EXERCISE 4.3

1. Find the cube roots of the following numbers by successive subtraction of numbers:
1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397, …
(i) 64
(ii) 512
(iii) 1728
Solution:
(i) 64
Let's perform subtraction
64 – 1 = 63
63 – 7 = 56
56 – 19 =37
37 – 37 = 0
Subtraction is performed 4 times.
∴ Cube root of 64 is 4.

(ii) 512
Let's perform subtraction
512 – 1 = 511
511 – 7 = 504
504 – 19 = 485
485 – 37 = 448
448 – 61 = 387
387 – 91 = 296
296 – 127 = 169
169 – 169 = 0
Subtraction is performed 8 times.
∴ Cube root of 512 is 8.

(iii) 1728
Let's perform subtraction
1728 – 1 = 1727
1727 – 7 = 1720
1720 – 19 = 1701
1701 – 37 = 1664
1664 – 91 = 1512
1512 – 127 = 1385

1385 – 169 = 1216
1216 – 217 = 999
999 – 271 = 728
728 – 331 = 397
397 – 397 = 0
Subtraction is performed 12 times.
∴ Cube root of 1728 is 12.

2. Using the method of successive subtraction examine whether or not the following numbers are perfect cubes:
(i) 130
(ii) 345
(iii) 792
(iv) 1331
Solution:
(i) 130
Let's perform subtraction
130 – 1 = 129
129 – 7 = 122
122 – 19 = 103
103 – 37 = 66
66 – 61 = 5
Next number to be subtracted is 91, which is greater than 5
∴130 is not a perfect cube.

(ii) 345
Let's perform subtraction
345 – 1 = 344
344 – 7 = 337
337 – 19 = 318
318 – 37 = 281
281 – 61 = 220
220 – 91 = 129
129 – 127 = 2
Next number to be subtracted is 169, which is greater than 2
∴ 345 is not a perfect cube

(iii) 792
Let's perform subtraction

792 – 1 = 791
791 – 7 = 784
784 – 19 = 765
765 – 37 = 728
728 – 61 = 667
667 – 91 = 576
576 – 127 = 449
449 – 169 = 280
280 – 217 = 63
Next number to be subtracted is 271, which is greater than 63
∴ 792 is not a perfect cube

(iv) 1331
Let's perform subtraction
1331 – 1 = 1330
1330 – 7 = 1323
1323 – 19 = 1304
1304 – 37 = 1267
1267 – 61 = 1206
1206 – 91 = 1115
1115 – 127 = 988
988 – 169 = 819
819 – 217 = 602
602 – 271 = 331
331 – 331 = 0
Subtraction is performed 11 times
Cube root of 1331 is 11
∴ 1331 is a perfect cube.

3. Find the smallest number that must be subtracted from those of the numbers in question 2 which are not perfect cubes, to make them perfect cubes. What are the corresponding cube roots?
Solution:
In previous question there are three numbers which are not perfect cubes.
(i) 130
Let's perform subtraction
130 – 1 = 129
129 – 7 = 122
122 – 19 = 103

103 – 37 = 66
66 – 61 = 5
Next number to be subtracted is 91, which is greater than 5
Since, 130 is not a perfect cube. So, to make it perfect cube we subtract 5 from the given number.
130 – 5 = 125 (which is a perfect cube of 5)

(ii) 345
Let's perform subtraction
345 – 1 = 344
344 – 7 = 337
337 – 19 = 318
318 – 37 = 281
281 – 61 = 220
220 – 91 = 129
129 – 127 = 2
Next number to be subtracted is 169, which is greater than 2
Since, 345 is not a perfect cube. So, to make it a perfect cube we subtract 2 from the given number.
345 – 2 = 343 (which is a perfect cube of 7)

(iii) 792
Let's perform subtraction
792 – 1 = 791
791 – 7 = 784
784 – 19 = 765
765 – 37 = 728
728 – 61 = 667
667 – 91 = 576
576 – 127 = 449
449 – 169 = 280
280 – 217 = 63
Next number to be subtracted is 271, which is greater than 63
Since, 792 is not a perfect cube. So, to make it a perfect cube we subtract 63 from the given number.
792 – 63 = 729 (which is a perfect cube of 9)

4. Find the cube root of each of the following natural numbers:
(i) 343 (ii) 2744

(iii) 4913 (iv) 1728
(v) 35937 (vi) 17576
(vii) 134217728 (viii) 48228544
(ix) 74088000 (x) 157464
(xi) 1157625 (xii) 33698267
Solution:
(i) 343
By using prime factorization method
$\sqrt[3]{343} = \sqrt[3]{(7\times7\times7)} = 7$

(ii) 2744
By using prime factorization method
$\sqrt[3]{2744} = \sqrt[3]{(2\times2\times2\times7\times7\times7)} = \sqrt[3]{(2^3\times7^3)} = 2\times7 = 14$

(iii) 4913
By using prime factorization method,
$\sqrt[3]{4913} = \sqrt[3]{(17\times17\times17)} = 17$

(iv) 1728
By using prime factorization method,
$\sqrt[3]{1728} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times3\times3\times3)} = \sqrt[3]{(2^3\times2^3\times3^3)} = 2\times2\times3 = 12$

(v) 35937
By using prime factorization method,
$\sqrt[3]{35937} = \sqrt[3]{(3\times3\times3\times11\times11\times11)} = \sqrt[3]{(3^3\times11^3)} = 3\times11 = 33$

(vi) 17576
By using prime factorization method,
$\sqrt[3]{17576} = \sqrt[3]{(2\times2\times2\times13\times13\times13)} = \sqrt[3]{(2^3\times13^3)} = 2\times13 = 26$

(vii) 134217728
By using prime factorization method
$\sqrt[3]{134217728} = \sqrt[3]{(2^{27})} = 2^9 = 512$

(viii) 48228544
By using prime factorization method
$\sqrt[3]{48228544} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times7\times7\times7\times13\times13\times13)} = \sqrt[3]{(2^3\times2^3\times7^3\times13^3)} = 2\times2\times7\times13 = 364$

(ix) 74088000
By using prime factorization method
$\sqrt[3]{74088000} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times3\times3\times3\times5\times5\times5\times7\times7\times7)} = \sqrt[3]{(2^3\times2^3\times3^3\times5^3\times7^3)} =$
$2\times2\times3\times5\times7 = 420$

(x) 157464
By using prime factorization method
$\sqrt[3]{157464} = \sqrt[3]{(2\times2\times2\times3\times3\times3\times3\times3\times3\times3\times3\times3)} = \sqrt[3]{(2^3\times3^3\times3^3\times3^3)} = 2\times3\times3\times3 = 54$

(xi) 1157625
By using prime factorization method
$\sqrt[3]{1157625} = \sqrt[3]{(3\times3\times3\times5\times5\times5\times7\times7\times7)} = \sqrt[3]{(3^3\times5^3\times7^3)} = 3\times5\times7 = 105$

(xii) 33698267
By using prime factorization method
$\sqrt[3]{33698267} = \sqrt[3]{(17\times17\times17\times19\times19\times19)} = \sqrt[3]{(17^3\times19^3)} = 17\times19 = 323$

5. Find the smallest number which when multiplied with 3600 will make the product a perfect cube. Further, find the cube root of the product.
Solution:
Firstly let's find the prime factors for 3600
$3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
$= 2^3 \times 3^2 \times 5^2 \times 2$
Since only one triples is formed and three factors remained ungrouped in triples.
The given number 3600 is not a perfect cube.
To make it a perfect cube we have to multiply it by $(2 \times 2 \times 3 \times 5) = 60$
$3600 \times 60 = 216000$
Cube root of 216000 is
$\sqrt[3]{216000} = \sqrt[3]{(60\times60\times60)} = \sqrt[3]{(60^3)} = 60$
∴ the smallest number which when multiplied with 3600 will make the product a perfect cube is 60 and the cube root of the product is 60.

6. Multiply 210125 by the smallest number so that the product is a perfect cube. Also, find out the cube root of the product.
Solution:
The prime factors of 210125 are
$210125 = 5 \times 5 \times 5 \times 41 \times 41$
Since, one triples remained incomplete, 210125 is not a perfect cube.
To make it a perfect cube we need to multiply the factors by 41, we will get 2 triples as

23 and 41^3.
And the product become:
$210125 \times 41 = 8615125$
$8615125 = 5 \times 5 \times 5 \times 41 \times 41 \times 41$
Cube root of product = $\sqrt[3]{8615125} = \sqrt[3]{}(5\times41) = 205$

7. What is the smallest number by which 8192 must be divided so that quotient is a perfect cube? Also, find the cube root of the quotient so obtained.
Solution:
The prime factors of 8192 are
$8192 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2 = 2^3\times2^3\times2^3\times2$
Since, one triples remain incomplete, hence 8192 is not a perfect cube.
So, we divide 8192 by 2 to make its quotient a perfect cube.
$8192/2 = 4096$
$4096 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2 = 2^3\times2^3\times2^3\times2^3$
Cube root of 4096 = $\sqrt[3]{4096} = \sqrt[3]{}(2^3\times2^3\times2^3\times2^3) = 2\times2\times2\times2 = 16$

8. Three numbers are in the ratio 1:2:3. The sum of their cubes is 98784. Find the numbers.
Solution:
Let us consider the ratio 1:2:3 as x, 2x and 3x
According to the question,
$X^3 + (2x)^3 + (3x)^3 = 98784$
$x^3 + 8x^3 + 27x^3 = 98784$
$36x^3 = 98784$
$x^3 = 98784/36$
$= 2744$
$x = \sqrt[3]{2744} = \sqrt[3]{}(2\times2\times2\times7\times7\times7) = 2\times7 = 14$
So, the numbers are,
$x = 14$
$2x = 2 \times 14 = 28$
$3x = 3 \times 14 = 42$

9. **The volume of a cube is 9261000 m^3. Find the side of the cube.**
Given, volume of cube = 9261000 m^3
Let us consider the side of cube be ‘a’ metre
So, $a^3 = 9261000$
$a = \sqrt[3]{9261000} = \sqrt[3]{}(2\times2\times2\times3\times3\times3\times5\times5\times5\times7\times7\times7) = \sqrt[3]{}(2^3\times3^3\times5^3\times7^3) = 2\times3\times5\times7 = 210$
∴ the side of cube = 210 metre