NCERT Solutions for Class-XI Maths

Chapter-9 Exercise-9.4

NCERT Math Class 11

1. Find the sum to n terms of the series $1\times2+2\times3+3\times4+4\times5+\ldots$

1. The given series is $1\times2+2\times3+3\times4+4\times5+\ldots n^{th}$ term, $a_n=n(n+1)$

$$\therefore S_n=\sum_{k=1}^{n}a_k=\sum_{k=1}^{n}k(k+1)$$

$$=\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}k$$

$$=\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$=\frac{n(n+1)}{2}\left(\frac{2n+1}{3}+1\right)$$

$$=\frac{n(n+1)}{2}\left(\frac{2n+4}{3}\right)$$

$$=\frac{n(n+1)(n+2)}{3}$$

2. Find the sum to n terms of the series $1\times2\times3+2\times3\times4+3\times4\times5+\ldots$

2. $1\times2\times3+2\times3\times4+3\times4\times5+\ldots$ is the given series.

The n^{th} term of the series is

$$a_n=n(n+1)(n+2)$$

$$=(n^2+n)(n+2)$$

$$=n^3+3n^2+2n$$

The sum of n terms of a series is given by the equation $S_n=\sum_{k=1}^{n}a_k$.

The sum of n terms of the given series is

$$S_n=\sum_{k=1}^{n}(k^3+3k^2+2k)$$

$$=\sum_{k=1}^{n}k^3+3\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k$$

$$=\left[\frac{n(n+1)}{2}\right]^2+\frac{3n(n+1)(2n+1)}{6}+\frac{2n(n+1)}{2}$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + 2n + 1 + 2\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{n^2 + n + 4n + 6}{2}\right]$$

$$= \frac{n(n+1)}{4}\left(n^2 + 5n + 6\right)$$

$$= \frac{n(n+1)}{4}\left(n^2 + 2n + 3n + 6\right)$$

$$= \frac{n(n+1)\left[n(n+2) + 3(n+2)\right]}{4}$$

$$= \frac{n(n+1)(n+2)(n+3)}{4}$$

Therefore, the sum of n terms of the series $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \ldots$ is $\frac{n(n+1)(n+2)(n+3)}{4}$.

3. Find the sum to n terms of the series $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots$

3. The given series is $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots n^{th}$ term,

$a_n = (2n+1)n^2 = 2n^3 + n^2$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$

$$= \sum_{k=1}^{n} = \left(2k^3 + k^2\right) = 2\sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n^2(n+1)^2}{2} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left[n(n+1) + \frac{2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2 + 3n + 2n + 1}{3}\right]$$

$=\frac{n(n+1)}{2}\left[\frac{3n^2+5n+1}{3}\right]$

$=\frac{n(n+1)(3n^2+5n+1)}{6}$

4. Find the sum to n terms of the series $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots$

4. $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots$ is the given series.

The n^{th} term of the series is

$a_n=\frac{1}{n(n+1)}$

By partial fractions the above equation can be written as

$a_n=\frac{1}{n}-\frac{1}{n+1}$

Therefore,

$a_1=\frac{1}{1}-\frac{1}{2}$

$a_2=\frac{1}{2}-\frac{1}{3}$

$a_3=\frac{1}{3}-\frac{1}{4}\ldots$

$a_n=\frac{1}{n}-\frac{1}{n+1}$

Add the above terms.

$a_1+a_2+\ldots+a_n=\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{n}\right]-\left[\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots\frac{1}{n+1}\right]$

The sum of n terms of a series is given by the equation $S_n=\sum_{k=1}^{n}a_k$.

The sum of n terms of the given series is

$S_n=1-\frac{1}{n+1}$

$=\frac{n+1-1}{n+1}$

$=\frac{n}{n+1}$

Therefore, the sum of n terms of the series $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots$ is $\frac{n}{n+1}$.

5. Find the sum to n terms of the series $5^2+6^2+7^2+\ldots+20^2$

5. The given series is $5^2+6^2+7^2+\ldots+20^2 n^{th}$ term,

$a_n = (n+4)^2 = n^2 + 8n + 16$

$= \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \left(k^2 + 8k + 16\right)$

$\therefore S_n = \sum_{k=1}^{n} k^2 + 8\sum_{k=1}^{n} k + \sum_{k=1}^{n} 16$

$= \dfrac{n(n+1)(2n+1)}{6} + \dfrac{8n(n+1)}{2} + 16n$

16^{th} term is $(16+4)^2 = 20^2$

$\therefore S_{10} = \dfrac{16(16+1)(2\times16+1)}{6} + \dfrac{8\times16\times(16+1)}{2} + 16\times16$

$\therefore 5^2 = \dfrac{(16)(17)(33)}{6} + \dfrac{(8)\times16\times(16+1)}{2} + 16\times16$

$= \dfrac{(16)(17)(33)}{6} + \dfrac{(8)(16)(17)}{2} + 256$

$= 1496 + 1088 + 256$

$= 2840$

$+6^2 + 7^2 + \ldots\ldots + 20^2 = 2840$

6. Find the sum to n terms of the series $3\times8 + 6\times11 + 9\times14 + \ldots$

6. $3\times8 + 6\times11 + 9\times14 + \ldots a_n$ is the given series.

The n^{th} term of the series is $a_n = \left(n^{th} \text{ term of } 3,6,9\ldots\right)\times\left(n^{th} \text{ term of } 8,11,14\ldots\right)$

$= (3n)(3n+5)$

$= 9n^2 + 15n$

The sum of n terms of a series is given by the equation $S_n = \sum_{k=1}^{n} a_k$.

The sum of n terms of the given series is

$S_n = \sum_{k=1}^{n} \left(9k^2 + 15k\right)$

$= 9\sum_{k=1}^{n} k^2 + 15\sum_{k=1}^{n} k$

$= 9\times\dfrac{n(n+1)(2n+1)}{6} + 15\times\dfrac{n(n+1)}{2}$

$= \dfrac{3n(n+1)(2n+1)}{2} + \dfrac{15n(n+1)}{2}$

$$= \frac{3n(n+1)}{2}(2n+1+5)$$

$$= \frac{3n(n+1)}{2}(2n+6)$$

$$= 3n(n+1)(n+3)$$

Therefore, the sum of n terms of the series $3\times8+6\times11+9\times14+\ldots$ is $3n(n+1)(n+3)$.

7. Find the sum to n terms of the series $1^2+(1^2+2^2)+(1^2+2^2+3^2)+\ldots$

7. The given series is $1^2+(1^2+2^2)+(1^2+2^2+3^3)+\ldots a_n$

$$= (1^2+2^2+3^3+\ldots\ldots+n^2)$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(2n^2+3n+1)}{6} = \frac{2^3+3n^2+n}{6}$$

$$= \frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n$$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$

$$= \sum_{k=1}^{n}\left(\frac{1}{3}k^3+\frac{1}{2}k^2+\frac{1}{6}k\right)$$

$$= \frac{1}{3}\sum_{k=1}^{n}k^3+\frac{1}{2}\sum_{k=1}^{n}k^2+\frac{1}{6}\sum_{k=1}^{n}k$$

$$= \frac{1}{3}\frac{n^2(n+1)^2}{(2)^2}+\frac{1}{2}\times\frac{n(n+1)(2n+1)}{6}+\frac{1}{6}\times\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{6}\left[\frac{n(n+1)}{2}+\frac{(2n+1)}{2}+\frac{1}{2}\right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n^2+n+2n+1+1}{2}\right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n^2+n+2n+2}{2}\right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n(n+1)+2(n+1)}{2}\right]$$

$$=\frac{n(n+1)}{6}\left[\frac{(n+1)(n+2)}{2}\right]$$

$$=\frac{n(n+1)^2(n+2)}{12}$$

8. Find the sum to n terms of the series whose n^{th} term is given by $n(n+1)(n+4)$

8. The n^{th} term of the series is

$$a_n = n(n+1)(n+4)$$

$$= n(n^2+5n+4)$$

$$= n^3+5n^2+4n$$

The sum of n terms of a series is given by the equation $S_n = \sum_{k=1}^{n} a_k$.

The sum of n terms of the given series is

$$S_n = \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$=\frac{n^2(n+1)^2}{4}+\frac{5n(n+1)(2n+1)}{6}+\frac{4n(n+1)}{2}$$

$$=\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{5(2n+1)}{3}+4\right]$$

$$=\frac{n(n+1)}{2}\left[\frac{3n^2+3n+20n+10+24}{6}\right]$$

$$=\frac{n(n+1)}{2}\left[\frac{3n^2+23n+34}{6}\right]$$

$$=\frac{n(n+1)(3n^2+23n+34)}{12}$$

Therefore, the sum of n^{th} terms of the series whose n^{th} term is given by $n(n+1)(n+4)$ is $\frac{n(n+1)(3n^2+23n+34)}{12}$.

9. Find the sum to n terms of the series whose n^{th} terms is given by n^2+2^n

9. $a_n = n^2 + 2^n$

$$\therefore S_n = \sum_{k=1}^{n} k^2 + 2^k = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 2^k$$

Consider $\sum_{k=1}^{n} 2^k = 2^1 + 2^2 + 2^3 + \ldots$

The above series $2, 2^2, 2^3 \ldots$ is a G.P. with both the first term and common ratio equal to 2.

$$\therefore \sum_{k=1}^{n} 2^k = \frac{(2)\left[(2)^n - 1\right]}{2-1} = 2\left(2^n - 1\right)$$

Therefore, from (1) and (2), we obtain

$$S_n = \sum_{k=1}^{n} k^2 + 2\left(2^n - 1\right) = \frac{n(n+1)(2n+1)}{6} + 2\left(2^n - 1\right)$$

10. Find the sum to n terms of the series whose n^{th} term is given by $(2n-1)^2$

10. The n^{th} term of the series is

$$a_n = (2n-1)^2$$

$$= 4n^2 - 4n + 1$$

The sum of n terms of a series is given by the equation $S_n = \sum_{k=1}^{n} a_k$.

The sum of first n terms of the given series is

$$S_n = \sum_{k=1}^{n}\left(4k^2 - 4k + 1\right)$$

$$= 4\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$$

$$= \frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n$$

$$= n\left[\frac{2\left(2n^2 + 3n + 1\right)}{3} - 2(n+1) + 1\right]$$

$$= n\left[\frac{4n^2 + 6n + 2 - 6n - 6 + 3}{3}\right]$$

$$= n\left[\frac{4n^2 - 1}{3}\right]$$

$$= \frac{n(2n+1)(2n-1)}{3}$$

Therefore, the sum of n terms of the series whose n^{th} term is given by $(2n-1)^2$ is $\frac{n(2n+1)(2n-1)}{3}$