

EXERCISE 14.2

Solving the following quadratic equations by factorization method:

(i) $x^2 + 10ix - 21 = 0$

(ii) $x^2 + (1 - 2i)x - 2i = 0$

(iii) $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

(iv) $6x^2 - 17ix - 12 = 0$

Solution:

(i) $x^2 + 10ix - 21 = 0$

Given: $x^2 + 10ix - 21 = 0$

$$x^2 + 10ix - 21 \times 1 = 0$$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$x^2 + 10ix - 21(-i^2) = 0$$

$$x^2 + 10ix + 21i^2 = 0$$

$$x^2 + 3ix + 7ix + 21i^2 = 0$$

$$x(x + 3i) + 7i(x + 3i) = 0$$

$$(x + 3i)(x + 7i) = 0$$

$$x + 3i = 0 \text{ or } x + 7i = 0$$

$$x = -3i \text{ or } -7i$$

∴ The roots of the given equation are $-3i, -7i$

(ii) $x^2 + (1 - 2i)x - 2i = 0$

Given: $x^2 + (1 - 2i)x - 2i = 0$

$$x^2 + x - 2ix - 2i = 0$$

$$x(x + 1) - 2i(x + 1) = 0$$

$$(x + 1)(x - 2i) = 0$$

$$x + 1 = 0 \text{ or } x - 2i = 0$$

$$x = -1 \text{ or } 2i$$

∴ The roots of the given equation are $-1, 2i$

(iii) $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

Given: $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

$$x^2 - (2\sqrt{3}x + 3ix) + 6\sqrt{3}i = 0$$

$$x^2 - 2\sqrt{3}x - 3ix + 6\sqrt{3}i = 0$$

$$x(x - 2\sqrt{3}) - 3i(x - 2\sqrt{3}) = 0$$

$$(x - 2\sqrt{3})(x - 3i) = 0$$

$$(x - 2\sqrt{3}) = 0 \text{ or } (x - 3i) = 0$$

$$x = 2\sqrt{3} \text{ or } x = 3i$$

∴ The roots of the given equation are $2\sqrt{3}$, $3i$

(iv) $6x^2 - 17ix - 12 = 0$

Given: $6x^2 - 17ix - 12 = 0$

$6x^2 - 17ix - 12 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$6x^2 - 17ix - 12(-i^2) = 0$

$6x^2 - 17ix + 12i^2 = 0$

$6x^2 - 9ix - 8ix + 12i^2 = 0$

$3x(2x - 3i) - 4i(2x - 3i) = 0$

$(2x - 3i)(3x - 4i) = 0$

$2x - 3i = 0$ or $3x - 4i = 0$

$2x = 3i$ or $3x = 4i$

$x = 3i/2$ or $x = 4i/3$

∴ The roots of the given equation are $3i/2$, $4i/3$

2. Solve the following quadratic equations:

(i) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

(ii) $x^2 - (5 - i)x + (18 + i) = 0$

(iii) $(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$

(iv) $x^2 - (2 + i)x - (1 - 7i) = 0$

(v) $ix^2 - 4x - 4i = 0$

(vi) $x^2 + 4ix - 4 = 0$

(vii) $2x^2 + \sqrt{15}ix - i = 0$

(viii) $x^2 - x + (1 + i) = 0$

(ix) $ix^2 - x + 12i = 0$

(x) $x^2 - (3\sqrt{2} - 2i)x - \sqrt{2}i = 0$

(xi) $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

(xii) $2x^2 - (3 + 7i)x + (9i - 3) = 0$

Solution:

(i) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Given: $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

$x^2 - (3\sqrt{2}x + 2ix) + 6\sqrt{2}i = 0$

$x^2 - 3\sqrt{2}x - 2ix + 6\sqrt{2}i = 0$

$x(x - 3\sqrt{2}) - 2i(x - 3\sqrt{2}) = 0$

$(x - 3\sqrt{2})(x - 2i) = 0$

$(x - 3\sqrt{2}) = 0$ or $(x - 2i) = 0$

$x = 3\sqrt{2}$ or $x = 2i$

∴ The roots of the given equation are $3\sqrt{2}, 2i$

(ii) $x^2 - (5 - i)x + (18 + i) = 0$

Given: $x^2 - (5 - i)x + (18 + i) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Here, $a = 1, b = -(5-i), c = (18+i)$

So,

$$\begin{aligned} x &= \frac{-(-5 - i) \pm \sqrt{(-5 - i)^2 - 4(1)(18 + i)}}{2(1)} \\ &= \frac{(5 - i) \pm \sqrt{(5 - i)^2 - 4(18 + i)}}{2} \\ &= \frac{(5 - i) \pm \sqrt{25 - 10i + i^2 - 72 - 4i}}{2} \\ &= \frac{(5 - i) \pm \sqrt{-47 - 14i + i^2}}{2} \end{aligned}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned} &= \frac{(5 - i) \pm \sqrt{-47 - 14i + (-1)}}{2} \\ &= \frac{(5 - i) \pm \sqrt{-48 - 14i}}{2} \\ &= \frac{(5 - i) \pm \sqrt{(-1)(48 + 14i)}}{2} \\ &= \frac{(5 - i) \pm \sqrt{i^2(48 + 14i)}}{2} \\ &= \frac{(5 - i) \pm i\sqrt{48 + 14i}}{2} \end{aligned}$$

We can write $48 + 14i = 49 - 1 + 14i$

So,

$$\begin{aligned} 48 + 14i &= 49 + i^2 + 14i \quad [\because i^2 = -1] \\ &= 7^2 + i^2 + 2(7)(i) \\ &= (7 + i)^2 \quad [\text{Since, } (a + b)^2 = a^2 + b^2 + 2ab] \end{aligned}$$

By using the result $48 + 14i = (7 + i)^2$, we get

$$\begin{aligned}
 &= \frac{(5 - i) \pm i\sqrt{(7 + i)^2}}{2} \\
 &= \frac{(5 - i) \pm i(7 + i)}{2} \\
 &= \frac{(5 - i) + i(7 + i)}{2} \text{ or } \frac{(5 - i) - i(7 + i)}{2} \\
 &= \frac{5 - i + 7i + i^2}{2} \text{ or } \frac{5 - i - 7i - i^2}{2} \\
 &= \frac{5 + 6i + (-1)}{2} \text{ or } \frac{5 - 8i - (-1)}{2} \\
 &= \frac{5 + 6i - 1}{2} \text{ or } \frac{5 - 8i + 1}{2} \\
 &= \frac{4 + 6i}{2} \text{ or } \frac{6 - 8i}{2} \\
 &= \frac{2(2 + 3i)}{2} \text{ or } \frac{2(3 - 4i)}{2}
 \end{aligned}$$

$$x = 2 + 3i \text{ or } 3 - 4i$$

∴ The roots of the given equation are $3 - 4i, 2 + 3i$

(iii) $(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$

Given: $(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Here, $a = (2 + i), b = -(5 - i), c = 2(1 - i)$

So,

$$\begin{aligned}
 x &= \frac{-(-(5 - i)) \pm \sqrt{(-(5 - i))^2 - 4(2 + i)(2(1 - i))}}{2(2 + i)} \\
 &= \frac{(5 - i) \pm \sqrt{(5 - i)^2 - 8(2 + i)(1 - i)}}{2(2 + i)} \\
 &= \frac{(5 - i) \pm \sqrt{25 - 10i + i^2 - 8(2 - 2i + i - i^2)}}{2(2 + i)}
 \end{aligned}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned}
 &= \frac{(5 - i) \pm \sqrt{25 - 10i + (-1) - 8(2 - i - (-1))}}{2(2 + i)} \\
 &= \frac{(5 - i) \pm \sqrt{24 - 10i - 8(3 - i)}}{2(2 + i)} \\
 &= \frac{(5 - i) \pm \sqrt{24 - 10i - 24 + 8i}}{2(2 + i)} \\
 &= \frac{(5 - i) \pm \sqrt{-2i}}{2(2 + i)}
 \end{aligned}$$

We can write $-2i = -2i + 1 - 1$

$$-2i = -2i + 1 + i^2 \text{ [Since, } i^2 = -1]$$

$$= 1 - 2i + i^2$$

$$= 1^2 - 2(1)(i) + i^2$$

$$= (1 - i)^2 \text{ [By using the formula, } (a - b)^2 = a^2 - 2ab + b^2]$$

By using the result $-2i = (1 - i)^2$, we get

$$\begin{aligned}
 x &= \frac{(5 - i) \pm \sqrt{(1 - i)^2}}{2(2 + i)} \\
 &= \frac{(5 - i) \pm (1 - i)}{2(2 + i)} \\
 &= \frac{(5 - i) + (1 - i)}{2(2 + i)} \text{ or } \frac{(5 - i) - (1 - i)}{2(2 + i)} \\
 &= \frac{5 - i + 1 - i}{2(2 + i)} \text{ or } \frac{5 - i - 1 + i}{2(2 + i)} \\
 &= \frac{6 - 2i}{2(2 + i)} \text{ or } \frac{4}{2(2 + i)} \\
 &= \frac{3 - i}{2 + i} \text{ or } \frac{2}{2 + i}
 \end{aligned}$$

Let us multiply and divide by $(2 - i)$, we get

$$\begin{aligned}
 &= \frac{3 - i}{2 + i} \times \frac{2 - i}{2 - i} \text{ or } \frac{2}{2 + i} \times \frac{2 - i}{2 - i} \\
 &= \frac{(3 - i)(2 - i)}{(2 + i)(2 - i)} \text{ or } \frac{2(2 - i)}{(2 + i)(2 - i)} \\
 &= \frac{6 - 3i - 2i + i^2}{2^2 - i^2} \text{ or } \frac{4 - 2i}{2^2 - i^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{6-5i+(-1)}{4-(-1)} \text{ or } \frac{4-2i}{4-(-1)} \\
 &= \frac{5-5i}{4+1} \text{ or } \frac{4-2i}{4+1} \\
 &= \frac{5(1-i)}{5} \text{ or } \frac{4-2i}{5}
 \end{aligned}$$

$$x = (1 - i) \text{ or } 4/5 - 2i/5$$

∴ The roots of the given equation are $(1 - i)$, $4/5 - 2i/5$

(iv) $x^2 - (2 + i)x - (1 - 7i) = 0$

Given: $x^2 - (2 + i)x - (1 - 7i) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Here, $a = 1$, $b = -(2+i)$, $c = -(1-7i)$

So,

$$\begin{aligned}
 x &= \frac{-(-(2+i)) \pm \sqrt{(-(2+i))^2 - 4(1)(-(1-7i))}}{2(1)} \\
 &= \frac{(2+i) \pm \sqrt{(2+i)^2 + 4(1-7i)}}{2} \\
 &= \frac{(2+i) \pm \sqrt{4+4i+i^2+4-28i}}{2} \\
 &= \frac{(2+i) \pm \sqrt{8-24i+i^2}}{2}
 \end{aligned}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned}
 &= \frac{(2+i) \pm \sqrt{8-24i+(-1)}}{2} \\
 &= \frac{(2+i) \pm \sqrt{7-24i}}{2}
 \end{aligned}$$

We can write $7 - 24i = 16 - 9 - 24i$

$$7 - 24i = 16 + 9(-1) - 24i$$

$$= 16 + 9i^2 - 24i \quad [\because i^2 = -1]$$

$$= 4^2 + (3i)^2 - 2(4)(3i)$$

$$= (4 - 3i)^2 \quad [\because (a - b)^2 = a^2 - b^2 + 2ab]$$

By using the result $7 - 24i = (4 - 3i)^2$, we get

$$\begin{aligned} x &= \frac{(2 + i) \pm \sqrt{(4 - 3i)^2}}{2} \\ &= \frac{(2 + i) \pm (4 - 3i)}{2} \\ &= \frac{(2 + i) + (4 - 3i)}{2} \text{ or } \frac{(2 + i) - (4 - 3i)}{2} \\ &= \frac{2 + i + 4 - 3i}{2} \text{ or } \frac{2 + i - 4 + 3i}{2} \\ &= \frac{6 - 2i}{2} \text{ or } \frac{-2 + 4i}{2} \\ &= \frac{2(3 - i)}{2} \text{ or } \frac{2(-1 + 2i)}{2} \end{aligned}$$

$$x = 3 - i \text{ or } -1 + 2i$$

∴ The roots of the given equation are $(-1 + 2i)$, $(3 - i)$

(v) $ix^2 - 4x - 4i = 0$

Given: $ix^2 - 4x - 4i = 0$

$ix^2 + 4x(-1) - 4i = 0$ [We know, $i^2 = -1$]

So by substituting $-1 = i^2$ in the above equation, we get

$$ix^2 + 4xi^2 - 4i = 0$$

$$i(x^2 + 4ix - 4) = 0$$

$$x^2 + 4ix - 4 = 0$$

$$x^2 + 4ix + 4(-1) = 0$$

$$x^2 + 4ix + 4i^2 = 0$$
 [Since, $i^2 = -1$]

$$x^2 + 2ix + 2ix + 4i^2 = 0$$

$$x(x + 2i) + 2i(x + 2i) = 0$$

$$(x + 2i)(x + 2i) = 0$$

$$(x + 2i)^2 = 0$$

$$x + 2i = 0$$

$$x = -2i, -2i$$

∴ The roots of the given equation are $-2i, -2i$

(vi) $x^2 + 4ix - 4 = 0$

Given: $x^2 + 4ix - 4 = 0$

$$x^2 + 4ix + 4(-1) = 0 \text{ [We know, } i^2 = -1\text{]}$$

So by substituting $-1 = i^2$ in the above equation, we get

$$x^2 + 4ix + 4i^2 = 0$$

$$x^2 + 2ix + 2ix + 4i^2 = 0$$

$$x(x + 2i) + 2i(x + 2i) = 0$$

$$(x + 2i)(x + 2i) = 0$$

$$(x + 2i)^2 = 0$$

$$x + 2i = 0$$

$$x = -2i, -2i$$

∴ The roots of the given equation are $-2i, -2i$

(vii) $2x^2 + \sqrt{15}ix - i = 0$

Given: $2x^2 + \sqrt{15}ix - i = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Here, $a = 2, b = \sqrt{15}i, c = -i$

So,

$$x = \frac{-\sqrt{15}i \pm \sqrt{(\sqrt{15}i)^2 - 4(2)(-i)}}{2(2)}$$

$$= \frac{-\sqrt{15}i \pm \sqrt{15i^2 + 8i}}{4}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{-\sqrt{15}i \pm \sqrt{15(-1) + 8i}}{4}$$

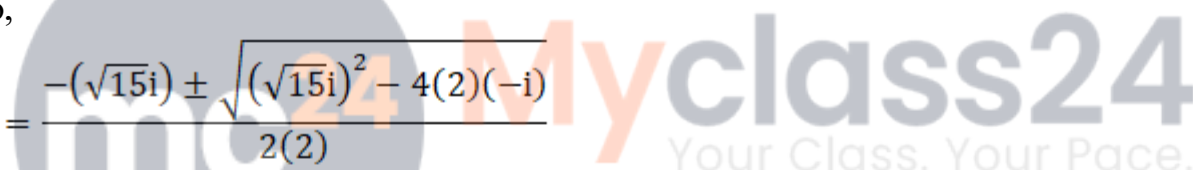
$$= \frac{-\sqrt{15}i \pm \sqrt{8i - 15}}{4}$$

$$= \frac{-\sqrt{15}i \pm \sqrt{(-1)(15 - 8i)}}{4}$$

$$= \frac{-\sqrt{15}i \pm \sqrt{i^2(15 - 8i)}}{4}$$

$$= \frac{-\sqrt{15}i \pm i\sqrt{15 - 8i}}{4}$$

We can write $15 - 8i = 16 - 1 - 8i$



$$\begin{aligned}
 15 - 8i &= 16 + (-1) - 8i \\
 &= 16 + i^2 - 8i \quad [\because i^2 = -1] \\
 &= 4^2 + (i)^2 - 2(4)(i) \\
 &= (4 - i)^2 \quad [\text{Since, } (a - b)^2 = a^2 - b^2 + 2ab]
 \end{aligned}$$

By using the result $15 - 8i = (4 - i)^2$, we get

$$\begin{aligned}
 x &= \frac{-\sqrt{15i} \pm i\sqrt{(4 - i)^2}}{4} \\
 &= \frac{-\sqrt{15i} \pm i(4 - i)}{4} \\
 &= \frac{-\sqrt{15i} + i(4 - i)}{4} \quad \text{or} \quad \frac{-\sqrt{15i} - i(4 - i)}{4} \\
 &= \frac{-\sqrt{15i} + 4i - i^2}{4} \quad \text{or} \quad \frac{-\sqrt{15i} - 4i + i^2}{4} \\
 &= \frac{-\sqrt{15i} + 4i - (-1)}{4} \quad \text{or} \quad \frac{-\sqrt{15i} - 4i + (-1)}{4} \\
 &= \frac{-\sqrt{15i} + 4i + 1}{4} \quad \text{or} \quad \frac{-\sqrt{15i} - 4i - 1}{4} \\
 &= \frac{1 + (4 - \sqrt{15})i}{4} \quad \text{or} \quad \frac{-1 - (4 + \sqrt{15})i}{4}
 \end{aligned}$$

\therefore The roots of the given equation are $[1 + (4 - \sqrt{15})i/4]$, $[-1 - (4 + \sqrt{15})i/4]$

(viii) $x^2 - x + (1 + i) = 0$

Given: $x^2 - x + (1 + i) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Here, $a = 1$, $b = -1$, $c = (1 + i)$

So,

$$\begin{aligned}
 x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1 + i)}}{2(1)} \\
 &= \frac{1 \pm \sqrt{1 - 4(1 + i)}}{2}
 \end{aligned}$$

$$= \frac{1 \pm \sqrt{1 - 4 - 4i}}{2}$$

$$= \frac{1 \pm \sqrt{-3 - 4i}}{2}$$

$$= \frac{1 \pm \sqrt{(-1)(3 + 4i)}}{2}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{1 \pm \sqrt{i^2(3 + 4i)}}{2}$$

$$= \frac{1 \pm i\sqrt{3 + 4i}}{2}$$

We can write $3 + 4i = 4 - 1 + 4i$

$$3 + 4i = 4 + i^2 + 4i \quad [\because i^2 = -1]$$

$$= 2^2 + i^2 + 2(2)(i)$$

$$= (2 + i)^2 \quad [\text{Since, } (a + b)^2 = a^2 + b^2 + 2ab]$$

By using the result $3 + 4i = (2 + i)^2$, we get

$$x = \frac{1 \pm i\sqrt{(2 + i)^2}}{2}$$

$$= \frac{1 \pm i(2 + i)}{2}$$

$$= \frac{1 + i(2 + i)}{2} \quad \text{or} \quad \frac{1 - i(2 + i)}{2}$$

$$= \frac{1 + 2i + i^2}{2} \quad \text{or} \quad \frac{1 - 2i - i^2}{2}$$

$$= \frac{1 + 2i + (-1)}{2} \quad \text{or} \quad \frac{1 - 2i - (-1)}{2}$$

$$= \frac{1 + 2i - 1}{2} \quad \text{or} \quad \frac{1 - 2i + 1}{2}$$

$$x = 2i/2 \quad \text{or} \quad (2 - 2i)/2$$

$$x = i \quad \text{or} \quad 2(1-i)/2$$

$$x = i \quad \text{or} \quad (1 - i)$$

\therefore The roots of the given equation are $(1-i), i$

(ix) $ix^2 - x + 12i = 0$

Given: $ix^2 - x + 12i = 0$

$$ix^2 + x(-1) + 12i = 0 \text{ [We know, } i^2 = -1]$$

so by substituting $-1 = i^2$ in the above equation, we get

$$ix^2 + xi^2 + 12i = 0$$

$$i(x^2 + ix + 12) = 0$$

$$x^2 + ix + 12 = 0$$

$$x^2 + ix - 12(-1) = 0$$

$$x^2 + ix - 12i^2 = 0 \text{ [Since, } i^2 = -1]$$

$$x^2 - 3ix + 4ix - 12i^2 = 0$$

$$x(x - 3i) + 4i(x - 3i) = 0$$

$$(x - 3i)(x + 4i) = 0$$

$$x - 3i = 0 \text{ or } x + 4i = 0$$

$$x = 3i \text{ or } -4i$$

∴ The roots of the given equation are $-4i, 3i$

$$(x) x^2 - (3\sqrt{2} - 2i)x - \sqrt{2}i = 0$$

$$\text{Given: } x^2 - (3\sqrt{2} - 2i)x - \sqrt{2}i = 0$$

We shall apply discriminant rule,

$$\text{Where, } x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\text{Here, } a = 1, b = -(3\sqrt{2} - 2i), c = -\sqrt{2}i$$

So,

$$x = \frac{-(-(3\sqrt{2} - 2i)) \pm \sqrt{(-(3\sqrt{2} - 2i))^2 - 4(1)(-\sqrt{2}i)}}{2(1)}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{(3\sqrt{2} - 2i)^2 + 4\sqrt{2}i}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 12\sqrt{2}i + 4i^2 + 4\sqrt{2}i}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i + 4i^2}}{2}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i + 4(-1)}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i - 4}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{14 - 8\sqrt{2}i}}{2}$$

We can write $14 - 8\sqrt{2}i = 16 - 2 - 8\sqrt{2}i$

$$14 - 8\sqrt{2}i = 16 + 2(-1) - 8\sqrt{2}i$$

$$= 16 + 2i^2 - 8\sqrt{2}i \text{ [Since, } i^2 = -1]$$

$$= 4^2 + (\sqrt{2}i)^2 - 2(4)(\sqrt{2}i)$$

$$= (4 - \sqrt{2}i)^2 \text{ [By using the formula, } (a - b)^2 = a^2 - 2ab + b^2]$$

By using the result $14 - 8\sqrt{2}i = (4 - \sqrt{2}i)^2$, we get

$$x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{(4 - \sqrt{2}i)^2}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm (4 - \sqrt{2}i)}{2}$$

$$= \frac{3\sqrt{2} - 2i}{2} \pm \frac{4 - \sqrt{2}i}{2}$$

∴ The roots of the given equation are $\frac{3\sqrt{2} - 2i}{2} \pm \frac{4 - \sqrt{2}i}{2}$

(xi) $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

Given: $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

$$x^2 - (\sqrt{2}x + ix) + \sqrt{2}i = 0$$

$$x^2 - \sqrt{2}x - ix + \sqrt{2}i = 0$$

$$x(x - \sqrt{2}) - i(x - \sqrt{2}) = 0$$

$$(x - \sqrt{2})(x - i) = 0$$

$$(x - \sqrt{2}) = 0 \text{ or } (x - i) = 0$$

$$x = \sqrt{2} \text{ or } x = i$$

∴ The roots of the given equation are $i, \sqrt{2}$

(xii) $2x^2 - (3 + 7i)x + (9i - 3) = 0$

Given: $2x^2 - (3 + 7i)x + (9i - 3) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 2, b = -(3 + 7i), c = (9i - 3)$

So,

$$\begin{aligned}
 x &= \frac{-(-(3+7i)) \pm \sqrt{(-(3+7i))^2 - 4(2)(9i-3)}}{2(2)} \\
 &= \frac{(3+7i) \pm \sqrt{(3+7i)^2 - 8(9i-3)}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{9+42i+49i^2-72i+24}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{33-30i+49i^2}}{4}
 \end{aligned}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned}
 &= \frac{(3+7i) \pm \sqrt{33-30i+49(-1)}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{33-30i-49}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{-16-30i}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{(-1)(16+30i)}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{i^2(16+30i)}}{4} \\
 &= \frac{(3+7i) \pm i\sqrt{16+30i}}{4}
 \end{aligned}$$

We can write $16+30i = 25-9+30i$

$$\begin{aligned}
 16+30i &= 25+9(-1)+30i \\
 &= 25+9i^2+30i \quad [\because i^2 = -1] \\
 &= 5^2+(3i)^2+2(5)(3i) \\
 &= (5+3i)^2 \quad [\because (a+b)^2 = a^2+b^2+2ab]
 \end{aligned}$$

By using the result $16+30i = (5+3i)^2$, we get

$$\begin{aligned}
 x &= \frac{(3+7i) \pm i\sqrt{(5+3i)^2}}{4} \\
 &= \frac{(3+7i) \pm i(5+3i)}{4}
 \end{aligned}$$

$$\begin{aligned} &= \frac{(3 + 7i) + i(5 + 3i)}{4} \text{ or } \frac{(3 + 7i) - i(5 + 3i)}{4} \\ &= \frac{3 + 7i + 5i + 3i^2}{4} \text{ or } \frac{3 + 7i - 5i - 3i^2}{4} \\ &= \frac{3 + 12i + 3i^2}{4} \text{ or } \frac{3 + 2i - 3i^2}{4} \\ &= \frac{3 + 12i + 3(-1)}{4} \text{ or } \frac{3 + 2i - 3(-1)}{4} \\ &= \frac{3 + 12i - 3}{4} \text{ or } \frac{3 + 2i + 3}{4} \\ &= \frac{12}{4}i \text{ or } \frac{6 + 2i}{4} \\ &= 3i \text{ or } \frac{6}{4} + \frac{2}{4}i \\ x &= 3i \text{ or } \frac{3}{2} + \frac{1}{2}i \\ &= 3i \text{ or } (3 + i)/2 \end{aligned}$$

∴ The roots of the given equation are $(3 + i)/2, 3i$

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