$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=k\,(\text{constant})$$

Putting the values from the equation of a given parallel plane,

$$\frac{A_1}{5}=\frac{B_1}{4}=\frac{C_1}{-11}=k$$

$$A_1=5k, B_1=4k, C_1=-11k$$

Putting in the equation of a plane

$$5kx+4ky-11kz+D_1=0$$

As the plane is passing through (1,-2,7), it must satisfy the plane,

$$5k.1+4k.(-2)-11k.7+D_1=0 \text{ (1)}$$

$$5k-8k-77k+D_1=0$$

$$D_1=80k$$

Putting value in equation (1), we have

$$5kx+4ky-11kz+80k=0$$

5x + 4y-11z + 80=0

So, the required equation of the plane is 5x + 4y-11z + 80=0.

12. Question

Find the equation of the plane passing through the point A(-1, -1, 2) and perpendicular to each of the planes 3x + 2y - 3z = 1 and 5x – 4y + z = 5.

Answer

Applying the condition of perpendicularity between planes

$$AA_1+BB_1+CC_1=0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of another plane.

$$3.A_1+2B_1-3C_1=0 \text{ (1)}$$

$$5.A_1-4B_1+C_1=0 \text{ (2)}$$

And plane passes through (-1,-1,2),

A(x + 1) + B(y + 1) + C(z-2)=0 (3)

On solving equation (1) and (2)

$$A=\frac{5B}{9} \text{ and } C=\frac{11B}{9}$$

Putting values in equation (3)

$$\frac{5B}{9}.(x+1)+B(y+1)+\frac{11B}{9}.(z-2)=0$$

B(5x + 5 + 9y + 9 + 11z-22)=0

5x + 9y + 11z-8=0

So, required equation of plane is 5x + 9y + 11z=8.

13. Question

Find the equation of the plane passing through the origin and perpendicular to each of the planes x + 2y - z = 1 and 3x - 4y + z = 5.

Answer

Applying condition of perpendicularity between planes,

$$AA_1+BB_1+CC_1=0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of other

plane.

$$1.A+2.B-1.C=0$$

$$A+2B-C=0 \text{ (1)}$$

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$$3.A-4.B+C=0$$

$$3A-4B+C=0 \quad (2)$$

And plane passes through (0, 0, 0),

A(x-0) + B(y-0) + C(z-0)=0

Ax + By + Cz=0 (3)

On solving equation (1) and (2)

$$A=\frac{B}{2} \text{ and } C=\frac{5B}{2}$$

Putting values in equation(3)

$$\frac{B}{2}.x+By+\frac{5B}{2}.z=0$$

B(x + 2y + 5z)=0

x + 2y + 5z=0

So, required equation of plane is x + 2y + 5z=0.

14. Question

Find the equation of the plane that contains the point A(1, -1, 2) and is perpendicular to both the planes 3x + 3y – 2z = 5 and x + 2y - 3z = 8. Hence, find the distance of the point P(-2, 5, 5) from the plane obtained above.

Answer

Applying condition of perpendicularity between planes,

$$AA_1+BB_1+CC_1=0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of other

plane.

$$3.A+3.B-2.C=0$$

$$3A+3B-2C=0 \quad (1)$$

$$1.A+2.B-3C=0$$

$$A+2B-3C=0 \quad (2)$$

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And plane contains the point (1,-1,2),

A(x-1) + B(y + 1) + C(z-2)=0 (3)

On solving equation (1) and (2)

$$A=\frac{-5B}{7} \text{ and } C=\frac{3B}{7}$$

Putting values in equation (3)

$$\frac{-5B}{7}.(x-1)+B(y+1)+\frac{3B}{7}.(z-2)=0$$

$$B\left(-5(x-1)+7(y+1)+3(z-2)\right)=0$$

-5x + 5 + 7y + 7 + 3z-6=0

-5x + 7y + 3z + 6=0

5x-7y-3z-6=0

For equation of plane Ax + By + Cz=D and point (x1,y1,z1), distance of a

point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{5.(-2)-7.5-3.5-6}{\sqrt{(5)^2+(-7)^2+(-3)^2)}}\right| \Rightarrow \left|\frac{-10-35-15-6}{\sqrt{25}+49+9}\right| = \left|\frac{-66}{\sqrt{83}}\right| \Rightarrow \frac{66}{\sqrt{83}}$$

15. Question

Find the equation of the plane passing through the points A(1, 1, 2) and B(2, -2, 2) and perpendicular to the plane 6x – 2y + 2z = 9.

Answer

Plane passes through (1,1,2) and (2,-2,2),

A(x-1) + B(y-1) + C(z-2)=0 (1)

A(x-2) + B(y + 2) + C(z-2)=0 (2)

Subtracting (1) from (2),

A(x-2-x + 1) + B(y + 2-y-1)=0

A-3B=0 (3)

Now plane is perpendicular to 6x-2y + 2z=9

6A-2B + 2C=0 (4)

Using (3) in (4)

18A-2B + 2C=0

16B + 2C=0

C=-8B

Putting values in equation (1)

3B(x-1) + B(y + 2)-8B(z-2)=0

B(3x-3 + y + 2-8z + 16)=0

3x + y-8z + 15=0

16. Question

Find the equation of the plane passing through the points A(-1, 1, 1) and B(1, -1, 1) and perpendicular to the plane x + 2y + 2z = 5.

Answer

Plane passes through (-1,1,1) and (1,-1,1),

A(x + 1) + B(y-1) + C(z-1)=0 (1)

A(x-1) + B(y + 1) + C(z-1)=0 (2)

Subtracting (1) from (2),

A(x-1-x-1) + B(y + 1-y + 1)=0

-2A + 2B=0

A=B (3)

Now plane is perpendicular to x + 2y + 2z=5

A + 2B + 2C=0 (4)

Using (3) in (4)

B + 2B + 2C=0

3B + 2C=0

$$C = \frac{-3}{2}B$$

Putting values in equation (1)

$$B(x+1)+B(y-1)+\frac{-3}{2}B(z-1)=0$$

B(2(x + 1) + 2(y-1)-3(z-1)=0

2x + 2y-3z + 2-2-3=0

2x + 2y-3z-3=0

17. Question

Find the equation of the plane through the points A(3, 4, 2) and B(7, 0, 6) and perpendicular to the plane 2x – 5y = 15.

HINT: The given plane is 2x – 5y + 0z = 15

Answer

Plane passes through (3,4,2) and (7,0,6),

A(x-3) + B(y-4) + C(z-2)=0 (1)

A(x-7) + B(y-0) + C(z-6)=0 (2)

Subtracting (1) from (2),

A(x-7-x + 3) + B(y-y + 4) + C(z-6-z + 2)=0

-4A + 4B-4C=0

A-B + C=0

B=A + C (3)

Now plane is perpendicular to 2x-5y=15

2A-5B=0 (4)

Using (3) in (4)

2A-5(A + C)=0

2A-5A-5C=0

-3A-5C=0

$$C=\frac{-3}{5}A$$

$$B=A+\frac{-3}{5}A\Rightarrow\frac{2}{5}A$$

Putting values in equation (1)

$$A\left(x-3\right)+\frac{2}{5}A\left(y-4\right)+\frac{-3}{5}A\left(z-2\right)=0$$

A(5(x-3) + 2(y-4)-3(z-2)=0

5x + 2y-3z-15-8 + 6=0

5x + 2y-3z-17=0

So, required equation of plane is 5x + 2y-3z-17=0.

18. Question

Find the equation of the plane through the points A(2, 1, -1) and B(-1, 3, 4) and perpendicular to the plane x – 2y + 4z = 10. Also, show that the plane thus obtained contains the line $\vec{r}=\left(-\hat{i}+3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}-2\hat{j}-5\hat{k}\right)$

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Answer

Plane passes through (2,1,-1) and (-1,3,4),

A(x-2) + B(y-1) + C(z + 1)=0 (1)

A(x + 1) + B(y-3) + C(z-4)=0 (2)

Subtracting (1) from (2),

A(x + 1-x + 2) + B(y-3-y + 1) + C(z-4-z-1)=0

3A-2B-5C=0 (3)

Now plane is perpendicular to x-2y + 4z=10

A-2B + 4C=0 (4)

Using (3) in (4)

2A-9C=0

$$C=\frac{2}{9}A$$

$$2B=A+4.\frac{2}{9}A\Rightarrow\left(\frac{9+8}{9}\right)A=\frac{17}{9}A$$

$$B=\frac{17}{18}A$$

Putting values in equation (1)

$$A\left(x-2\right)+\frac{17}{18}A\left(y-1\right)+\frac{2}{9}A\left(z+1\right)=0$$

A(18(x-2) + 17(y-1) + 4(z + 1)=0

18x + 17y + 4z-36-17 + 4=0

18x + 17y + 4z-49=0

So, the required equation of plane is 18x + 17y + 4z-49=0

If plane contains $\vec{r} = -\hat{i} + 3\hat{j} + 4\hat{k} + (3\hat{i} - 2\hat{j} - 5\hat{k})$ then $(-1, 3, 4)$ satisfies plane and normal vector of plane is perpendicular to vector of line

LHS=18(-1) + 17.3 + 4.4-49

=-18 + 51 + 16-49

=-2 + 2=0=RHS

In vector form normal of plane

$\vec{n} = 18\hat{i} + 17\hat{j} + 4\hat{k}$

LHS=18.3 + 17(-2) + 4.(-5)=54-34-20=0=RHS

Hence line is contained in plane.

Exercise 28G

1. Question

Find the angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$.

Answer

Given - $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) = 4$

To find – The angle between the line and the plane

Direction ratios of the line = (1, - 1, 1)

Direction ratios of the normal of the plane = (2, - 1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{1 \times 2 + (-1) \times (-1) + 1 \times 1}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{2^2 + 1^2 + 1^2}}\right)$

$= \sin^{-1}\left(\frac{2 + 1 + 1}{\sqrt{3}\sqrt{6}}\right)$

$= \sin^{-1}\left(\frac{4}{3\sqrt{2}}\right)$

$= \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

2. Question

Find the angle between the line $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$.

Answer

Given - $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 3$

To find – The angle between the line and the plane

Direction ratios of the line = (3, - 1, 2)

Direction ratios of the normal of the plane = (1, 1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{3 \times 1 + (-1) \times 1 + 2 \times 1}{\sqrt{3^2 + 1^2 + 2^2}\sqrt{1^2 + 1^2 + 1^2}}\right)$

$= \sin^{-1}\left(\frac{3 - 1 + 2}{\sqrt{14}\sqrt{3}}\right)$

$= \sin^{-1}\left(\frac{4}{\sqrt{42}}\right)$

3. Question

Find the angle between the line $\vec{r} = (3\hat{i} + \hat{k}) + \lambda(\hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 1$.

Answer

Given - $\vec{r} = (3\hat{i} + \hat{k}) + \lambda(\hat{j} + \hat{k})$ and $\vec{r}.(2\hat{i} - \hat{j} + 2\hat{k}) = 1$

To find – The angle between the line and the plane

Direction ratios of the line = (0, 1, 1)

Direction ratios of the normal of the plane = (2, - 1, 2)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{0 \times 2 + 1 \times (-1) + 1 \times 2}{\sqrt{0^2 + 1^2 + 1^2}\sqrt{2^2 + 1^2 + 2^2}}\right)$

$= \sin^{-1}\left(\frac{-1 + 2}{3\sqrt{2}}\right)$

$= \mathbf{\sin^{-1}\left(\frac{1}{3\sqrt{2}}\right)}$

4. Question

Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and the plane 3x + 4y + z + 5 = 0.

Answer

Given - $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and $3x + 4y + z + 5 = 0$

To find – The angle between the line and the plane

Direction ratios of the line = (3, - 1, 2)

Direction ratios of the normal of the plane = (3, 4, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{3 \times 3 + (-1) \times 4 + 2 \times 1}{\sqrt{3^2 + 1^2 + 2^2}\sqrt{3^2 + 4^2 + 1^2}}\right)$

$= \sin^{-1}\left(\frac{9 - 4 + 2}{\sqrt{14}\sqrt{26}}\right)$

$= \sin^{-1}\left(\frac{7}{\sqrt{2}\sqrt{7} \times \sqrt{2} \times \sqrt{13}}\right)$

$= \mathbf{\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)}$

5. Question

Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane 10x + 2y – 11z = 3.

Answer

Given - $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and $10x + 2y - 11z = 3$

To find – The angle between the line and the plane

Direction ratios of the line = (2, 3, 6)

Direction ratios of the normal of the plane = (10, 2, - 11)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{2\times 10 + 3\times 2 + 6\times(-11)}{\sqrt{2^2+3^2+6^2}\sqrt{10^2+2^2+11^2}}\right)$$

$$= \sin^{-1}\left(\frac{20 + 6 - 66}{7\times 15}\right)$$

$$= \sin^{-1}\left(\frac{-40}{7\times 15}\right)$$

$$= \mathbf{\sin^{-1}\left(-\frac{8}{21}\right)}$$

6. Question

Find the angle between the line joining the points A(3, - 4, - 2) and B(12, 2, 0) and the plane 3x – y + z = 1.

Answer

Given - A = (3, - 4, - 2) , B = (12, 2, 0) and 3x - y + z = 1

To find – The angle between the line joining the points A and B and the plane

Tip – If P = (a, b, c) and Q = (a', b', c'), then the direction ratios of the line PQ is given by ((a' - a), (b' - b), (c' - c))

The direction ratios of the line AB can be given by

((12 - 3), (2 + 4), (0 + 2))

= (9, 6, 2)

Direction ratios of the normal of the plane = (3, - 1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{9\times 3 + 6\times(-1) + 2\times 1}{\sqrt{9^2+6^2+2^2}\sqrt{3^2+1^2+1^2}}\right)$$

$$= \sin^{-1}\left(\frac{27 - 6 + 2}{11\times\sqrt{11}}\right)$$

$$= \mathbf{\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)}$$

7. Question

If the plane 2x – 3y – 6z = 13 makes an angle $\sin^{-1}(\lambda)$ with the x - axis, then find the value of λ.

Answer

Given - $y = z = 0$ and $2x - 3y - 6z = 13$

To find – The angle between the line and the plane

Direction ratios of the line = (1, 0, 0)

Direction ratios of the normal of the plane = (2, - 3, - 6)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{1\times 2 + 0\times(-3) + 0\times(-6)}{\sqrt{1^2+0^2+0^2}\sqrt{2^2+3^2+9^2}}\right)$$

$$= \mathbf{\sin^{-1}\left(\frac{2}{7}\right)}$$

8. Question

Show that the line $\vec{r} = \left(2\hat{i}+5\hat{j}+7\hat{k}\right)+\lambda\left(\hat{i}+3\hat{j}+4\hat{k}\right)$ is parallel to the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}-\hat{k}\right)=7$. Also, find the distance between them.

Answer

Given - $\vec{r} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and $\vec{r}.\left(\hat{i} + \hat{j} - \hat{k}\right) = 7$

To prove – The line and the plane are parallel &

To find – The distance between them

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Direction ratios of the line = (1, 3, 4)

Direction ratios of the normal of the plane = (1, 1, - 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{1\times 1 + 3\times 1 + 4\times(-1)}{\sqrt{1^2+3^2+4^2}\sqrt{1^2+1^2+1^2}}\right)$

$= \sin^{-1}\left(\frac{1+3-4}{\sqrt{26}\sqrt{3}}\right)$

$= \sin^{-1}(0)$

$= 0$

Hence, **the line and the plane are parallel.**

Now, the equation of the plane may be written as x + y - z = 7.

Tip – If ax + by + c + d = 0 be a plane and $\vec{r} = (a'\hat{i} + b'\hat{j} + c'\hat{k}) + \lambda(a''\hat{i} + b''\hat{j} + c''\hat{k})$ be a line vector, then the distance between them is given by $\left|\frac{a\times a' + b\times b' + c\times c' + d}{\sqrt{a^2+b^2+c^2}}\right|$

The distance between the plane and the line

$= \left|\frac{1\times 2 + 1\times 5 - 1\times 7 - 7}{\sqrt{1^2+1^2+1^2}}\right|$

$= \left|\frac{2+5-7-7}{\sqrt{3}}\right|$

$= \frac{7}{\sqrt{3}}$ **units**

9. Question

Find the value of m for which the line $\vec{r} = (\hat{i} + 2\hat{k}) + \lambda(2\hat{i} - m\hat{j} - 3\hat{k})$ is parallel to the plane $\vec{r}\cdot(m\hat{i} + 3\hat{j} + \hat{k}) = 4$.

Answer

Given - $\vec{r} = (\hat{i} + 2\hat{k}) + \lambda(2\hat{i} - m\hat{j} - 3\hat{k})$ and $\vec{r}.(m\hat{i} + 3\hat{j} + \hat{k}) = 4$ and they are parallel

To find – The value of m

Direction ratios of the line = (2, - m, - 3)

Direction ratios of the normal of the plane = (m, 3, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

$\therefore \sin^{-1}\left(\frac{2\times m + (-m)\times 3 + (-3)\times 1}{\sqrt{2^2+m^2+3^2}\sqrt{m^2+3^2+1^2}}\right) = 0$

$\Rightarrow \sin^{-1}\left(\frac{2m-3m-3}{\sqrt{13+m^2}\sqrt{10+m^2}}\right) = 0$

$\Rightarrow \frac{-m-3}{\sqrt{13+m^2}\sqrt{10+m^2}} = 0$

$\Rightarrow \mathbf{m = -3}$

10. Question

Find the vector equation of a line passing through the origin and perpendicular to the plane $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k}) = 3$.

Answer

Given - $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 3$

To find – The vector equation of the line passing through the origin and perpendicular to the given plane

Tip – The equation of a plane can be given by $\vec{r}.\hat{n} = d$ where $\hat{n}$ is the normal of the plane

A line parallel to the given plane will be in the direction of the normal and will have the direction ratios same as that of the normal.

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given

by $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ where λ is any scalar constant

The required equation will be $\vec{r} = (0.\hat{i} + 0.\hat{j} + 0.\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$= \boldsymbol{\lambda(\hat{i} + 2\hat{j} + 3\hat{k})}$ **for some scalar λ**

11. Question

Find the vector equation of the line passing through the point with position vector $\left(\hat{i}-2\hat{j}+5\hat{k}\right)$ and perpendicular to the plane

$\vec{r}\cdot\left(2\hat{i}-3\hat{j}-\hat{k}\right)=0.$

Answer

Given - $\vec{r}.(2\hat{i} - 3\hat{j} - \hat{k}) = 0$ and the vector has position vector $(\hat{i} - 2\hat{j} + 5\hat{k})$

To find – The vector equation of the line passing through (1, - 2, 5) and perpendicular to the given plane

Tip – The equation of a plane can be given by $\vec{r}.\hat{n} = d$ where $\hat{n}$ is the normal of the plane

A line parallel to the given plane will be in the direction of the normal and will have the direction ratios same as that of the normal.

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ where λ is any scalar constant

The required equation will be $\boldsymbol{\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} - 3\hat{j} - \hat{k})}$ **for some scalar λ**

12. Question

Show that the equation ax + by + d = 0 represents a plane parallel to the z - axis. Hence, find the equation of a plane which is parallel to the z - axis and passes through the points A(2, - 3, 1) and B(- 4, 7, 6).

Answer

Given – The equation of the plane is given by ax + by + d = 0

To prove – The plane is parallel to z - axis

Tip – If ax + by + cz + d is the equation of the plane then its angle with the z - axis will be given by $\sin^{-1}\left(\frac{c}{\sqrt{a^2+b^2+c^2}}\right)$

Considering the equation, the direction ratios of its normal is given by (a, b, 0)

The angle the plane makes with the z - axis = $\sin^{-1}[0/\sqrt{(a^2 + b^2)}] = 0$

Hence, **the plane is parallel to the z - axis**

To find – Equation of the plane parallel to z - axis and passing through points A = (2, - 3, 1) and B = (- 4, 7, 6)

The given equation ax + by + d = 0 passes through (2, - 3, 1) & (- 4, 7, 6)

$\therefore 2a - 3b + d = 0$.....(i)

$\therefore -4a + 7b + d = 0$.....(ii)

Solving (i) and (ii),

$\therefore \frac{a}{\begin{vmatrix}-3 & 1\\ 7 & 1\end{vmatrix}} = \frac{b}{\begin{vmatrix}1 & 2\\ 1 & -4\end{vmatrix}} = \alpha$ [α → arbitrary constant]

$\therefore a = -10\alpha$

$\therefore b = -6\alpha$

Substituting the values of a and b in eqn (i), we get,

- 2X10α + 3X6α + d = 0 i.e. d = - 2α

Putting the value of a, b and d in the equation ax + by + d = 0,

(- 10α)x + (- 6α)y + (- 2α) = 0

i.e. **5x + 3y + 1 = 0**

13. Question

Find the equation of the plane passing through the points (1, 2, 3) and (0, - 1, 0) and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}.$

Answer

Given – A plane passes through points (1, 2, 3) and (0, - 1, 0) and is parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

To find – Equation of the plane

Tip – If a plane passes through points (a', b', c'), then its equation may be given as a(x - a') + b(y - b') + c(z - c') = 0

Taking points (1, 2, 3):

a(x - 1) + b(y - 2) + c(z - 3) = 0.....(i)

The plane passes through (0, - 1, 0):

a(0 - 1) + b(- 1 - 2) + c(0 - 3) = 0

i.e. a + 3b + 3c = 0.....(ii)

The plane is parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

Tip – The normal of the plane will be normal to the given line since both the line and plane are parallel.

Direction ratios of the line is (2, 3, - 3)

Direction ratios of the normal of the plane is (a, b, c)

So, 2a + 3b - 3c = 0.....(iii)

Solving equations (ii) and (iii),

$\therefore \frac{a}{\begin{vmatrix} 3 & 3 \\ 3 & -3 \end{vmatrix}} = -\frac{b}{\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix}} = \alpha$ [α → arbitrary constant]

$\therefore a = -18\alpha$

$\therefore b = 9\alpha$

$\therefore c = -3\alpha$

Putting these values in equation (i) we get,

$-18\alpha(x-1) + 9\alpha(y-2) - 3\alpha(z-3) = 0$

$\Rightarrow 18(x-1) - 9(y-2) + 3(z-3) = 0$

$\Rightarrow 6(x-1) - 3(y-2) + (z-3) = 0$

$\Rightarrow 6x - 3y + z - 3 = 0$

$\Rightarrow \mathbf{6x - 3y + z = 3}$

14. Question

Find the equation of a plane passing through the point (2, - 1, 5), perpendicular to the plane x + 2y - 3z = 7 and parallel to the line $\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$.

Answer

Given – A plane passes through (2, - 1, 5), perpendicular to the plane x + 2y - 3z = 7 and parallel to the line $\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$

To find – The equation of the plane

Let the equation of the required plane be ax + by + cz + d = 0.....(a)

The plane passes through (2, - 1, 5)

So, 2a - b + 5c + d = 0.....(i)

The direction ratios of the normal of the plane is given by (a, b, c)

Now, this plane is perpendicular to the plane x + 2y - 3z = 7 having direction ratios (1, 2, - 3)

So, a + 2b - 3c = 0.....(ii)

This plane is also parallel to the line having direction ratios (3, - 1, 1)

So, the direction of the normal of the required plane is also at right angles to the given line.

So, 3a - b + c = 0.....(iii)

Solving equations (ii) and (iii),

$\therefore \frac{a}{\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix}} = -\frac{b}{\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}} = \alpha$ [α → arbitrary constant]

$\therefore a = -\alpha$

$\therefore b = -10\alpha$

$\therefore c = -7\alpha$

Putting these values in equation (i) we get,

2X(- α) - (- 10α) + 5(- 7α) + d = 0 i.e. d = 27α

Substituting all the values of a, b, c and d in equation (a) we get,

$-\alpha x - 10\alpha y - 7\alpha z + 27\alpha = 0$

$\Rightarrow \mathbf{x + 10y + 7z + 27 = 0}$

15. Question

Find the equation of the plane passing through the intersection of the planes5x - y + z = 10 and x + y - z = 4 and parallel to the line with direction ratios2, 1, 1. Find also the perpendicular distance of (1, 1, 1) from this plane.

Answer

Given – A plane passes through the intersection of 5x - y + z = 10 and x + y - z = 4 and parallel to the line with direction ratios (2, 1, 1)

To find – Equation of the plane

Tip – If ax + by + cz + d = 0 and a'x + b'y + c'z + d' = 0 be two planes, then the equation of the plane passing through their intersection will be given by

(ax + by + cz + d) + λ(a'x + b'y + c'z + d') = 0, where λ is any scalar constant

So, the equation of the plane maybe written as

(5x - y + z - 10) + λ(x + y - z - 4) = 0

⇒ (5 + λ)x + (- 1 + λ)y + (1 - λ)z + (- 10 - 4λ) = 0

This is plane parallel to a line with direction ratios (2, 1, 1)

So, the normal of this line with direction ratios ((5 + λ), (- 1 + λ), (1 - λ)) will be perpendicular to the given line.

Hence,

2(5 + λ) + (- 1 + λ) + (1 - λ) = 0

⇒ λ = - 5

The equation of the plane will be

(5 + (- 5))x + (- 1 + (- 5))y + (1 - X(- 5))z + (- 10 - 4X(- 5)) = 0

⇒ - 6y + 6z + 10 = 0

⇒ **3y - 3z = 5**

To find – Perpendicular distance of point (1, 1, 1) from the plane

Formula to be used - If ax + by + c + d = 0 be a plane and (a', b', c') be the point, then the distance between them is given by $\left|\frac{a\times a' + b\times b' + c\times c' + d}{\sqrt{a^2+b^2+c^2}}\right|$

The distance between the plane and the line

$= \left|\frac{0\times 2 + 3\times 1 - 3\times 1 - 5}{\sqrt{0^2 + 3^2 + 3^2}}\right|$

$= \left|\frac{3 - 3 - 5}{2\sqrt{3}}\right|$

$= \frac{5}{2\sqrt{3}}$ **units**

Exercise 28H

1. Question

Find the vector and Cartesian equations of the plane passing through the origin and parallel to the vectors $\left(\hat{i} + \hat{j} - \hat{k}\right)$ and $\left(3\hat{i} - \hat{k}\right)$.

Answer

Given - $\vec{r} = \hat{i} + \hat{j} - \hat{k}$ & $\vec{r'} = 3\hat{i} - \hat{k}$ are two lines to which a plane is parallel and it passes through the origin.

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\therefore \vec{r} \times \vec{r'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

$= \hat{i}(-1 - 0) + \hat{j}(-3 + 1) + \hat{k}(0 - 3)$

$= -\hat{i} - 2\hat{j} - 3\hat{k}$

The plane passes through origin (0, 0, 0).

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by $\vec{r} = \left(a\hat{i} + b\hat{j} + c\hat{k}\right) + \lambda\left(a'\hat{i} + b'\hat{j} + c'\hat{k}\right)$ where λ is any scalar constant

The required plane will be

$\vec{r} = (0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}) + \lambda'(-\hat{i} - 2\hat{j} - 3\hat{k})$

$\Rightarrow \vec{r} = \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

The vector equation : $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 0$

The Cartesian equation : **x + 2y + 3z = 0**

3. Question

Find the vector and Cartesian equations of the plane passing through the point(3, - 1, 2) and parallel to the lines $\vec{r} = (-\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + \hat{k}) + \mu(-5\hat{i} + 4\hat{j})$.

Answer

Given - $\vec{r} = (-\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$ & $\vec{r} = (\hat{i} - 3\hat{j} + \hat{k}) + \mu(-5\hat{i} + 4\hat{j})$. A plane is parallel to both these lines and passes through (3, - 1, 2).

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\vec{R} = 2\hat{i} - 5\hat{j} - \hat{k}$ & $\vec{R'} = -5\hat{i} + 4\hat{j}$, where the two vectors represent the directions

$\therefore \vec{R} \times \vec{R'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{bmatrix}$$

$= \hat{i}(0 + 4) + \hat{j}(5 - 0) + \hat{k}(8 - 25)$

$= 4\hat{i} + 5\hat{j} - 17\hat{k}$

The equation of the plane maybe represented as 4x + 5y - 17z + d = 0

Now, this plane passes through the point (3, - 1, 2)

Hence,

4 × 3 + 5 × (- 1) – 17 × 2 + d = 0

⇒ d = 27

The Cartesian equation of the plane : **4x + 5y - 17z + 27 = 0**

The vector equation : $\vec{r}.(4\hat{i} + 5\hat{j} - 17\hat{k}) + 27 = 0$

3. Question

Find the vector equation of a plane passing through the point (1, 2, 3) and parallel to the lines whose direction ratios are 1, - 1, - 2, and - 1, 0, 2.

Answer

Given – The lines have direction ratios of (1, - 1, - 2) and (- 1, 0, 2). The plane parallel to these lines passes through (1, 2, 3)

To find – The vector equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\vec{R} = \hat{i} - \hat{j} - 2\hat{k}$ & $\vec{R'} = -\hat{i} + 2\hat{k}$, where the two vectors represent the directions

$\therefore \vec{R} \times \vec{R'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{bmatrix}$$

$= \hat{i}(-2 - 0) + \hat{j}(2 - 2) + \hat{k}(0 - 1)$

$= -2\hat{i} - \hat{k}$

The equation of the plane maybe represented as - 2x - z + d = 0

Now, this plane passes through the point (1, 2, 3)

Hence,

(- 2) × 1 - 3 + d = 0

⇒ d = 5

The Cartesian equation of the plane : - 2x - z + 5 = 0 i.e. 2x + z = 5

The vector equation : $\vec{r}.(2\hat{i} + \hat{k}) = 5$

4. Question

Find the Cartesian and vector equations of a plane passing through the point (1, 2, - 4) and parallel to the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and $\frac{x-1}{1}=\frac{y+3}{1}=\frac{z}{-1}$.

Answer

Given - $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ & $\frac{x-1}{1}=\frac{y+3}{1}=\frac{z}{-1}$. A plane is parallel to both these lines and passes through (1, 2, - 4).

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

The direction ratios of the given lines are (2, 3, 6) and (1, 1, - 1)

$\therefore \vec{R} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ & $\overrightarrow{R'} = \hat{i} + \hat{j} - \hat{k}$

$\therefore \vec{R} \times \overrightarrow{R'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \hat{i}(-3-6) + \hat{j}(6+2) + \hat{k}(2-3)$$

$$= -9\hat{i} + 8\hat{j} - \hat{k}$$

The equation of the plane maybe represented as - 9x + 8y - z + d = 0

Now, this plane passes through the point (1, 2, - 4)

Hence,

(- 9) × 1 + 8 × 2 - (- 4) + d = 0

⇒ d = - 11

The Cartesian equation of the plane : - 9x + 8y - z - 11 = 0 i.e. **9x - 8y + z + 11 = 0**

The vector equation : $\vec{r}.(9\hat{i} - 8\hat{j} + \hat{k}) + 11 = 0$

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5. Question

Find the vector equation of the plane passing through the point $(3\hat{i}+4\hat{j}+2\hat{k})$ and parallel to the vectors $(\hat{i}+2\hat{j}+3\hat{k})$ and $(\hat{i}-\hat{j}+\hat{k})$.

Answer

Given - $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$ & $\overrightarrow{r'} = \hat{i} - \hat{j} + \hat{k}$ are two lines to which a plane is parallel and it passes through the point $3\hat{i} + 4\hat{j} + 2\hat{k}$

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\therefore \vec{r} \times \overrightarrow{r'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \hat{i}(2+3) + \hat{j}(3-1) + \hat{k}(-1-2)$$

$$= 5\hat{i} + 2\hat{j} - 3\hat{k}$$

The equation of the plane maybe represented as 5x + 2y - 3z + d = 0

Now, this plane passes through the point (3, 4, 2)

Hence,

5 × 3 + 2 × 4 – 3 × 2 + d = 0

⇒ d = - 17

The Cartesian equation of the plane : 5x + 2y - 3z - 17 = 0 i.e. 5x + 2y - 3z = 17

The vector equation : $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$

Exercise 28I

1. Question

Show that the lines $\bar{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\bar{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ are coplanar.

Also find the equation of the plane containing these lines.

Answer

Given : Equations of lines -

$\bar{r_1} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$\bar{r_2} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$

To Prove : $\bar{r_1}$ & $\bar{r_2}$ are coplanar.

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Coplanarity of two lines :

If two lines $\bar{r_1} = \bar{a} + \lambda\bar{b}$ & $\bar{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.\left(\bar{b} \times \bar{d}\right) = \bar{c}.\left(\bar{b} \times \bar{d}\right)$

4) Equation of plane :

If two lines $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$

Where,

$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Answer :

Given equations of lines are

$\bar{r_1} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$\bar{r_2} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$

Let, $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$\overline{a_1} = 2\hat{j} - 3\hat{k}$

$\overline{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$= \hat{i}(8 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$

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$\therefore (\bar{b_1} \times \bar{b_2}) = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$

Therefore,

$\bar{a_1}.(\bar{b_1} \times \bar{b_2}) = (0 \times (-1)) + (2 \times 2) + ((-3) \times (-1))$

$= 0 + 4 + 3$

$= 7$

$\therefore \bar{a_1}.(\bar{b_1} \times \bar{b_2}) = 7$ ..... eq(1)

And

$\bar{a_2}.(\bar{b_1} \times \bar{b_2}) = (2 \times (-1)) + (6 \times 2) + (3 \times (-1))$

$= -2 + 12 - 3$

$= 7$

$\therefore \bar{a_2}.(\bar{b_1} \times \bar{b_2}) = 7$ ..... eq(2)

From eq(1) and eq(2)

$\bar{a_1}.(\bar{b_1} \times \bar{b_2}) = \bar{a_2}.(\bar{b_1} \times \bar{b_2})$

Hence lines $\bar{r_1}$ & $\bar{r_2}$ are coplanar.

Equation of plane containing lines $\bar{r_1}$ & $\bar{r_2}$ is

$\bar{r}.(\bar{b_1} \times \bar{b_2}) = \bar{a_1}.(\bar{b_1} \times \bar{b_2})$

Now,

$\bar{b_1} \times \bar{b_2} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$

From eq(1)

$\bar{a_1}.(\bar{b_1} \times \bar{b_2}) = 7$

Therefore, equation of required plane is

$\bar{r}.(-\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 7$

$\therefore \bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = -7$

$\therefore \bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) + 7 = 0$

$\bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) + 7 = 0$

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2. Question

Find the vector and Cartesian forms of the equations of the plane containing the two lines $\bar{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and .

$\bar{r} = (9\hat{\imath} + 5\hat{\jmath} - \hat{k}) + \mu(-2\hat{\imath} + 3\hat{\jmath} + 8\hat{k})$..

Answer

Given : Equations of lines -

$\bar{r_1} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

$\bar{r_2} = (9\hat{\imath} + 5\hat{\jmath} - \hat{k}) + \mu(-2\hat{\imath} + 3\hat{\jmath} + 8\hat{k})$

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If two lines $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

Given equations of lines are

$\bar{r_1} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

$\bar{r_2} = (9\hat{\imath} + 5\hat{\jmath} - \hat{k}) + \mu(-2\hat{\imath} + 3\hat{\jmath} + 8\hat{k})$

Let, $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$\overline{a_1} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

$\overline{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

$\overline{a_2} = 9\hat{\imath} + 5\hat{\jmath} - k$

$\overline{b_2} = -2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$= \hat{\imath}(24 - 18) - \hat{\jmath}(16 + 12) + \hat{k}(6 + 6)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (1 \times 6) + (2 \times (-28)) + ((-4) \times 12)$

= 6 - 56 - 48

= - 98

$\therefore \overline{a_1}.(\overline{b_1} \times \overline{b_2}) = -98$ .....eq(1)

Equation of plane containing lines $\bar{r_1}$ & $\bar{r_2}$ is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Now,

$\overline{b_1} \times \overline{b_2} = 6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}$

From eq(1)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = -98$

Therefore, equation of required plane is

$\bar{r}.(6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}) = -98$

$\therefore \bar{r}.(6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}) + 98 = 0$

This vector equation of plane.

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.(\overline{b_1} \times \overline{b_2}) = (x \times 6) + (y \times (-28)) + (z \times 12)$

= 6x - 28y + 12z

Therefore, equation of plane is

6x - 28y + 12z = -98

6x - 28y + 12z + 98 = 0

This Cartesian equation of plane.

3. Question

mc24 Myclass24 Your Class. Your Pace.

Find the vector and Cartesian equations of a plane containing the two lines $\bar{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\bar{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$. Also show that the lines $\bar{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + \rho\ (3\hat{i} - 2\hat{j} + 5\hat{k})$ lies in the plane.

Answer

Given : Equations of lines -

$\overline{r_1} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$

$\overline{r_2} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$

To Prove : $\overline{r_1}$ & $\overline{r_2}$ are coplanar.

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Coplanarity of two lines :

If two lines $\overline{r_1} = \bar{a} + \lambda\bar{b}$ & $\overline{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.(\bar{b} \times \bar{d}) = \bar{c}.(\bar{b} \times \bar{d})$

4) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Answer :

Given equations of lines are

$\overline{r_1} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$

$\overline{r_2} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$

Let, $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$\overline{a_1} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\overline{b_1} = \hat{i} + 2\hat{j} + 5\hat{k}$

$\overline{a_2} = 3\hat{i} + 3\hat{j} + 2\hat{k}$

$\overline{b_2} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix}$$

$= \hat{i}(10 + 10) - \hat{j}(5 - 15) + \hat{k}(-2 - 6)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (2 \times 20) + (1 \times 10) + ((-3) \times (-8))$

= 40 + 10 + 24

= 74

$\therefore \overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 74$ ..... eq(1)

And

$\overline{a_2}.(\overline{b_1} \times \overline{b_2}) = (3 \times 20) + (3 \times 10) + (2 \times (-8))$

= 60 + 30 - 16

= 74

$\therefore \overline{a_2}.(\overline{b_1} \times \overline{b_2}) = 74$ ..... eq(2)

From eq(1) and eq(2)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = \overline{a_2}.(\overline{b_1} \times \overline{b_2})$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar.

Equation of plane containing lines $\overline{r_1}$ & $\overline{r_2}$ is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Now,

$\overline{b_1} \times \overline{b_2} = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$

From eq(1)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 74$

Therefore, equation of required plane is

$\bar{r}.(20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}) = 74$

$\therefore \bar{r}.(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k}) = 37$

$\therefore \bar{r}.(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k}) - 37 = 0$

This vector equation of plane.

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.(\overline{b_1} \times \overline{b_2}) = (x \times 20) + (y \times 10) + (z \times (-8))$

= 20x + 10y - 8z

Therefore, equation of plane is

20x + 10y - 8z = 74

20x + 10y - 8z - 74 = 0

10x + 5y - 4z - 37 = 0

This Cartesian equation of plane.

4. Question

Prove that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar. Also find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$

Line 2 : $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Answer :

Given lines –

Line 1 : $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$

Line 2 : $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$

Here, $x_1 = 0$, $y_1 = 2$, $z_1 = -3$, $a_1 = 1$, $b_1 = 2$, $c_1 = 3$

$x_2 = 2$, $y_2 = 6$, $z_2 = 3$, $a_2 = 2$, $b_2 = 3$, $c_2 = 4$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 2-0 & 6-2 & 3+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$=\begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$=2(8-9)-4(4-6)+6(3-4)$$

$$=2(-1)-4(-2)+6(-1)$$

= - 2 + 8 - 6

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\therefore \begin{vmatrix} x-0 & y-2 & z+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}=0$$

$$\therefore (x-0)\times(8-9)-(y-2)\times(4-6)+(z+3)\times(3-4)=0$$

$$\therefore -1(x)-(y-2)(-2)+(z+3)(-1)=0$$

-x + 2y - 4 - z - 3 = 0

- x + 2y - z - 7 = 0

x - 2y + z + 7 = 0

Therefore, equation of plane is

x - 2y + z + 7 = 0

5. Question

Prove that the lines $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ and $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ are coplanar. Also find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$

Line 2 : $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$

Line 2 : $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$

Here, $x_1 = 2$, $y_1 = 4$, $z_1 = 6$, $a_1 = 1$, $b_1 = 4$, $c_1 = 7$

$x_2 = -1$, $y_2 = -3$, $z_2 = -5$, $a_2 = 3$, $b_2 = 5$, $c_2 = 7$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} -1-2 & -3-4 & -5-6 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & -7 & -11 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$

$$= -3(28-35) - (-7)(7-21) - 11(5-12)$$

$$= -3(-7) + 7(-14) - 11(-7)$$

= 21 - 98 + 77

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Hence, given two lines are coplanar.

Equation of plane passing through line 1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-2 & y-4 & z-6 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$$\therefore (x-2) \times (28-35) - (y-4) \times (7-21) + (z-6) \times (5-12) = 0$$

$$\therefore -7(x-2) - (y-4)(-14) + (z-6)(-7) = 0$$

-7x + 14 + 14y - 56 – 7z + 42 = 0

- 7x + 14y – 7z = 0

x – 2y + z = 0

Therefore, equation of plane is

mc24 Myclass24 Your Class. Your Pace.

x – 2y + z = 0

6. Question

Show that the lines $\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z-3}{-5}$ and $\frac{x-8}{7} = \frac{2y-8}{2} = \frac{z-5}{3}$ are coplanar. Find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z+3}{-5}$ or $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$

Line 2 : $\frac{x-8}{7} = \frac{2y-8}{2} = \frac{z-5}{3}$ or $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

mc24 Myclass24 Your Class. Your Pace.

Answer :

Given lines –

Line 1 : $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$

Line 2 : $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$

Here, $x_1 = 5$, $y_1 = 7$, $z_1 = -3$, $a_1 = 4$, $b_1 = 4$, $c_1 = -5$

$x_2 = 8$, $y_2 = 4$, $z_2 = 5$, $a_2 = 7$, $b_2 = 1$, $c_2 = 3$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$= 3(12+5) - (-3)(12+35) + 8(4-28)$$

$$= 3(17) + 3(47) + 8(-24)$$

= 51 + 141 - 192

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-5 & y-7 & z+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = 0$$

$\therefore (x-5) \times (12+5) - (y-7) \times (12+35) + (z+3) \times (4-28) = 0$

$\therefore 17(x-5) - 47(y-7) + (z+3)(-24) = 0$

17x - 85 - 47y + 329 - 24z - 72 = 0

17x - 47y - 24z + 172 = 0

Therefore, equation of plane is

17x - 47y - 24z + 172 = 0

7. Question

Show that the lines $\frac{x-1}{-3} = \frac{y-3}{2} = \frac{z-2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z-7}{2}$ are coplanar. Find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

Line 2 : $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

Line 2 : $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

Here, $x_1 = -1$, $y_1 = 3$, $z_1 = -2$, $a_1 = -3$, $b_1 = 2$, $c_1 = 1$

$x_2 = 0$, $y_2 = 7$, $z_2 = -7$, $a_2 = 1$, $b_2 = -3$, $c_2 = 2$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0+1 & 7-3 & -7+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 4 & -5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$= 1(4+3) - 4(-6-1) - 5(9-2)$

$= 1(7) - 4(-7) - 5(7)$

= 7 + 28 - 35

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$\therefore (x+1) \times (4+3) - (y-3) \times (-6-1) + (z+2) \times (9-2) = 0$

$\therefore 7(x+1) - (y-3)(-7) + (z+2)(7) = 0$

7x + 7 + 7y - 21 + 7z + 14 = 0

7x + 7y + 7z = 0

x + y + z = 0

Therefore, equation of plane is

8. Question

Show that the lines $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{-1}$ are coplanar. Also find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z}{-1}$

Line 2 : $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{-1}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z}{-1}$

Line 2 : $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{-1}$

Here, $x_1 = 1$, $y_1 = 3$, $z_1 = 0$, $a_1 = 2$, $b_1 = -1$, $c_1 = -1$

$x_2 = 4$, $y_2 = 1$, $z_2 = 1$, $a_2 = 3$, $b_2 = -2$, $c_2 = -1$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 4-1 & 1-3 & 1-0 \\ 2 & -1 & -1 \\ 3 & -2 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -2 & 1 \\ 2 & -1 & -1 \\ 3 & -2 & -1 \end{vmatrix}$$

$$= 3(1-2) - (-2)(-2+3) + 1(-4+3)$$

$$= 3(-1) + 2(1) + 1(-1)$$

$= -2$

$$\therefore \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \neq 0$$

Hence, given two lines are not coplanar.

9. Question

Find the equation of the plane which contains two parallel lines given by $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$ and $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$.

Answer

Given : Equations of lines –

Line 1 : $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$

Line 2 : $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$

To Find : Equation of plane.

Formulae :

Equation of plane :

The equation of plane containing two parallel lines $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

& $\frac{x-x_2}{a} = \frac{y-y_2}{b} = \frac{z-z_2}{c}$ is given by,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a & b & c \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$

Line 2 : $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$

Here, $x_1 = 3$, $y_1 = -2$, $z_1 = 0$, a= 1 , b = -4 , c = 5

$x_2 = 4$, $y_2 = 3$, $z_2 = 2$

Therefore, equation of plane containing line 1 & line 2 is given by,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a & b & c \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-3 & y+2 & z-0 \\ 4-3 & 3+2 & 2-0 \\ 1 & -4 & 5 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-3 & y+2 & z \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix} = 0$$

$$\therefore (x-3) \times (25+8) - (y+2) \times (5-2) + (z) \times (-4-5) = 0$$

$$\therefore 33(x-3) - (y+2)(3) + (z)(-9) = 0$$

$33x - 99 - 3y - 6 - 9z = 0$

$33x - 3y - 9z - 105 = 0$

$11x - y - 3z = 35$

Therefore, equation of plane is

Exercise 28J

1. Question

Find the direction ratios of the normal to the plane x + 2y - 3z = 5.

Answer

Given :

Equation of plane : x + 2y – 3z = 5

To Find : direction ratios of normal

Answer :

Given equation of plane : x + 2y – 3z = 5

It can be written as

$(x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = 5$

Comparing with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Therefore, normal vector is $\bar{n} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

Hence, direction ratios of normal are (1, 2, -3).

2. Question

Find the direction cosines of the normal to the plane 2x + 3y - z = 4.

Answer

Given :

Equation of plane : 2x + 3y – z = 4

To Find : Direction cosines of the normal i.e. l, m & n

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

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Answer :

For the given equation of plane

2x + 3y – z = 4

Direction ratios of normal vector are (2, 3, -1)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{2^2 + 3^2 + (-1)^2}$$

$$= \sqrt{4 + 9 + 1}$$

$$= \sqrt{14}$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{2}{\sqrt{14}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{3}{\sqrt{14}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{-1}{\sqrt{14}}$$

$$(l, m, n) = \left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$$

3. Question

Find the direction cosines of the normal to the plane y = 3.

Answer

Given :

Equation of plane : y = 3

To Find : Direction cosines of the normal i.e. l, m & n

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

y = 3

Direction ratios of normal vector are (0, 1, 0)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{0^2 + 1^2 + 0^2}$$

$$= \sqrt{0 + 1 + 0}$$

$$= \sqrt{1}$$

$$= 1$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{1} = 0$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{1}{1} = 1$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{1} = 0$$

$(l, m, n) = (0,1,0)$

4. Question

Find the direction cosines of the normal to the plane 3x + 4 = 0.

Answer

Given :

Equation of plane : 3x + 4 = 0

To Find : Direction cosines of the normal i.e. l, m & n

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

-3x = 4

Direction ratios of normal vector are (-3, 0, 0)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{(-3)^2 + 0^2 + 0^2}$$

$$= \sqrt{9 + 0 + 0}$$

$$= \sqrt{9}$$

$$= 3$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{3} = -1$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{3} = 0$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{3} = 0$$

$(l, m, n) = (-1, 0, 0)$

5. Question

Write the equation of the plane parallel to XY-plane and passing through the point (4, -2, 3).

Answer

Given :

Point : (4, -2, 3)

To Find : equation of plane

Formula :

1) Equation of plane :

Equation of plane passing through point A with position vector $\bar{a}$ and perpendicular to vector $\bar{n}$ is given by,

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

Position vector for given point A ≡ (4, -2, 3) is

$\bar{a} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

As required plane is parallel to XY plane, therefore Z-axis is perpendicular to the plane.

$\therefore \bar{n} = \hat{k}$

Therefore, equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{k}) = (4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}).(\hat{k})$

$\therefore (x \times 0) + (y \times 0) + (z \times 1) = (4 \times 0) + (-2 \times 0) + (3 \times 1)$

$\therefore z = 3$

This is required equation of plane.

6. Question

Write the equation of the plane parallel to YZ-plane and passing through the point(-3, 2, 0).

Answer

Given :

Point : (-3, 2, 0)

To Find : equation of plane

Formula :

1) Equation of plane :

Equation of plane passing through point A with position vector $\bar{a}$ and perpendicular to vector $\bar{n}$ is given by,

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

Position vector for given point A ≡ (-3, 2, 0) is

$\bar{a} = -3\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$

As required plane is parallel to YZ plane, therefore X-axis is perpendicular to the plane.

$\therefore \bar{n} = \hat{\imath}$

Therefore, equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath}) = (-3\hat{\imath} + 2\hat{\jmath} + 0\hat{k}).(\hat{\imath})$

$\therefore (x \times 1) + (y \times 0) + (z \times 0) = (-3 \times 1) + (2 \times 0) + (0 \times 0)$

$\therefore x = -3$

This is required equation of plane.

7. Question

Write the general equation of a plane parallel to the x-axis.

Answer

Let, normal vector of plane be

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

Equation of plane is given by,

$\bar{r}.\bar{n} = d$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(a\hat{\imath} + b\hat{\jmath} + c\hat{k}) = d$

$\therefore ax + by + cz = d$

As the required plane is parallel to the given plane, hence normal vector of plane is perpendicular to x-axis.

$\therefore \bar{n}.\hat{\imath} = 0$

$\therefore (a\hat{\imath} + b\hat{\jmath} + c\hat{k}).\hat{\imath} = 0$

a = 0

Therefore, equation of plane is

by + cz = d

8. Question

Write the intercept cut off by the plane 2x + y - z = 5 on the x-axis.

Answer

Given :

Equation of plane : 2x + y – z = 5

To Find : Intercept made by the plane with the X-axis.

Formula :

If $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

2x + y – z = 5

Dividing above equation throughout by 5

$\therefore \frac{2x}{5} + \frac{y}{5} + \frac{-z}{5} = 1$

$\therefore \frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$

Comparing above equation with

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

We get,

a = 5/2

Therefore, intercepts made by plane with X-axis are

X-intercept = 5/2

9. Question

Write the intercepts made by the plane4x - 3y + 2z = 12 on the coordinate axes.

Answer

Given :

Equation of plane : 4x – 3y + 2z = 12

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

4x – 3y + 2z = 12

Dividing above equation throughout by 12

$\therefore \frac{4x}{12}+\frac{-3y}{12}+\frac{2z}{12}=1$

$\therefore \frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$

This is the equation of plane in intercept form.

Comparing above equation with

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

We get,

a = 3

b = -4

c = 6

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = 3

Y-intercept = -4

Z-intercept = 6

10. Question

Reduce the equation 2x – 3y + 5z + 4 = 0 to intercept form and find the intercepts made by it on the coordinate axes.

Answer

Given :

Equation of plane : 2x – 3y + 5z + 4 = 0

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

2x – 3y + 5z = -4

Dividing above equation throughout by -4

$$\therefore \frac{2x}{-4}+\frac{-3y}{-4}+\frac{5z}{-4}=1$$

$$\therefore \frac{x}{-2}+\frac{y}{4/3}+\frac{z}{-4/5}=1$$

This is the equation of plane in intercept form.

Comparing above equation with

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

We get,

a = -2

$$b=\frac{4}{3}$$

$$c=\frac{-4}{5}$$

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = -2

$$Y-intercept=\frac{4}{3}$$

$$Z-intercept=-\frac{4}{5}$$

11. Question

Find the equation of a plane passing through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c).

Answer

Given : Plane is passing through points

A ≡ (a, 0, 0)

B ≡ (0, b, 0)

C ≡ (0, 0, c)

To Find : Equation of plane

Formulae :

Equation of plane making intercepts (a, b, c) on X, Y & Z axes respectively is given by,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Answer : As plane is passing through points A ≡ (a, 0, 0),

B ≡ (0, b, 0) & C ≡ (0, 0, c)

Therefore, intercepts made by it on X, Y & Z axes respectively are

a, b & c.

hence, equation of plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

12. Question

Write the value of k for which the planes 2x – 5y + kz = 4 and x + 2y – z = 6 are perpendicular to each other.

Answer

Given : equations of perpendicular planes-

2x – 5y + kz = 4

x + 2y – z = 6

To Find : k

Formulae :

Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$$\vec{n}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}$$

Answer :

For given planes –

2x – 5y + kz = 4

x + 2y – z = 6

normal vectors are

$\overline{n_1} = 2\hat{\imath} - 5\hat{\jmath} + k\hat{k}$

$\overline{n_2} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

As given vectors are perpendicular, hence their normal vectors are also perpendicular to each other.

$\therefore \overline{n_1}.\overline{n_2} = 0$

$\therefore (2\hat{\imath} - 5\hat{\jmath} + k\hat{k}).(\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 0$

(2×1) + (-5×2) + (k×(-1)) = 0

2 – 10 – k = 0

- 8 – k = 0

k = -8

13. Question

Find the angle between the planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7.

Answer

Given : equations of planes-

2x + y – 2z = 5

3x – 6y – 2z = 7

To Find : angle between two planes

Formulae :

1) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

2) Angle between two planes :

The angle Θ between the planes $\bar{r}.\overline{n_1} = p_1$ and $\bar{r}.\overline{n_2} = p_2$ is given by

$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$

Answer :

For given planes

2x + y – 2z = 5

3x – 6y – 2z = 7

Normal vectors are

$\overline{n_1} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and

$\overline{n_2} = 3\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$

$\therefore |\overline{n_1}| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$

$\therefore |\overline{n_2}| = \sqrt{3^2 + (-6)^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$

Therefore, angle between two planes is

$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$

$\therefore \cos\theta = \frac{(2\hat{\imath} + \hat{\jmath} - 2\hat{k}).(3\hat{\imath} - 6\hat{\jmath} - 2\hat{k})}{3 \times 7}$

$\therefore \cos\theta = \frac{(2 \times 3) + (1 \times (-6)) + ((-2) \times (-2))}{21}$

$\therefore \cos\theta = \frac{6 - 6 + 4}{21}$

$\therefore \cos\theta = \frac{4}{21}$

$\therefore \theta = cos^{-1}\left(\frac{4}{21}\right)$

14. Question

Find the angle between the planes $\vec{r}\cdot(\hat{i}+\hat{j})=1$ and $\vec{r}\cdot(\hat{i}+\hat{k})=3$.

Answer

Given : equations of planes-

$\bar{r}.(\hat{\imath}+\hat{\jmath})=1$

$\bar{r}.(\hat{\jmath}+\hat{k})=3$

To Find : angle between two planes

Formulae :

Angle between two planes :

The angle Θ between the planes $\bar{r}.\overline{n_1}=p_1$ and $\bar{r}.\overline{n_2}=p_2$ is given by

$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$

Answer :

For given planes

$\bar{r}.(\hat{\imath}+\hat{\jmath})=1$

$\bar{r}.(\hat{\jmath}+\hat{k})=3$

Normal vectors are

$\overline{n_1}=\hat{\imath}+\hat{\jmath}$ and

$\overline{n_2}=\hat{\jmath}+\hat{k}$

$\therefore |\overline{n_1}| = \sqrt{1^2+1^2+0^2} = \sqrt{1+1+0} = \sqrt{2}$

$\therefore |\overline{n_2}| = \sqrt{0^2+1^2+1^2} = \sqrt{0+1+1} = \sqrt{2}$

Therefore, angle between two planes is

$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$

$\therefore \cos\theta = \frac{(\hat{\imath}+\hat{\jmath}).(\hat{\jmath}+\hat{k})}{\sqrt{2}\times\sqrt{2}}$

$\therefore \cos\theta = \frac{(1\times 0)+(1\times 1)+(0\times 1)}{2}$

$\therefore \cos\theta = \frac{0+1+0}{2}$

$\therefore \cos\theta = \frac{1}{2}$

$\therefore \theta = cos^{-1}\left(\frac{1}{2}\right)$

$\therefore \theta = \frac{\pi}{3}$

15. Question

Find the angle between the planes $\vec{r}\cdot(3\hat{i}-4\hat{j}+5\hat{k})=0$ and $\vec{r}\cdot(2\hat{i}-\hat{j}-2\hat{k})=7$.

Answer

Given : equations of planes-

$\bar{r}.(3\hat{\imath}-4\hat{\jmath}+5\hat{k})=0$

$\bar{r}.(2\hat{\imath}-\hat{\jmath}-2\hat{k})=7$

To Find : angle between two planes

Formulae :

Angle between two planes :

The angle Θ between the planes $\bar{r}.\overline{n_1} = p_1$ and $\bar{r}.\overline{n_2} = p_2$ is given by

$$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$$

Answer :

For given planes

$\bar{r}.(3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}) = 0$

$\bar{r}.(2\hat{\imath} - \hat{\jmath} - 2\hat{k}) = 7$

Normal vectors are

$\overline{n_1} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and

$\overline{n_2} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$

$\therefore |\overline{n_1}| = \sqrt{3^2 + (-4)^2 + 5^2} = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$

$\therefore |\overline{n_2}| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$

Therefore, angle between two planes is

$$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$$

$$\therefore \cos\theta = \frac{(3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}).(2\hat{\imath} - \hat{\jmath} - 2\hat{k})}{5\sqrt{2} \times 3}$$

$$\therefore \cos\theta = \frac{(3 \times 2) + ((-4) \times (-1)) + (5 \times (-2))}{15\sqrt{2}}$$

$$\therefore \cos\theta = \frac{6 + 4 - 10}{2}$$

$\therefore \cos\theta = 0$

$\therefore \theta = cos^{-1}(0)$

$$\therefore \theta = \frac{\pi}{2}$$

16. Question

Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the planes 10x + 2y – 11z = 3.

Answer

Given :

Equation of line : $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$

Equation of plane : 10x + 2y – 11z = 3

To Find : angle between line and plane

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$\bar{b} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

3) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r}.\bar{n} = p$, then

$$\sin\theta = \frac{\bar{b}.\bar{n}}{|\bar{b}|.|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$$

Parallel vector to the line is

$$\bar{b}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$$

$$\therefore |\bar{b}|=\sqrt{2^2+3^2+6^2}=\sqrt{4+9+36}=\sqrt{49}=7$$

For given equation of plane,

$10x + 2y - 11z = 3$

normal vector to the plane is

$$\bar{n}=10\hat{\imath}+2\hat{\jmath}-11\hat{k}$$

$$\therefore |\bar{n}|=\sqrt{10^2+2^2+(-11)^2}=\sqrt{100+4+121}=\sqrt{225}=15$$

Therefore, angle between given line and plane is

$$\sin\theta=\frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

$$\therefore \sin\theta=\frac{(2\hat{\imath}+3\hat{\jmath}+6\hat{k})\,.\,(10\hat{\imath}+2\hat{\jmath}-11\hat{k})}{7\times 15}$$

$$\therefore \sin\theta=\frac{(2\times 10)+(3\times 2)+(6\times(-11))}{105}$$

$$\therefore \sin\theta=\frac{20+6-66}{105}$$

$$\therefore \sin\theta=\frac{-40}{105}$$

$$\therefore \sin\theta=\frac{-8}{21}$$

$$\therefore \theta=sin^{-1}\left(\frac{-8}{21}\right)$$

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17. Question

Find the angle between the line $\bar{r}=(\hat{\imath}+\hat{\jmath}-2\hat{k})+\lambda(\hat{\imath}-\hat{\jmath}+\hat{k})$ and the plane $\bar{r}\cdot(2\hat{\imath}-\hat{\jmath}+\hat{k})=4$.

Answer

Given :

Equation of line : $\bar{r}=(\hat{\imath}+\hat{\jmath}-2\hat{k})+\lambda(\hat{\imath}-\hat{\jmath}+\hat{k})$

Equation of plane : $\bar{r}.(2\hat{\imath}-\hat{\jmath}+\hat{k})=4$

To Find : angle between line and plane

Formulae :

1) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r}=\bar{a}+\lambda\bar{b}$ and the plane $\bar{r}.\bar{n}=p$, then

$$\sin\theta=\frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\bar{r}=(\hat{\imath}+\hat{\jmath}-2\hat{k})+\lambda(\hat{\imath}-\hat{\jmath}+\hat{k})$$

Parallel vector to the line is

$$\bar{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$$

$$\therefore |\bar{b}|=\sqrt{1^2+(-1)^2+1^2}=\sqrt{1+1+1}=\sqrt{3}$$

For given equation of plane,

$\bar{r}.(2\hat{\imath}-\hat{\jmath}+\hat{k})=4$

normal vector to the plane is

$\bar{n}=2\hat{\imath}-\hat{\jmath}+\hat{k}$

$\therefore |\bar{n}|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{4+1+1}=\sqrt{6}$

Therefore, angle between given line and plane is

$$\sin\theta=\frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.|\bar{n}|}$$

$$\therefore \sin\theta=\frac{(\hat{\imath}-\hat{\jmath}+\hat{k}).(2\hat{\imath}-\hat{\jmath}+\hat{k})}{\sqrt{3}\times\sqrt{6}}$$

$$\therefore \sin\theta=\frac{(1\times2)+((-1)\times(-1))+(1\times1)}{\sqrt{18}}$$

$$\therefore \sin\theta=\frac{2+1+1}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{4}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{2\times2}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{2\sqrt{2}}{3}$$

$$\therefore \theta=sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

18. Question

Find the value of λ such that the line $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{4}$ is perpendicular to the plane 3x - y – 2z = 7.

Answer

Given :

Equation of line : $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{4}$

Equation of plane : 3x – y – 2z = 7

To Find : λ

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$\bar{b}=a_1\hat{\imath}+b_1\hat{\jmath}+c_1\hat{k}$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

Answer :

For given equation of line,