :

If θ is a angle between the line $\bar{r}=\bar{a}+\lambda\bar{b}$ and the plane $\bar{r}.\bar{n}=p$, then

$$\sin\theta=\frac{\bar{b}.\bar{n}}{|\bar{b}|.|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{\lambda}$$

Parallel vector to the line is

$\bar{b}=2\hat{\imath}+3\hat{\jmath}+\lambda\hat{k}$

For given equation of plane,

$\bar{r}.\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=4$

normal vector to the plane is

$\bar{n}=2\hat{\imath}+3\hat{\jmath}+4\hat{k}$

Therefore, angle between given line and plane is

$$\sin\theta=\frac{\bar{b}.\bar{n}}{|\bar{b}|.|\bar{n}|}$$

As given line is parallel too the given plane, angle between them is 0.

$\therefore\theta=0$

$\therefore\sin\theta=0$

$\therefore\bar{b}.\bar{n}=0$

$\therefore\left(2\hat{\imath}+3\hat{\jmath}+\lambda\hat{k}\right).\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=0$

$\therefore(2\times2)+(3\times3)+(\lambda\times4)=0$

4 + 9 + 4 λ = 0

13 + 4λ = 0

4λ = -13

$$\therefore\lambda=-\frac{13}{4}$$

$$\lambda=-\frac{13}{4}$$

25. Question

Write the angle between the line

$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$ and the plane x + y + 4 = 0.

Answer

Given :

Equation of line : $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$

Equation of plane : x + y + 4 = 0

To Find : angle between line and plane

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$\bar{b} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

3) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r}.\bar{n} = p$, then

$$\sin\theta = \frac{\bar{b}.\bar{n}}{|\bar{b}|.|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$$

Parallel vector to the line is

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$

For given equation of plane,

x + y + 4 = 0

normal vector to the plane is

$\bar{n} = \hat{\imath} + \hat{\jmath} + 0\hat{k}$

$\therefore |\bar{n}| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{1+1+0} = \sqrt{2}$

Therefore, angle between given line and plane is

$$\sin\theta = \frac{\bar{b}.\bar{n}}{|\bar{b}|.|\bar{n}|}$$

$$\therefore \sin\theta = \frac{(2\hat{\imath} + \hat{\jmath} - 2\hat{k}).(\hat{\imath} + \hat{\jmath} + 0\hat{k})}{3 \times \sqrt{2}}$$

$$\therefore \sin\theta = \frac{(2 \times 1) + (1 \times 1) + ((-2) \times 0)}{3\sqrt{2}}$$

$$\therefore \sin\theta = \frac{2+1-0}{3\sqrt{2}}$$

$$\therefore \sin\theta = \frac{3}{3\sqrt{2}}$$

$$\therefore \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\theta = \frac{\pi}{4}$$

26. Question

Write the equation of a plane passing through the point (2, -1, 1) and parallel to the plane 3x + 2y - z = 7.

Answer

Given :

A ≡ (2, -1, 1)

Plane parallel to the required plane : 3x + 2y - z = 7

To Find : Equation of plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

For point A ≡ (2, -1, 1), position vector is

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

As required plane is parallel to 3x + 2y - z = 7.

Therefore, normal vector of given plane is also perpendicular to required plane

$\bar{n} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$

Now, $\bar{a}.\bar{n} = (2 \times 3) + ((-1) \times 2) + (1 \times (-1))$

= 6 - 2 - 1

= 3

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right) = 3$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right)$

= 3x + 2y - z

Therefore, equation of the plane is

3x + 2y - z = 3

3x + 2y - z - 3 = 0

Objective Questions

1. Question

Mark against the correct answer in each of the following:

The direction cosines of the perpendicular from the origin to the plane $\vec{r}\cdot\left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) + 1 = 0$ are

A. $\frac{6}{7}, \frac{3}{7}, \frac{-2}{7}$

B. $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

C. $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$

D. None of these

Answer

Given: Equation of plane is $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})+1=0$

Formula Used: Equation of a plane is $\hat{u}.\vec{r}=p$ where $\hat{u}$ is the unit vector normal to the plane, $\vec{r}$ represents a point on the plane and p is the distance of the plane from the origin.

Explanation:

The equation of the given plane is $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=-1$ ... (1)

Now, $|6\hat{i}-3\hat{j}+2\hat{k}|=\sqrt{36+9+4}$

= 7

$\therefore \frac{6}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}$ is a unit vector.

(1) can be rewritten as

$\vec{r}.\left(\frac{6}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}\right)=-\frac{1}{7}$

$\Rightarrow \vec{r}.\left(\frac{-6}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{2}{7}\hat{k}\right)=\frac{1}{7}$

which is of the form $\hat{u}.\vec{r}=p$

Perpendicular vector from the origin to the plane is

$\hat{u}=\frac{-6}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{2}{7}\hat{k}$

So, direction cosines of the vector perpendicular from the origin to the plane is $\left(\frac{-6}{7},\frac{3}{7},\frac{-2}{7}\right)$

2. Question

Mark against the correct answer in each of the following:

The direction cosines of the normal to the plane 5y + 4 = 0 are

A. $0, \frac{-4}{5}, 0$

B. 0, 1, 0

C. 0, -1, 0

D. None of these

Answer

Given: Equation of plane is 5y + 4 = 0

Formula Used: Equation of a plane is lx + my + nz = p where (l, m, n) are the direction cosines of the normal to the plane and (x, y, z) is a point on the plane and p is the distance of plane from origin.

Explanation:

Given equation is 5y = -4

Dividing by -5,

$-y=\frac{4}{5}$

which is of the form lx + my + nz = p where l = 0, m = -1, n = 0

Therefore, direction cosines of the normal to the plane is (0, -1, 0)

3. Question

Mark against the correct answer in each of the following:

The length of perpendicular from the origin to the plane $\overline{r}\cdot(3\hat{i}-4\hat{j}-12\hat{k})+39=0$ is

A. 3 units

B. $\frac{13}{5}$ units

C. $\frac{5}{3}$ units

D. None of these

Answer

Given: Equation of plane is $\vec{r}.(3\hat{\imath}-4\hat{\jmath}-12\hat{k})+39=0$

Formula Used: Equation of a plane is $\hat{u}.\vec{r}=p$ where $\hat{u}$ is the unit vector normal to the plane, $\vec{r}$ represents a point on the plane and p is the distance of the plane from the origin.

Explanation:

Given equation is $\vec{r}.(3\hat{\imath}-4\hat{\jmath}-12\hat{k})=-39$ ... (1)

Now, $|3\hat{\imath}-4\hat{\jmath}-12\hat{k}|=\sqrt{9+16+144}=\sqrt{169}$

= 13

Dividing (1) by 13 and multiplying by -1,

$\vec{r}.\left(\frac{-3}{13}\hat{\imath}+\frac{4}{13}\hat{\jmath}+\frac{12}{13}\hat{k}\right)=3$

which is of the form $\hat{u}.\vec{r}=p$

Therefore, length of perpendicular from origin to plane is 3 units.

4. Question

Mark against the correct answer in each of the following:

The equation of a plane passing through the point A(2, -3, 7) and making equal intercepts on the axes, is

A. x + y + z = 3

B. x + y + z = 6

C. x + y + z = 9

D. x + y + z = 4

Answer

Given: A(2, -3, 7) is a point on the plane making equal intercepts on the axes.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Explanation:

Let the equation of the plane be

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ ... (1)

Here a = b = c = p (let's say)

Since (2, -3, 7) is a point on the plane,

(1) becomes

$\frac{2-3+7}{p}=1$

p = 6

Therefore equation of the plane is

x + y + z = 6

5. Question

Mark against the correct answer in each of the following:

A plane cuts off intercepts 3, -4, 6 on the coordinate axes. The length of perpendicular from the origin to this plane is

A. $\frac{5}{\sqrt{29}}$ units

B. $\frac{8}{\sqrt{29}}$ units

C. $\frac{6}{\sqrt{29}}$ units

D. $\frac{12}{\sqrt{29}}$ units

Answer

Given: Plane makes intercepts 3, -4 and 6 with the coordinate axes.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Normal Form of a plane ⇒ lx + my + nz = p where (l, m, n) is the direction cosines and p is the distance of perpendicular to the plane from the origin.

Explanation:

Equation of the given plane is

$$\frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$$

i.e., 4x – 3y + 2z = 12 ... (1)

which is of the form ax + by + cz = d

Direction ratios are (4, -3, 12)

So, $\sqrt{4^2+(-3)^2+2^2}=\sqrt{16+9+4}$

$= \sqrt{29}$

Dividing (1) by 13,

$$\frac{4}{\sqrt{29}}x-\frac{3}{\sqrt{29}}+\frac{2}{\sqrt{29}}=\frac{12}{\sqrt{29}}$$

which is in the normal form

Therefore length of perpendicular from the origin is $\frac{12}{\sqrt{29}}$ units

6. Question

Mark against the correct answer in each of the following:

If the line $\frac{x+1}{3}=\frac{y-2}{4}=\frac{z+6}{5}$ is parallel to the plane 2x – 3y + kz = 0, then the value of k is

A. $\frac{5}{6}$

B. $\frac{6}{5}$

C. $\frac{3}{4}$

D. $\frac{4}{5}$

Answer

Given:

1. Equation of line is $\frac{x+1}{3}=\frac{y-2}{4}=\frac{z+6}{5}$

2. Equation of plane is 2x – 3y + kz = 0

Formula Used: If two direction ratios are perpendicular, then

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

Explanation:

Direction ratios of given line is (3, 4, 5)

Direction ratios of given plane is (2, -3, k)

Since the given line is parallel to the plane, the normal to the plane is perpendicular to the line.

So direction ratio of line is perpendicular to direction ratios of plane.

⇒ 3× 2 + 4× -3+ 5× k = 0

⇒ 6 – 12 + 5k = 0

$$\Rightarrow k=\frac{6}{5}$$

Therefore, $k=\frac{6}{5}$

7. Question

Mark against the correct answer in each of the following:

If O is the origin and P(1, 2, -3) is a given point, then the equation of the plane through P and perpendicular to OP is

A. x + 2y – 3z = 14

B. x – 2y + 3z = 12

C. x – 2y – 3z = 14

D. None of these

Answer

Given: P(1, 2, -3) is a point on the plane. OP is perpendicular to the plane.

Explanation:

Let equation of plane be ax + by + cz = d ... (1)

Substituting point P,

⇒ a + 2b -3c = d ... (2)

$\overrightarrow{OP} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

Since OP is perpendicular to the plane, direction ratio of the normal is (1, 2, -3)

Substituting in (2)

1 + 4 + 9 = d

d = 14

Substituting the direction ratios and value of 'd' in (1), we get

x + 2y – 3z = 14

Therefore equation of plane is x + 2y – 3z = 14

8. Question

Mark against the correct answer in each of the following:

If the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane 2x – 4y + z = 7, then the value of k is

A. -7

B. 7

C. 4

D. -4

Answer

Given: Equation of plane is 2x – 4y + z = 7

Line $\frac{(x-4)}{1} = \frac{(y-2)}{1} = \frac{(z-k)}{2}$ lies on the given plane.

Formula Used: Equation of a line is

$$\frac{(x-x_1)}{b_1} = \frac{(y-y_1)}{b_2} = \frac{(z-z_1)}{b_3} = \lambda$$

Where (x_1, y_1, z_1) is a point on the line and b_1, b_2, b_3 : direction ratios of line.

Explanation:

Let $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2} = \lambda$

So the given line passes through the point (4, 2, k)

Since the line lies on the given plane, (4, 2, k) is a point on the plane.

Therefore, substituting the point on the equation for the plane,

⇒ 8 – 8 + k = 7

⇒ k = 7

9. Question

Mark against the correct answer in each of the following:

The plane 2x + 3y + 4z =12 meets the coordinate axes in A, B and C. The centroid of ΔABC is

A. (2, 3, 4)

B. (6, 4, 3)

C. $\left(2, \frac{4}{3}, 1\right)$

D. None of these

Answer

Given: The plane 2x + 3y + 4z = 12 meets coordinate axes at A, B and C.

To find: Centroid of ΔABC

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Centroid of a triangle = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Explanation:

Equation of given plane is 2x + 3y + 4z = 12

Dividing by 12,

$\frac{x}{6}+\frac{y}{4}+\frac{z}{3}=1$

Therefore the intercepts on x, y and z-axis are 6, 6 and 3 respectively.

So, the vertices of ΔABC are (6, 0, 0), (0, 4, 0) and (0, 0, 3)

Centroid = $\left(\frac{6+0+0}{3}, \frac{0+4+0}{3}, \frac{0+0+3}{3}\right)$

= (2, 4/3, 1)

Therefore, the centroid of ΔABC is (2, 4/3, 1)

10. Question

Mark against the correct answer in each of the following:

If a plane meets the coordinate axes in A, B and C such that the centroid of ΔABC is (1, 2, 4), then the equation of the plane is

A. x + 2y + 4z = 6

B. 4x + 2y + z = 12

C. x + 2y + 4z = 7

D. 4x + 2y + z = 7

Answer

Given: Centroid of ΔABC is (1, 2, 4)

To find: Equation of plane.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Centroid of a triangle = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Explanation:

Let the equation of plane be

$\frac{x}{A}+\frac{y}{B}+\frac{z}{C}=1$ ... (1)

Therefore, A = 3a, B = 3b, C = 3c where (a, b, c) is the centroid of the triangle with vertices (A, 0, 0), (0, B, 0) and (0, 0, C)

Substituting in (1),

$\Rightarrow \frac{x}{3a}+\frac{y}{3b}+\frac{z}{3c}=1$

Here a = 1, b = 2 and c = 4

$\Rightarrow \frac{x}{3}+\frac{y}{6}+\frac{z}{12}=1$

Multiplying by 12,

4x + 2y + z = 12

Therefore equation of required plane is 4x + 2y + z = 12

11. Question

Mark against the correct answer in each of the following:

The equation of a plane through the point A(1, 0, -1) and perpendicular to the line $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+7}{-3}$ is

A. 2x + 4y - 3z = 3

B. 2x - 4y + 3z = 5

C. 2x + 4y - 3z = 5

D. x + 3y + 7z = -6

Answer

Given: Plane passes through the point A(1, 0, -1).

Plane is perpendicular to the line

$$\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+7}{-3}$$

To find: Equation of the plane.

Formula Used: Equation of a plane is ax + by + cz = d where (a, b, c) are the direction ratios of the normal to the plane.

Explanation:

Let the equation of the plane be

ax + by + cz = d ... (1)

Substituting point A,

a - z = d

Since the given line is perpendicular to the plane, it is the normal.

Direction ratios of line is 2, 4, -3

Therefore, 2 + 3 = d

d = 5

So the direction ratios of perpendicular to plane is 2, 4, -3 and d = 5

Substituting in (1),

2x + 4y - 3z = 5

Therefore, equation of plane is 2x + 4y - 3z = 5

12. Question

Mark against the correct answer in each of the following:

The line $\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{-3}$ meets the plane 2x + 3y - z = 14 in the point

A. (2, 5, 7)

B. (3, 5, 7)

C. (5, 7, 3)

D. (6, 5, 3)

Answer

Given: Line $\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{-3}$ meets plane 2x + 3y - z = 14

To find: Point of intersection of line and plane.

Explanation:

Let the equation of the line be

$$\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{-3}=\lambda$$

Therefore, any point on the line is $(2\lambda + 1, 4\lambda + 2, -3\lambda + 3)$

Since this point also lies on the plane,

$2(2\lambda + 1) + 3(4\lambda + 2) - (-3\lambda + 3) = 14$

$4\lambda + 2 + 12\lambda + 6 + 3\lambda - 3 = 14$

$19\lambda + 5 = 14$

$$\lambda = \frac{19}{19} = 1$$

Therefore the required point is (3, 5, 7).

13. Question

Mark against the correct answer in each of the following:

The equation of the plane passing through the points A(2, 2, 1) and B(9, 3, 6) and perpendicular to the plane 2x + 6y + 6z = 1, is

A. x + 2y - 3z + 5 = 0

B. 2x - 3y + 4z - 6 = 0

C. 4x + 5y - 6z + 3 = 0

D. 3x + 4y - 5z - 9 = 0

Answer

Given: Plane passes through A(2, 2, 1) and B(9, 3, 6). Plane is perpendicular to 2x + 6y + 6z = 1

To find: Equation of the plane

Formula Used: Equation of a plane is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where a:b:c is the direction ratios of the normal to the plane.

(x_1, y_1, z_1) is a point on the plane.

Explanation:

Let the equation of plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Since (2, 2, 1) is a point in the plane,

a(x - 2) + b(y - 2) + c(z - 1) = 0 ... (1)

Since B(9, 3, 6) is another point on the plane,

a(9 - 2) + b(3 - 2) + c(6 - 1) = 0

7a + b + 5c = 0 ... (1)

Since this plane is perpendicular to the plane 2x + 6y + 6z = 1, the direction ratios of the normal to the plane will also be perpendicular.

So, 2a + 6b + 6c = 0 ⇒ a + 3b + 3c = 0 ... (2)

Solving (1) and (2),

$$\frac{a}{\begin{vmatrix}1 & 5\\3 & 3\end{vmatrix}} = \frac{-b}{\begin{vmatrix}7 & 5\\1 & 3\end{vmatrix}} = \frac{c}{\begin{vmatrix}7 & 1\\1 & 3\end{vmatrix}}$$

$$\frac{a}{-12} = \frac{b}{-16} = \frac{c}{20}$$

$$\frac{a}{3} = \frac{b}{4} = \frac{c}{-5}$$

a : b : c = 3 : 4 : -5

Substituting in (1),

3x - 6 + 4y - 8 - 5z + 5 = 0

3x + 4y - 5z - 9 = 0

Therefore the equation of the plane is 3x + 4y - 5z - 9 = 0

14. Question

Mark against the correct answer in each of the following:

The equation of the plane passing through the intersection of the planes 3x - y + 2z - 4 = 0 and x + y + z - 2 = 0 and passing through the point A(2, 2, 1) is given by

A. 7x + 5y - 4z - 8 = 0

B. 7x - 5y + 4z - 8 = 0

C. 5x - 7y + 4z - 8 = 0

D. 5x + 7y - 4z + 8 = 0

Answer

Given: Plane passes through the intersection of planes 3x - y + 2z - 4 = 0 and x + y + z - 2 = 0. Point A(2, 2, 1) lies on the plane.

To find: Equation of the plane.

Formula Used: Equation of plane passing through the intersection of 2 planes P_1 and P_2 is given by $P_1 + \lambda P_2 = 0$

Explanation:

Equation of plane is

$3x - y + 2z - 4 + \lambda (x + y + z - 2) = 0$ ... (1)

Since A(2, 2, 1) lies on the plane,

$6 - 2 + 2 - 4 + \lambda (2 + 2 + 1 - 2) = 0$

$2 + 3\lambda = 0$

$\lambda = \frac{-2}{3}$

Substituting in (1) and multiplying by 3,

$9x - 3y + 6z - 12 - 2 (x + y + z - 2) = 0$

$9x - 3y + 6z - 12 - 2x - 2y - 2z + 4 = 0$

$7x - 5y + 4z - 8 = 0$

Therefore the equation of the plane is $7x - 5y + 4z - 8 = 0$

15. Question

Mark against the correct answer in each of the following:

The equation of the plane passing through the points A(0, -1, 0), B(2, 1, -1) and C(1, 1, 1) is given by

A. $4x + 3y - 2z - 3 = 0$

B. $4x - 3y + 2z + 3 = 0$

C. $4x - 3y + 2z - 3 = 0$

D. None of these

Answer

Given: Plane passes through A(0, -1, 0), B(2, 1, -1) and C(1, 1, 1)

To find: Equation of the plane

Formula Used: Equation of a plane is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where a:b:c is the direction ratios of the normal to the plane.

(x_1, y_1, z_1) is a point on the plane.

Explanation:

Let the equation of plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Substituting point A,

$ax + b(y + 1) + cz = 0$ ... (1)

Substituting points B and C,

$2a + 2b - c = 0$ and $a + 2b + c = 0$

Solving,

$$\frac{a}{\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix}}$$

$$\frac{a}{4} = \frac{b}{-3} = \frac{c}{2}$$

Therefore, a : b : c = 4 : -3 : 2

Substituting in (1),

$4x - 3 (y + 1) + 2z = 0$

$4x - 3y + 2z - 3 = 0$

Therefore equation of plane is $4x - 3y + 2z - 3 = 0$

16. Question

Mark against the correct answer in each of the following:

If the plane $2x - y + z = 0$ is parallel to the line $\frac{2x - 1}{2} = \frac{2 - y}{2} = \frac{z + 1}{a}$, then the value of a is

A. -4

B. -2

C. 4

D. 2

Answer

Given: Plane 2x – y + z = 0 is parallel to the line

$$\frac{2x-1}{2}=\frac{2-y}{2}=\frac{z+1}{a}.$$

To find: value of a

Formula Used: If two lines with direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then

$a_1b_1 + a_2b_2 + a_3b_3 = 0$

Explanation:

Since the plane is parallel to the line, the normal to the plane will be perpendicular to the line.

Equation of the line can be rewritten as

$$\frac{x-\frac{1}{2}}{1}=\frac{y-2}{-2}=\frac{z-(-1)}{a}$$

Direction ratio of the normal to the plane is 2 : -1 : 1

Direction ratio of line is 1 : -2 : a

Therefore,

2 + 2 + a = 0

a = -4

Therefore, a = -4

17. Question

Mark against the correct answer in each of the following:

The angle between the line $\frac{x+1}{1}=\frac{y}{2}=\frac{z-1}{1}$ and a normal to the plane x – y + z = 0 is

A. 0°

B. 30°

C. 45°

D. 90°

Answer

Given: Equation of line is $\frac{x+1}{1}=\frac{y}{2}=\frac{z-1}{1}$

Equation of plane is x – y + z = 0

To find: Angle between a line and the normal to a plane.

Formula Used: If θ is the angle between two lines with direction ratios $b_1:b_2:b_3$ and $c_1:c_2:c_3$, then

$$\cos\theta = \frac{b_1c_1+b_2c_2+b_3c_3}{\sqrt{b_1^2+b_2^2+b_3^2}\times\sqrt{c_1^2+c_2^2+c_3^2}}$$

Explanation:

Direction ratios of given line is 1 : 2 : 1

Direction ratios of the normal to the plane is 1 : -1 : 1

Therefore,

$$\cos\theta = \frac{1-2+1}{\sqrt{1+4+1}\times\sqrt{1+1+1}}$$

cos θ = 0

θ = 90°

Therefore angle between them is 90°

18. Question

Mark against the correct answer in each of the following:

The point of intersection of the line $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}$ and the plane 2x - y + 3z - 1 = 0, is

A. (-10, 10, 3)

B. (10, 10, -3)

C. (10, -10, 3)

D. (10, -10, -3)

Answer

Given: Line $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}$ meets plane 2x - y + 3z - 1 = 0

To find: Point of intersection of line and plane.

Explanation:

Let the equation of the line be

$$\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}=\lambda$$

Therefore, any point on the line is (3λ + 1, 4λ - 2, -2λ + 3)

Since this point also lies on the plane,

2(3λ + 1) - (4λ -2) + 3(-2λ +3) = 1

6λ + 2 - 4λ + 2 - 6λ + 9 = 1

-4λ = -12

λ = 3

Therefore required point is (10, 10, -3)

19. Question

Mark against the correct answer in each of the following:

The equation of a plane passing through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c) is given by

A. ax + by + cz = 0

B. ax + by + cz = 1

C. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$

D. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Answer

Given: Plane passes through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c)

To find: Equation of plane.

Explanation:

The given points lie on the co-ordinate axes.

Therefore, the plane makes intercepts of a, b and c on the x, y and z-axis respectively.

Equation of the plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

20. Question

Mark against the correct answer in each of the following:

If θ is the angle between the planes 2x - y + 2z = 3 and 6x - 2y + 3z = 5, then cos θ = ?

A. $\frac{11}{20}$

B. $\frac{12}{23}$

C. $\frac{17}{25}$

D. $\frac{20}{21}$

Answer

Given: Equation of two planes are 2x - y + 2z = 3 and 6x - 2y + 3z = 5

To find: cos θ where θ: angle between the planes

Formula Used: Angle between two planes $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$ is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where θ : angle between the planes,

Explanation:

Here $a_1 = 2$, $b_1 = -1$, $c_1 = 2$

$a_2 = 6$, $b_2 = -2$, $c_2 = 3$

$$\Rightarrow \cos\theta = \frac{12 + 2 + 6}{\sqrt{4 + 1 + 4} \times \sqrt{36 + 4 + 9}}$$

$$\Rightarrow \cos\theta = \frac{20}{3 \times 7}$$

$$\Rightarrow \cos\theta = \frac{20}{21}$$

Therefore, $\cos\theta = \frac{20}{21}$

21. Question

Mark against the correct answer in each of the following:

The angle between the planes 2x - y + z = 6 and x + y + 2z = 7, is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer

Given: Equation of two planes are 2x - y + z = 6 and x + y + 2z = 7

To find: Angle between the two planes

Formula Used: Angle between two planes $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$ is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where θ : angle between the planes,

Explanation:

Here $a_1 = 2$, $b_1 = -1$, $c_1 = 1$

$a_2 = 1$, $b_2 = 1$, $c_2 = 2$

$$\Rightarrow \cos\theta = \frac{2 - 1 + 2}{\sqrt{4 + 1 + 1} \times \sqrt{1 + 1 + 4}}$$

$$\Rightarrow \cos\theta = \frac{3}{\sqrt{6} \times \sqrt{6}}$$

$$\Rightarrow \cos\theta = \frac{3}{6}$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$\Rightarrow \theta = \frac{\pi}{3}$

Therefore angle between the planes is $\frac{\pi}{3}$

22. Question

Mark against the correct answer in each of the following:

The angle between the planes $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 4$ and $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 3$, is

A. $\cos^{-1}\left(\frac{16}{21}\right)$

B. $\cos^{-1}\left(\frac{4}{21}\right)$

C. $\cos^{-1}\left(\frac{3}{4}\right)$

D. $\cos^{-1}\left(\frac{1}{4}\right)$

Answer

Given: Equation of two planes are $\vec{r}.(3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}) = 4$ and $\vec{r}.(2\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 3$

To find: Angle between the two planes

Formula Used: Angle between two planes $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$ is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where θ : angle between the planes.

Explanation:

Since $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, the given equation of planes can rewritten as:

$3x - 6y + 2z = 4$ and $2x - y + 2z = 3$

Here $a_1 = 3, b_1 = -6, c_1 = 2$

$a_2 = 2, b_2 = -1, c_2 = 2$

$$\Rightarrow \cos\theta = \frac{6 + 6 + 4}{\sqrt{9 + 36 + 4} \times \sqrt{4 + 1 + 4}}$$

$$\Rightarrow \cos\theta = \frac{16}{7 \times 3}$$

$$\Rightarrow \cos\theta = \frac{16}{21}$$

$$\Rightarrow \theta = \cos^{-1}\frac{16}{21}$$

Therefore angle between the planes is $\cos^{-1}\frac{16}{21}$

23. Question

Mark against the correct answer in each of the following:

The equation of the plane through the points A(2, 3, 1) and B(4, -5, 3), parallel to the x-axis, is

A. $x + y - 3z = 2$

B. $y + 4z = 7$

C. $y + 3z = 6$

D. $x + 5y - 3z = 4$

Answer

Given: Plane passes through the points A(2, 3, 1) and B(4, -5, 3) and is parallel to x-axis

To find: Equation of plane

Formula Used: Equation of a plane parallel o x-axis is

$b(y - y_1) + c(z - z_1) = 0$

Explanation:

Let the equation of the plane be

$b(y - y_1) + c(z - z_1) = 0$

Since A(2, 3, 1) lies on the plane,

$b(y - 3) + c(z - 1) = 0$ ... (1)

Since B(4, -5, 3) lies on the plane,

$b(-5 - 3) + c(3 - 1) = 0$

$-8b + 2c = 0$ or $-4b + c = 0$

$b : c = 1 : 4$

Substituting in (1),

$y - 3 + 4z - 4 = 0$

$y + 4z = 7$

The equation of the plane is $y + 4z = 7$

24. Question

Mark against the correct answer in each of the following:

A variable plane moves so that the sum of the reciprocals of its intercepts on the coordinate axes is (1/2). Then, the plane passes through the point

A. (0, 0, 0)

B. (1, 1, 1)

C. $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

D. (2, 2, 2)

Answer

Given: Variable plane moves so that the sum of the reciprocals of its intercepts on the coordinate axes is (1/2)

Formula Used: Equation of a plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Explanation:

Let the intercepts made by the plane on the co-ordinate axes be a, b and c.

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$$

Let the equation of the plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

On solving for each of the given options,

$(0, 0, 0) \Rightarrow LHS \neq RHS$

$(1, 1, 1) \Rightarrow LHS \neq RHS$

$$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \Rightarrow LHS = \frac{1}{2a} + \frac{1}{2b} + \frac{1}{2c} = \frac{1}{2} \times \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = \frac{1}{4} \neq RHS$$

$$(2, 2, 2) \Rightarrow LHS = \frac{2}{a} + \frac{2}{b} + \frac{2}{c} = 2 \times \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 1 = RHS$$

Therefore, plane passes through the point (2, 2, 2)

25. Question

Mark against the correct answer in each of the following:

The equation of a plane which is perpendicular to $\left(2\hat{i} - 3\hat{j} + \hat{k}\right)$ and at a distance of 5 units from the origin is

A. $2x - 3y + z = 5$

B. $2x - 3y + z = 5\sqrt{14}$

C. $\frac{x}{2}-\frac{y}{3}+\frac{z}{1}=5$

D. $\frac{x}{2}-\frac{y}{3}+\frac{z}{1}=\frac{5}{\sqrt{14}}$

Answer

Given: Plane is perpendicular to $(2\hat{i}-3\hat{j}+\hat{k})$ and is at a distance of 5 units from origin.

To find: Equation of plane

Formula Used: Equation of a plane is lx + my + nz = p where p is the distance from the origin and l, m and n are the direction cosines of the normal to the plane

Explanation:

Direction ratio of normal to the plane is 2:-3:1

$|2\hat{i}-3\hat{j}+\hat{k}|=\sqrt{4+9+1}=\sqrt{14}$

Therefore, direction cosines of the normal to the plane is

$l=\frac{2}{\sqrt{14}}, m=\frac{-3}{\sqrt{14}}, n=\frac{1}{\sqrt{14}}$

Since the equation of a plane is lx + my + nz = p where p is the distance from the origin,

$2x - 3y + z = 5\sqrt{14}$

Therefore, equation of the plane is $2x - 3y + z = 5\sqrt{14}$

26. Question

Mark against the correct answer in each of the following:

The equation of the plane passing through the point A(2, 3,4) and parallel to the plane 5x - 6y + 7z = 3, is

A. 5x - 6y + 7z = 20

B. 7x - 6y + 5z = 72

C. 20x – 18y + 14z = 11

D. 10x – 18y + 28z = 13

Answer

Given: Point A(2, 3, 4) lies on a plane which is parallel to 5x – 6y + 7z = 3

To find: Equation of the plane

Formula Used: Equation of a plane is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where a:b:c is the direction ratios of the normal to the plane

(x_1, y_1, z_1) is a point on the plane.

Explanation:

Since the plane (say P_1) is parallel to the plane 5x – 6y + 7z = 3 (say P_2), the direction ratios of the normal to P_1 is same as the direction ratios of the normal to P_2.

i.e., direction ratios of P_1 is 5 : -6 : 7

Let the equation of the required plane be

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Here a = 5, b = -6 and c = 7

Since (2, 3, 4) lies on the plane,

5(x - 2) - 6 (y - 3) + 7 (z - 4) = 0

5x – 6y + 7z – 10 + 18 - 28 = 0

5x – 6y + 7z = 20

The equation of the plane is 5x – 6y + 7z = 20

27. Question

Mark against the correct answer in each of the following:

The foot of the perpendicular from the point A(7, 14, 5) to the plane 2x + 4y – z = 2 is

A. (3, 1, 8)

B. (1, 2, 8)

C. (3, -3, 5)

D. (5, -3, -4)

Answer

Given: Perpendicular dropped from A(7, 14, 5) on to the plane 2x + 4y - z = 2

To find: co-ordinates of the foot of perpendicular

Formula Used: Equation of a line is

$$\frac{x-x_1}{b_1}=\frac{y-y_2}{b_2}=\frac{z-z_3}{b_3}=\lambda$$

Where $b_1:b_2:b_3$ is the direction ratio and (x_1, x_2, x_3) is a point on the line.

Explanation:

Let the foot of the perpendicular be (a, b, c)

Since this point lies on the plane,

2a + 4b - c = 2 ... (1)

Direction ratio of the normal to the plane is 2 : 4 : -1

Direction ratio perpendicular = direction ratio of normal to the plane

So, equation of the perpendicular is

$$\frac{x-x_1}{2}=\frac{y-y_1}{4}=\frac{z-z_1}{-1}=\lambda$$

Since (a, b, c) is a point on the perpendicular,

$$\frac{x-a}{2}=\frac{y-b}{4}=\frac{z-c}{-1}=\lambda$$

(7, 14, 5) is a point on the perpendicular.

$$\frac{7-a}{2}=\frac{14-b}{4}=\frac{5-c}{-1}=\lambda$$

So, a = 7 - 2λ, b = 14 - 4λ, c = 5 + λ

Substituting in (1),

14 - 4λ + 56 - 16λ - 5 - λ = 2

21λ = 70 - 7 = 63

λ = 3

Therefore, foot of the perpendicular is (1, 2, 8)

28. Question

Mark against the correct answer in each of the following:

The equation of the plane which makes with the coordinate axes, a triangle with centroid (α, β, γ) is given by

A. αx + βy + γz = 1

B. αx + βy + γz = 3

C. $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

D. $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

Answer

Given: Centroid of triangle is (α, β, γ)

To find: Equation of plane.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Centroid of a triangle = $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$

Explanation:

Let the equation of plane be

$\frac{x}{A}+\frac{y}{B}+\frac{z}{C}=1$ ... (1)

Therefore, A = 3α, B = 3β, C = 3γ where (a, b, c) is the centroid of the triangle with vertices (A, 0, 0), (0, B, 0) and (0, 0, C)

Substituting in (1),

$\Rightarrow \frac{x}{3a}+\frac{y}{3b}+\frac{z}{3c}=1$

Here a = α, b = β and c = γ

$\Rightarrow \frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

Therefore equation of required plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

29. Question

Mark against the correct answer in each of the following:

The intercepts made by the plane $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=12$ are

A. 2, -3, 4

B. 2, -3, -6

C. 6, -4, 3

D. -6, 4, 3

Answer

Given: Equation of plane is $\vec{r}.\left(2\hat{\imath}-3\hat{\jmath}+4\hat{k}\right)=12$

To find: Intercepts made by the plane.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Explanation:

The equation of the plane can be written as

2x - 3y + 4z = 12

Dividing by 12,

$\frac{x}{6}+\frac{y}{-4}+\frac{z}{3}=1$ which is of the form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Therefore the intercepts made by the plane are 6, -4, 3

30. Question

Mark against the correct answer in each of the following:

The angle between the line $\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+4}{-3}$ and the plane 2x - 3y + z = 5 is

A. $\cos^{-1}\left(\frac{5}{14}\right)$

B. $\sin^{-1}\left(\frac{5}{14}\right)$

C. $\cos^{-1}\left(\frac{3}{7}\right)$

D. $\sin^{-1}\left(\frac{3}{7}\right)$

Answer

Given: Equation of line is $\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+4}{-3}$

Equation of the plane is 2x - 3y + z = 5

To find: angle between line and plane

Formula Used: If θ is the angle between a line with direction ratio $b_1:b_2:b_3$ and a plane with direction ratio of normal $n_1:n_2:n_3$, then

$$\sin\theta = \frac{n_1b_1 + n_2b_2 + n_3b_3}{\sqrt{n_1^2 + n_2^2 + n_3^2} \times \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Explanation:

Here direction ratio of the line is 1 : -2 : -3

Direction ratio of normal to the plane is 2 : -3 : 1

Therefore,

$$\sin\theta = \frac{2 + 6 - 3}{\sqrt{1 + 4 + 9} \times \sqrt{4 + 9 + 1}}$$

$$\sin\theta = \frac{5}{\sqrt{14} \times \sqrt{14}}$$

$$\theta = \sin^{-1}\frac{5}{14}$$

Therefore, angle between the line and plane is $\sin^{-1}\frac{5}{14}$

31. Question

Mark against the correct answer in each of the following:

The angle between the line $\vec{r}\cdot(\hat{i}+\hat{j}-3\hat{k})+\lambda(2\hat{i}+2\hat{j}+\hat{k})$ and the plane $\vec{r}\cdot(6\hat{i}-3\hat{j}+2\hat{k})=5$, is

A. $\cos^{-1}\left(\frac{8}{21}\right)$

B. $\cos^{-1}\left(\frac{5}{21}\right)$

C. $\sin^{-1}\left(\frac{5}{21}\right)$

D. $\sin^{-1}\left(\frac{8}{21}\right)$

Answer

Given: Equation of line is $\vec{r}.(\hat{\imath} + \hat{\jmath} - 3\hat{k}) + \lambda(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$

Equation of plane is $\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 5$

To find: angle between line and plane

Formula Used: If θ is the angle between a line with direction ratio b_1:b_2:b_3 and a plane with direction ratio of normal n_1:n_2:n_3, then

$$\sin\theta = \frac{n_1b_1 + n_2b_2 + n_3b_3}{\sqrt{n_1^2 + n_2^2 + n_3^2} \times \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Explanation:

Here direction ratio of the line is 2 : 2 : 1

Direction ratio of normal to the plane is 6 : -3 : 2

Therefore,

$$\sin\theta = \frac{12 - 6 + 2}{\sqrt{4 + 4 + 1} \times \sqrt{36 + 9 + 4}}$$

$$\sin\theta = \frac{8}{3 \times 7}$$

$$\theta = \sin^{-1}\frac{8}{21}$$

Therefore, angle between the line and plane is $\sin^{-1}\frac{8}{21}$

32. Question

Mark against the correct answer in each of the following:

The distance of the point $(\hat{i}+2\hat{j}+5\hat{k})$ from the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})+17=0$, is

A. $\frac{25}{\sqrt{2}}$ units

B. $\frac{25}{\sqrt{3}}$ units

C. 25√2 units

D. 25√3 units

Answer

Given: Point is at $(\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ and equation of plane is $\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) + 17 = 0$

To find: distance of point from plane

Formula Used: Perpendicular distance from (x_1, y_1, z_1) to the plane ax + by + cz + d = 0 is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Explanation:

The point is at (1, 2, 5) and equation of plane is x + y + z + 17 = 0

Distance $= \frac{1+2+5+17}{\sqrt{1+1+1}}$

$= \frac{25}{\sqrt{3}}$

Therefore, distance $= \frac{25}{\sqrt{3}}$ units

33. Question

Mark against the correct answer in each of the following:

The distance between the parallel planes 2x – 3y + 6z = 5 and 6x – 9y + 18z + 20 = 0, is

A. $\frac{5}{3}$ units

B. 5√3 units

C. $\frac{8}{5}$ units

D. 8√5 units

Answer

Given: The equations of the parallel planes are 2x – 3y + 6z = 5 and 6x – 9y + 18z + 20 = 0

To find: distance between the planes

Formula Used: Distance between two parallel planes ax + by + cz + d_1 = 0 and $ax_1 + by_1 + cz_1 + d_1 = 0$ is

$$\left|\frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}}\right|$$

Explanation:

The equations of the parallel planes are:

2x – 3y + 6z – 5 = 0

$2x - 3y + 6z + \frac{20}{3} = 0$

Therefore distance between them is

$= \left|\frac{\frac{20}{3} + 5}{\sqrt{4 + 9 + 36}}\right|$

$= \left|\frac{35}{3 \times \sqrt{49}}\right|$

$= \frac{5}{3}$

Therefore distance between the planes is $\frac{5}{3}$ units

34. Question

Mark against the correct answer in each of the following:

The distance between the planes x + 2y - 2z + 1 = 0 and 2x + 4y - 4z - 4z + 5 = 0, is

A. 4 units

B. 2 units

C. $\frac{1}{2}$ units

D. $\frac{1}{4}$ units

Answer

Given: The equations of the planes are x + 2y – 2z + 1 = 0 and 2x + 4y – 4z – 4z + 5 = 0

To find: distance between the planes

Formula Used: Distance between two parallel planes ax + by + cz + d_1 = 0 and ax_1 + by_1 + cz_1 + d_1 = 0 is

$$\left|\frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}}\right|$$

Explanation:

The equations of the planes are:

x + 2y – 2z + 1 = 0 and 2x + 4y – 4z – 4z + 5 = 0

Multiplying the equation of first plane by 2,

2x + 4y - 4z + 2 = 0

Therefore distance between them is

$$= \left|\frac{\frac{20}{3} + 5}{\sqrt{4 + 9 + 36}}\right|$$

$$= \left|\frac{35}{3 \times \sqrt{49}}\right|$$

$$= \frac{5}{3}$$

Therefore distance between the planes is $\frac{5}{3}$ units

35. Question

Mark against the correct answer in each of the following:

The image of the point P(1, 3, 4) in the plane 2x – y + z + 3 = 0, is

A. (3, -5, 2)

B. (3, 5, -2)

C. (3, 5, 2)

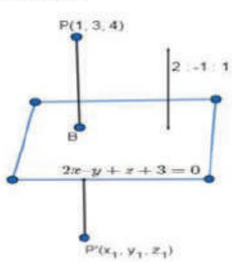
D. (-3, 5, 2)

Answer

Given: Equation of plane is 2x – y + z + 3 = 0. P is at (1, 3, 4)

To find: image of P

Explanation:



From the figure, P' is the image of P and B is the midpoint of PP'

If B is (a, b, c), then