

EXERCISE 29.7

Evaluate the following limits:

$$1. \lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

The limit $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

Let us consider the limit:

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{5x} = \frac{1}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

Now let us multiply and divide the expression by 3, we get

$$\begin{aligned} &= \frac{1}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \times 3 \\ &= \frac{3}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \end{aligned}$$

Now, put $3x = y$

$$= \frac{3}{5} \lim_{y \rightarrow 0} \frac{\sin y}{y} \quad \left[\text{We know that, } \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

So,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 3x}{5x} &= \frac{3}{5} \lim_{y \rightarrow 0} \frac{\sin y}{y} \\ &= \frac{3}{5} \times 1 \\ &= \frac{3}{5} \end{aligned}$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} \frac{\sin 3x}{5x} = \frac{3}{5}$$

$$2. \lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$

The limit $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$

We know, $1^\circ = \frac{x}{180}$ radians

So,

$$x^\circ = \frac{\pi x}{180} \text{ radians}$$

Let us consider the limit,

$$\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} = \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{180}}{x}$$

Now let us multiply and divide the expression by $\frac{\pi}{180}$, we get

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{180} \times \frac{\pi}{180}}{x \times \frac{\pi}{180}} \\ &= \frac{\pi}{180} \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \end{aligned}$$

Now, put $\frac{\pi x}{180} = y$

$$= \frac{\pi}{180} \lim_{y \rightarrow 0} \frac{\sin y}{y} \quad \left[\text{We know that, } \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} &= \frac{\pi}{180} \lim_{y \rightarrow 0} \frac{\sin y}{y} \\ &= \frac{\pi}{180} \times 1 \\ &= \frac{\pi}{180} \end{aligned}$$

\therefore The value of $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180}$

3.
$$\lim_{x \rightarrow 0} \frac{x^2}{\sin x^2}$$

Solution:

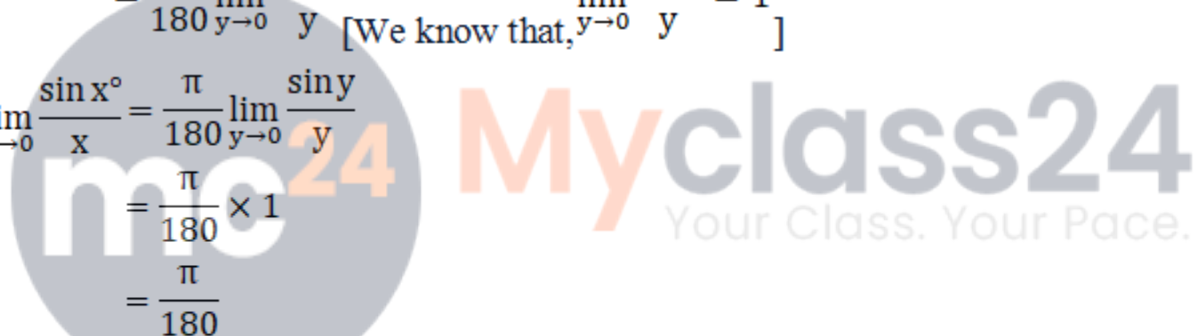
Given:
$$\lim_{x \rightarrow 0} \frac{x^2}{\sin x^2}$$

The limit

Let us consider the limit and divide the expression by x^2 , we get

$$\lim_{x \rightarrow 0} \frac{x^2}{\sin x^2} = \lim_{x \rightarrow 0} \frac{1}{\frac{\sin x^2}{x^2}}$$

Now, put $x^2 = y$



$$\lim_{x \rightarrow 0} \frac{1}{\frac{\sin x^2}{x^2}} = \frac{1}{\lim_{y \rightarrow 0} \frac{\sin y}{y}} \quad \left[\text{We know that, } \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

$$= \frac{1}{1}$$

$$= 1$$

∴ The value of $\lim_{x \rightarrow 0} \frac{x^2}{\sin x^2} = 1$

4. $\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x}$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x}$

The limit

Let us consider the limit

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin x \cos x}{x}$$

$$= \frac{1}{3} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \cos x$$

We know,

$$\lim_{x \rightarrow 0} A(x) \cdot B(x) = \lim_{x \rightarrow 0} A(x) \times \lim_{x \rightarrow 0} B(x)$$

So,

$$= \frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \cos x \quad \left[\text{We know that, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \frac{1}{3} \times 1 \times \cos 0$$

$$= \frac{1}{3} \times 1 \times 1 \quad \left[\text{Since, } \cos 0 = 1 \right]$$

$$= \frac{1}{3}$$

∴ The value of $\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \frac{1}{3}$

$$5. \lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x}$

The limit $\lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x}$

We know that, $\sin 3x = 3 \sin x - 4 \sin^3 x$

So,

$$\lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

Now multiply and divide the expression by 3, we get

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 3x}{x} &= \lim_{x \rightarrow 0} \frac{\sin 3x \times 3}{3x} \\ &= 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \end{aligned}$$

Now, put $3x = y$

$$= 3 \lim_{y \rightarrow 0} \frac{\sin y}{y} \quad \left[\text{We know that, } \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x} &= 3 \lim_{y \rightarrow 0} \frac{\sin y}{y} \\ &= 3 \times 1 \\ &= 3 \end{aligned}$$

\therefore The value of $\lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x} = 3$

