

## EXERCISE 30.4

**Differentiate the following functions with respect to x:**

**1.  $x^3 \sin x$**

**Solution:**

Let us consider  $y = x^3 \sin x$

We need to find  $dy/dx$

We know that  $y$  is a product of two functions say  $u$  and  $v$  where,

$u = x^3$  and  $v = \sin x$

$\therefore y = uv$

Now let us apply product rule of differentiation.

By using product rule, we get

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} \dots \text{Equation (1)}$$

As,  $u = x^3$

$$\frac{du}{dx} = 3x^{3-1} = 3x^2 \dots \text{Equation (2) } \left\{ \text{Since, } \frac{d}{dx}(x^n) = nx^{n-1} \right\}$$

As,  $v = \sin x$

$$\frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x \dots \text{Equation (3) } \left\{ \text{Since, } \frac{d}{dx}(\sin x) = \cos x \right\}$$

From equation (1), we can find  $dy/dx$

$$\frac{dy}{dx} = x^3 \frac{dv}{dx} + \sin x \frac{du}{dx}$$

$$\frac{dy}{dx} = x^3 \cos x + 3x^2 \sin x \quad \{ \text{Using equation 2 \& 3} \}$$

$$\therefore \frac{dy}{dx} = x^3 \cos x + 3x^2 \sin x$$

**2.  $x^3 e^x$**

**Solution:**

Let us consider  $y = x^3 e^x$

We need to find  $dy/dx$

We know that  $y$  is a product of two functions say  $u$  and  $v$  where,

$u = x^3$  and  $v = e^x$

$\therefore y = uv$

Now let us apply product rule of differentiation.

By using product rule, we get

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} \dots \text{Equation (1)}$$

As,  $u = x^3$

$$\frac{du}{dx} = 3x^{3-1} = 3x^2 \dots \text{Equation (2)} \left\{ \frac{d}{dx}(x^n) = nx^{n-1} \right\}$$

As,  $v = e^x$

$$\frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x \dots \text{Equation (3)} \left\{ \text{Since, } \frac{d}{dx}(e^x) = e^x \right\}$$

Now from equation (1), we can find  $dy/dx$

$$\frac{dy}{dx} = x^3 \frac{dv}{dx} + e^x \frac{du}{dx}$$

$$\frac{dy}{dx} = x^3 e^x + 3x^2 e^x \quad \{\text{Using equation 2 \& 3}\}$$

$$\therefore \frac{dy}{dx} = x^2 e^x (3 + x)$$

### 3. $x^2 e^x \log x$

**Solution:**

Let us consider  $y = x^2 e^x \log x$

We need to find  $dy/dx$

We know that  $y$  is a product of two functions say  $u$  and  $v$  where,

$$u = x^2 \text{ and } v = e^x, w = 1/x$$

$$\therefore y = uv$$

Now let us apply product rule of differentiation.

By using product rule, we get

$$\frac{dy}{dx} = uw \frac{dv}{dx} + vw \frac{du}{dx} + uv \frac{dw}{dx} \dots \text{equation 1}$$

As,  $u = x^2$

$$\frac{du}{dx} = 2x^{2-1} = 2x \dots \text{Equation (2)} \left\{ \text{Since, } \frac{d}{dx}(x^n) = nx^{n-1} \right\}$$

As,  $v = e^x$

$$\frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x \dots \text{Equation (3)} \left\{ \text{Since, } \frac{d}{dx}(e^x) = e^x \right\}$$

As,  $w = \log x$

$$\frac{dw}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x} \dots \text{Equation (4)} \left\{ \text{Since, } \frac{d}{dx}(\log_e x) = \frac{1}{x} \right\}$$

Now, from equation 1, we can find  $dy/dx$

$$\frac{dy}{dx} = x^2 \log x \frac{dv}{dx} + e^x \log x \frac{du}{dx} + x^2 e^x \frac{dw}{dx}$$

$$\frac{dy}{dx} = x^2 e^x \log x + 2xe^x \log x + x^2 e^x \frac{1}{x} \quad \{\text{Using equation 2, 3 \& 4}\}$$

$$\therefore \frac{dy}{dx} = xe^x(1 + x \log x + 2 \log x)$$

#### 4. $x^n \tan x$

##### Solution:

Let us consider  $y = x^n \tan x$

We need to find  $dy/dx$

We know that  $y$  is a product of two functions say  $u$  and  $v$  where,

$$u = x^n \text{ and } v = \tan x$$

$$\therefore y = uv$$

Now let us apply product rule of differentiation.

By using product rule, we get

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} \dots \text{Equation 1}$$

$$\text{As, } u = x^n$$

$$\frac{du}{dx} = nx^{n-1} \dots \text{Equation 2 } \left\{ \text{Since, } \frac{d}{dx}(x^n) = nx^{n-1} \right\}$$

$$\text{As, } v = \tan x$$

$$\frac{dv}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x \dots \text{Equation 3 } \left\{ \text{Since, } \frac{d}{dx}(\tan x) = \sec^2 x \right\}$$

Now, from equation 1, we can find  $dy/dx$

$$\frac{dy}{dx} = x^n \frac{dv}{dx} + \tan x \frac{du}{dx}$$

$$\frac{dy}{dx} = x^n \sec^2 x + nx^{n-1} \tan x \quad \{\text{Using equation 2 \& 3}\}$$

$$\therefore \frac{dy}{dx} = x^{n-1}(n \tan x + x \sec^2 x)$$

#### 5. $x^n \log_a x$

##### Solution:

Let us consider  $y = x^n \log_a x$

We need to find  $dy/dx$

We know that  $y$  is a product of two functions say  $u$  and  $v$  where,

$$u = x^n \text{ and } v = \log_a x$$

$$\therefore y = uv$$

Now let us apply product rule of differentiation.

By using product rule, we get

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} \dots \text{Equation (1)}$$

As,  $u = x^n$

$$\frac{du}{dx} = nx^{n-1} \dots \text{Equation (2) } \left\{ \text{Since, } \frac{d}{dx}(x^n) = nx^{n-1} \right\}$$

As,  $v = \log_a x$

$$\frac{dv}{dx} = \frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a} \dots \text{Equation (3) } \left\{ \text{Since, } \frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a} \right\}$$

Now, from equation 1, we can find  $dy/dx$

$$\frac{dy}{dx} = x^n \frac{dv}{dx} + \log_a x \frac{du}{dx}$$

$$\frac{dy}{dx} = x^n \frac{1}{x \log_e a} + nx^{n-1} \log_a x \quad \left\{ \text{Using equation 2 \& 3} \right\}$$

$$\therefore \frac{dy}{dx} = x^{n-1} \left( n \log_a x + \frac{1}{\log a} \right)$$



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