Exercise 5(D)

Solution 1:

$$a^3 - 27 = (a)^3 - (3)^3$$

$$= (a-3)\left[(a)^2 + a \times 3 + (3)^2\right] \quad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$

$$= (a-3)\left[a^2 + 3a + 9\right]$$

Solution 2:

$$1 - 8a^3 = (1)^3 - (2a)^3$$

$$= (1-2a)\left[(1)^2 + 1 \times 2a + (2a)^2\right]$$

$$\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$

$$= (1-2a)\left[1 + 2a + 4a^2\right]$$

Solution 3:

$$64 - a^3b^3 = (4)^3 - ab^3$$

$$= (4-ab)\left[(4)^2 + 4 \times ab + (ab)^2\right]$$

$$\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$

$$= (4-ab)\left[16 + 4ab + a^2b^2\right]$$

Solution 4:

$$a^6 + 27b^3 = (a^2)^3 + (3b)^3$$

$$= (a^2 + 3b)\left[(a^2)^2 - a^2 \times 3b + (3b)^2\right]$$

$$\left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)\right]$$

$$= (a^2 + 3b)\left[a^4 - 3a^2b + 9b^2\right]$$

Solution 5:

$$3x^7y - 81x^4y^4 = 3xy\left(x^6 - 27x^3y^3\right)$$

$$= 3xy\left((x^2)^3 - (3xy)^3\right)$$

$$= 3xy\left(x^2 - 3xy\right)\left[(x^2)^2 + x^2 \times 3xy + (3xy)^2\right]$$

$$\left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$$

$$= 3xy\left(x^2 - 3xy\right)\left[x^4 + 3x^3y + 9x^2y^2\right]$$

$$= 3xy\left\{x(x - 3y)x^2\left[x^2 + 3xy + 9y^2\right]\right\}$$

$$= 3x^4y(x - 3y)\left[x^2 + 3xy + 9y^2\right]$$

Solution 6:

$$a^3 - \frac{27}{a^3} = (a)^3 - \left(\frac{3}{a}\right)^3$$

$$= \left(a - \frac{3}{a}\right)\left(a^2 + a \times \frac{3}{a} + \left(\frac{3}{a}\right)^2\right)$$

$$\left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$$

$$= \left(a - \frac{3}{a}\right)\left(a^2 + 3 + \frac{9}{a^2}\right)$$

Solution 7:

$$a^3 + 0.064 = (a)^3 + (0.4)^3$$

$$= (a + 0.4)\left[(a)^2 - a \times 0.4 + (0.4)^2\right]$$

$$\left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)\right]$$

$$= (a + 0.4)\left[a^2 - 0.4a + 0.16\right]$$

Solution 8:

$$a^4 - 343a = a(a^3 - 7^3)$$

$$= a(a-7)\left[(a)^2 + a \times 7 + (7)^2\right]$$

$$\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$

$$= a(a-7)\left[a^2 + 7a + 49\right]$$

Solution 9:

$= (x - y)^3 - (2x)^3$

$= (x - y - 2x)[(x - y)^2 + 2x(x - y) + (2x)^2]$

[Using identity $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$]

$= (-x - y)[x^2 + y^2 - 2xy + 2x^2 - 2xy + 4x^2]$

$=-(x + y)\ [7x^2 - 4xy + y^2]$

Solution 10:

$$\frac{8a^3}{27} - \frac{b^3}{8} = \left(\frac{2a}{3}\right)^3 - \left(\frac{b}{2}\right)^3$$

$$= \left(\frac{2a}{3} - \frac{b}{2}\right)\left[\left(\frac{2a}{3}\right)^2 + \frac{2a}{3} \times \frac{b}{2} + \left(\frac{b}{2}\right)^2\right]$$

$[\because a^3 - b^3 = (a - b)(a^2 = ab + b^2)]$

$$= \left(\frac{2a}{3} - \frac{b}{2}\right)\left[\frac{4a^2}{9} + \frac{ab}{3} + \frac{b^2}{4}\right]$$

Solution 11:

We know that,

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2) \quad ....(1)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) \quad ....(2)$$

$$a^6 - b^6 = (a^3)^2 - (b^3)^2$$

$$= (a^3 + b^3)(a^3 - b^3)$$

$$= (a+b)(a^2 - ab + b^2)(a-b)(a^2 + ab + b^2) \quad \text{[from (1) and (2)]}$$

$$= (a+b)(a-b)(a^2 - ab + b^2)(a^2 + ab + b^2)$$

Solution 12:

We know that,

$a^3+b^3=(a+b)(a^2-ab+b^2)$ ....(1)

$a^3-b^3=(a-b)(a^2+ab+b^2)$ ....(2)

$a^6-7a^3-8=a^6-8a^3+a^3-8$

$=a^3(a^3-8)+1(a^3-8)$

$=(a^3+1)(a^3-8)$

$=(a^3+1^3)(a^3-2^3)$

$=(a+1)(a^2-a+1)(a-2)(a^2+2a+4)$ [from (1) and (2)]

$=(a+1)(a-2)(a^2-a+1)(a^2+2a+4)$

Solution 13:

We know that,

$a^3-b^3=(a-b)(a^2+ab+b^2)$ ....(1)

$a^3-27b^3+2a^2b-6ab^2$

$=(a)^3-(3b)^3+2ab(a-3b)$

$=(a-3b)\left[a^2+a\times 3b+(3b)^2\right]+2ab(a-3b)$ [from (1)]

$=(a-3b)\left[a^2+3ab+9b^2\right]+2ab(a-3b)$

$=(a-3b)\left[a^2+3ab+9b^2+2ab\right]$

$=(a-3b)\left[a^2+5ab+9b^2\right]$

Solution 14:

We know that,

$a^3-b^3=(a-b)(a^2+ab+b^2)$ ....(1)

$8a^3-b^3-4ax+2bx=\left[(2a)^3-(b)^3\right]-2x(2a-b)$

$=(2a-b)\left[(2a)^2+2a\times b+(b)^2\right]-2x(2a-b)$

[from (1)]

$=(2a-b)\left[4a^2+2ab+b^2\right]-2x(2a-b)$

$=(2a-b)\left[4a^2+2ab+b^2-2x\right]$

Solution 15

We know that,

$$a^3 - b^3 = (a-b)(a^2+ab+b^2) \quad ....(1)$$

$$a - b - a^3 + b^3 = a - b - (a^3 - b^3)$$

$$= (a-b) - (a-b)[a^2+ab+b^2] \quad \text{[from (1)]}$$

$$= (a-b)[1 - a^2 - ab - b^2]$$

Solution 16:

$= 2(x^3 + 27y^3 - 2x - 6y)$

$= 2\{[(x)^3+(3y)^3] - 2(x + 3y)\}$

[Using identity $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$]

$=2\{[(x + 3y)(x^2 - 3xy + 9y^2)] - 2(x + 3y)\}$

$=2(x + 3y)(x^2 - 3xy + 9y^2 - 2)$

Solution 17:

(i) $(13^3 - 5^3)$

[Using identity $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$]

$=(13 - 5)(13^2 + 13 \times 5 + 5^2)$

$=8(169 + 65 + 25)$

Therefore, the number is divisible by 8.

(ii) $(35^3 + 27^3)$

[Using identity $(a^3 + b^3)=(a + b)(a^2 - ab + b^2)$]

$=(35 + 27)(35^2 + 35\times 27 + 27^2)$

$=62 \times (35^2 + 35 \times 27 + 27^2)$

Therefore, the number is divisible by 62.