EXERCISE 4.1

1. Which of the following numbers are not perfect cubes? Give reasons in support of your answer:
(i) 648
(ii) 729
(iii) 8640
(iv) 8000
Solution:

(i) We have,

648 = 2 × 2 × 2 × 3 × 3 × 3 × 3

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, one factor 3 is left without grouping.

648 = (2 × 2 × 2) × (3 × 3 × 3) × 3

Thus, 648 is not a perfect cube.

(ii) We have,

729 = 3 × 3 × 3 × 3 × 3 × 3

3	729
3	243
3	81
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that no factor is left.

729 = (3 × 3 × 3) × (3 × 3 × 3)

Thus, 729 is a perfect cube.

(iii) We have,

8640 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5

2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

After grouping the prime factors in triplets, it's seen that one factor 5 is left without grouping.

8640 = (2 × 2 × 2) × (2 × 2 × 2) × (3 × 3 × 3) × 5

Thus, 8640 is not a perfect cube.

(iv) We have,

8000 = 2 × 2 × 2 × 2 × 2 × 2 × 5 × 5 × 5

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

After grouping the prime factors in triplets, it's seen that no factor is left.

8000 = (2 × 2 × 2) × (2 × 2 × 2) × (5 × 5 × 5)

Thus, 8000 is a perfect cube.

2. Show that each of the following numbers is a perfect cube. Also, find the number whose cube is the given number:
(i) 1728
(ii) 5832
(iii) 13824
(iv) 35937
Solution:

(i) We have,

1728 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that no factor is left without grouping.

1728 = (2 × 2 × 2) × (2 × 2 × 2) × (3 × 3 × 3)

Thus, 1728 is a perfect cube and its cube root is 2 × 2 × 3 = 12.

(ii) We have,

5832 = 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that no factor is left without grouping.

5832 = (2 × 2 × 2) × (3 × 3 × 3) × (3 × 3 × 3)

Thus, 5832 is a perfect cube, and its cube root is 2 × 3 × 3 = 18

(iii) We have,

13824 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that no factor is left without grouping.

13824 = (2 × 2 × 2) × (2 × 2 × 2) × (2 × 2 × 2) × (3 × 3 × 3)

Thus, 13824 is a perfect cube, and its cube root is 2 × 2 × 2 × 3 = 24.

(iv) We have,

35937 = 3 × 3 × 3 × 11 × 11 × 11

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

After grouping the prime factors in triplets, it's seen that no factor is left without grouping.

35937 = (3 × 3 × 3) × (11 × 11 × 11)

Thus, 35937 is a perfect cube, and its cube root is 3 × 11 = 33

3. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube:
(i) 243
(ii) 3072
(iii) 11979
(iv) 19652
Solution:

(i) We have,

243 = 3 × 3 × 3 × 3 × 3

3	243
3	81
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that factors 3 × 3 are left.

243 = (3 × 3 × 3) × 3 × 3

So, in order to complete in a group of 3's, one more factor of 3 is needed.

Thus, the smallest number which should be multiplied to 243 in order to make it a perfect cube is 3.

(ii) We have,

3072 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3

2	3072
2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

After grouping the prime factors in triplets, it's seen that factor 2 × 3 are left ungrouped.

3072 = (2 × 2 × 2) × (2 × 2 × 2) × (2 × 2 × 2) × 2 × 3

So, in order to complete them in a group of 3's we need the factors 2 × 2 × 3 × 3 to be multiplied.

i.e. the factor needed is 2 × 2 × 3 × 3 = 36

Thus, the smallest number which should be multiplied to 3072 in order to make it a perfect cube is 36.

(iii) We have,

11979 = 3 × 3 × 11 × 11 × 11

3	11979
3	3993
11	1331
11	121
11	11
	1

After grouping the prime factors in triplets, it's seen that factors 3 × 3 are left without grouping in 3's.

11979 = 3 × 3 × (11 × 11 × 11)

So, in order to complete in a group of 3's, one more factor of 3 is needed.

Thus, the smallest number which should be multiplied by 11979 in order to make it a perfect cube is 3.

(iv) We have,

19652 = 2 × 2 × 17 × 17 × 17

2	19652
2	9826
17	4913
17	289
17	17
	1

After grouping the prime factors in triplets, it’s seen that factors 2 × 2 are left ungrouped in 3’s.

19652 = 2 × 2 × (17 × 17 × 17)

So, in order to complete it in a triplet, one more 2 is needed.

Thus, the smallest number which should be multiplied by 19652 in order to make it a perfect cube is 2.

4. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube:
(i) 1536
(ii) 10985
(iii) 28672
(iv) 13718
Solution:

(i) We have,

1536 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

After grouping the prime factors in triplets, it's seen that one factor 3 is left without grouping.

1536 = (2 × 2 × 2) × (2 × 2 × 2) × (2 × 2 × 2) × 3

So, in order to make it a perfect cube, it must be divided by 3.

Thus, the smallest number by which 1536 must be divided to obtain a perfect cube is 3.

(ii) We have,

10985 = 5 × 13 × 13 × 13

5	10985
13	2197
13	169
13	13
	1

After grouping the prime factors in triplet, it's seen that one factor, 5, is left without grouping.

10985 = 5 × (13 × 13 × 13)

So, it must be divided by 5 in order to get a perfect cube.

Thus, the required smallest number is 5.

(iii) We have,

28672 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 7

2	28672
2	14336
2	7168
2	3584
2	1792
2	896
2	448
2	224
2	112
2	56
2	28
7	14
7	7
	1

After grouping the prime factors in triplets, it's seen that one factor, 7, is left without grouping.

28672 = (2 × 2 × 2) × (2 × 2 × 2) × (2 × 2 × 2) × (2 × 2 × 2) × 7

So, it must be divided by 7 in order to get a perfect cube.

Thus, the required smallest number is 7.

(iv) 13718 = 2 × 19 × 19 × 19

2	13718
19	6859
19	361
19	19
	1

After grouping the prime factors in triplets, it's seen that one factor 2 is left without grouping.

13718 = 2 × (19 × 19 × 19)

So, it must be divided by 2 in order to get a perfect cube.

Thus, the required smallest number is 2.

5. Rahul makes a cuboid of plasticine of sides 3 cm × 3 cm × 5 cm. How many such cuboids will he need to form a cube?

Solution:

Given,

Cuboid with dimensions 3 cm × 3 cm × 5 cm

To form a cube, the number of such cuboids required are

= (15/3) x (15/3) x (15/5)

= 5 × 5 × 3

= 75

Thus, the number of cuboids required to make a cube is 75.

6. Find the volume of a cubical box whose surface area is 486 cm^2.

Solution:

Given,

The surface area of a cubical box = 486 cm^2

We know that,

Surface area of a cubical box = 6 x $(side)^2$

So,

Side = √(486/6) = 9 cm

Now, volume = $(Side)^3$

= $(9)^3$

= 9 × 9 × 9 = 729 cm^3

Thus, the volume of the cubical box is 729 cm^3.

7. Which of the following are cubes of even natural numbers or odd natural numbers:

(i) 125

(ii) 512

(iii) 1000

(iv) 2197

(v) 4096

(vi) 6859

Solution:

We know that,

The cube of an even number is even, and the cube of an odd number is odd.

Hence,

125, 2197, and 6859 are cubes of an odd number, and 512, 1000, and 4096 are cubes of an even number.

8. Write the ones digit of the cube of each of the following numbers:
(i) 231
(ii) 358
(iii) 419
(iv) 725
(v) 854
(vi) 987
(vii) 752
(viii) 893
Solution:

We know that,

The cube of numbers having 1, 4, 5, 6 or 9 in unit place will end in 1, 4, 5, 6 or 9

And, if the numbers have:

2 in unit place, then its cube ends in 8

8 in unit place, then its cube ends in 2

3 in unit place, then its cube ends in 7

7 in unit place, then its cube ends in 3

0 in unit place, then its cube ends in 0.

So now,

(i) Unit digit of number 231 is 1, hence its cube will end in 1.

(ii) Unit digit of number 358 is 8, hence its cube will end in 2.

(iii) Unit digit of number 419 is 9, hence its cube will end in 9.

(iv) Unit digit of number 725 is 5, hence its cube will end in 5.

(v) Unit digit of number 854 is 4, hence its cube will end in 4.

(vi) Unit digit of number 987 is 7, hence its cube will end in 3.

(vii) Unit digit of number 752 is 2, hence its cube will end in 8.

(viii) Unit digit of number 893 is 3, hence its cube will end in 7.

9. Find the cubes of the following numbers:
(i) -13
(ii) $3\frac{1}{5}$

(iii) $-5\frac{1}{7}$

Solution:

(i) Cube of -13 = $(-13)^3$

= (-13) × (-13) × (-13)

= -2197

(ii) Cube of $3\frac{1}{5}$

= $(16/5)^3$ = (16 x 16 x 16)/ (5 x 5 x 5)

= 4096/125

=

$$32\frac{96}{125}$$

(iii) Cube of $-5\frac{1}{7}$

= $(-36/7)^3$ = (-36 x -36 x -36)/ (7 x 7 x 7)

= -46656/49

=

$$-136\frac{8}{343}$$

Exercise 4.2

1. Find the cube root of each of the following numbers by prime factorisation:
(i) 12167
(ii) 35937
(iii) 42875
(iv) 21952
(v) 373248
(vi) 32768
(vii) 262144
(viii) 157464
Solution:

(i) $\sqrt[3]{12167}$

23	12167
23	529
23	23
	1

$= \sqrt[3]{23 \times 23 \times 23}$

$= (23^3)^{\frac{1}{3}} = 23^{\frac{1}{3} \times 3}$

$= 23^1 = 23$

Thus, the cube root of 12167 is 23.

(ii) $\sqrt[3]{35937}$

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

$= \sqrt[3]{3 \times 3 \times 3 \times 11 \times 11 \times 11}$

$= 3 \times 11 = 33$

Thus, the cube root of 35937 is 33.

(iii) $\sqrt[3]{42875}$

5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

$= \sqrt[3]{5\times5\times5\times7\times7\times7}$

$= 3 \times 7 = 35$

Thus, the cube root of 42875 is 35.

(iv) $\sqrt[3]{21952}$

2	21952
2	1076
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}$

$= 2 \times 2 \times 7 = 28$

Thus, the cube root of 21952 is 28.

(v) $\sqrt[3]{373248}$

2	373248
2	186624
2	93312
2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$= \sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2\times3\times3\times3\times3\times3\times3}$$

$$= 2 \times 2 \times 2 \times 3 \times 3 = 72$$

Thus, the cube root of 373248 is 72.

(vi) $\sqrt[3]{32768}$

2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$= \sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2 \times2\times2\times2\times2}$$

$$= 2 \times 2 \times 2 \times 2 \times 2 = 32$$

Thus, the cube root of 32768 is 32.

(vii) $\sqrt[3]{262144}$

2	262144
2	131072
2	65536
2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$= \sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2 \times2\times2\times2\times2\times2\times2\times2}$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

Thus, the cube root of 262144 is 64.

(viii) $\sqrt[3]{157464}$

2	157464
2	78732
2	39366
3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$= \sqrt[3]{2\times2\times2\times3\times3\times3\times3\times3\times3\times3\times3\times3}$

$= 2 \times 3 \times 3 \times 3 = 54$

Thus, the cube root of 157464 is 54.

2. Find the cube root of each of the following cube numbers through estimation.
(i) 19683
(ii) 59319
(iii) 85184
(iv) 148877
Solution:

(i) 19683

Grouping in 3’s from right to left, we have 19,683

In the first group, 683, the unit digit is 3

So, the cube root will end with 7

And in the second group, 19

Cubing $2^3 = 8$ and $3^3 = 27$

As, $8 < 19 < 27$

The ten's digit of the cube root will be 2

Thus, the cube root of 19683 is 27.

(ii) 59319

Grouping in 3's from right to left, we have 59,319

In first group 319, unit digit is 9

So, the unit digit of its cube root will be 9

And in the second group, 59

Cubing $3^3 = 27$ and $4^3 = 64$

As, $27 < 59 < 64$

The ten's digit of the cube root will be 3

Thus, the cube root of 59319 is 39.

(iii) 85184

Grouping in 3's from right to left, we have 85,184

In the first group 184, the unit digit is 4

So, the unit digit of its cube root will be 4

And in the second group, 85

Cubing $4^3 = 64$ and $5^3 = 125$

As, $64 < 85 < 125$

The ten's digit of the cube root will be 4

Thus, the cube root of 85184 is 44.

(iv) 148877

Grouping in 3's, from right to left, we have 148,877

In the first group, 877, the unit digit is 7

So, the unit digit of the cube root will be 3

And in the second group, 148

Cubing $5^3 = 125$, $6^3 = 216$

$125 < 148 < 216$

The ten's digit of cube root will be 5

Thus, the cube root of 148877 is 53.

3. Find the cube root of each of the following numbers:

(i) -250047 (ii) -64/1331

(iii) $4\frac{17}{27}$ (iv) $5\frac{1182}{2197}$

Solution:

(i) $\sqrt[3]{-250047} = -\sqrt[3]{250047}$

3	250047
3	83349
3	27783
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$= \sqrt[3]{3\times3\times3\times3\times3\times3\times7\times7\times7}$

$= 3 \times 3 \times 7 = 63$

$= -\sqrt[3]{250047} = -63$

Thus, the cube root of -250047 is -63.

(ii) $\sqrt[3]{\frac{-64}{1331}} = -\sqrt[3]{\frac{64}{1331}}$

Performing prime factorization for both the numerator and denominator, we have

2	64
2	32
2	16
2	8
2	4
2	2
	1

11	1331
11	121
11	11
	1

$= \sqrt[3]{\frac{2\times2\times2\times2\times2\times2}{11\times11\times11}}$

$= \frac{\sqrt[3]{2\times2\times2\times2\times2\times2}}{\sqrt[3]{11\times11\times11}}$

$= \frac{2\times2}{11} = \frac{4}{11}$

$\therefore \sqrt[3]{\frac{-64}{1331}} = \frac{-4}{11}$

(iii) $\sqrt[3]{4\frac{17}{27}} = \sqrt[3]{\frac{108+17}{27}} = \sqrt[3]{\frac{125}{27}}$

$= \sqrt[3]{\frac{5\times5\times5}{3\times3\times3}} = \frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{3\times3\times3}} = \frac{5}{3}$

$\therefore \sqrt[3]{4\frac{17}{27}} = \frac{5}{3}$

(iv) $5\frac{1182}{2197} = \frac{5\times2197+1182}{2197}$

$= \frac{10985+1182}{2197} = \frac{12167}{2197}$

$= \frac{\sqrt{23\times23\times23}}{\sqrt{13\times13\times13}} = \frac{23}{13}$

23	12167
23	529
23	23
	1

13	2197
13	169
13	13
	1

$\therefore \sqrt[3]{5\frac{1182}{2197}} = \frac{23}{13}$

4. Evaluate the following:

(i) $\sqrt[3]{512 \times 729}$

(ii) $\sqrt[3]{(-1331) \times 3375}$

Solution:

(i) $\sqrt[3]{512\times729}$

$= \sqrt[3]{512} \times \sqrt[3]{729}$

$= \sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2} \times \sqrt[3]{3\times3\times3\times3\times3\times3}$

$= (2 \times 2 \times 2) \times (3 \times 3) = 72$

$\therefore \sqrt[3]{512\times729} = 72$

3	729
3	243
3	81
3	27
3	9
3	3
	1

(ii) $\sqrt[3]{(-1331)\times(3375)}$

$= \sqrt[3]{-1331} \times \sqrt[3]{3375}$

$= -\sqrt[3]{1331} \times \sqrt[3]{3375}$

$= -\sqrt[3]{11\times11\times11} \times \sqrt[3]{3\times3\times3\times5\times5\times5}$

$= -11 \times 3 \times 5 = -11 \times 15 = -165$

$\therefore \sqrt[3]{(-1331)\times(3375)} = -165$

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

5. Find the cube root of the following decimal numbers:
(i) 0.003375
(ii) 19.683
Solution:

(i) $\sqrt[3]{0.003375} = \sqrt[3]{\frac{3375}{1000000}}$

$= \frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}}$

$= \frac{\sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5}}{\sqrt[3]{10 \times 10 \times 10 \times 10 \times 10 \times 10}}$

$= \frac{3 \times 5}{10 \times 10} = \frac{15}{100} = 0.15$

10	1000000
10	100000
10	10000
10	1000
10	100
10	10
	1

3	337
3	112
3	375
5	125
5	25
5	5
	1

(ii) $\sqrt[3]{19.683} = \sqrt[3]{\frac{19683}{1000}}$

$= \frac{\sqrt[3]{19683}}{\sqrt[3]{1000}}$

$= \frac{\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}}{\sqrt[3]{10 \times 10 \times 10}}$

$= \frac{3 \times 3 \times 3}{10} = \frac{27}{10} = 2.7$

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

6. $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

Solution:

$$\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$$
$$= \sqrt[3]{3\times3\times3} + \sqrt[3]{0.2\times0.2\times0.2} + \sqrt[3]{0.4\times0.4\times0.4}$$
$$= 3 + 0.2 + 0.4 = 3.6$$

7. Multiply 6561 by the smallest number so that the product is a perfect cube. Also, find the cube root of the product.
Solution:

Performing prime factorisation of 6561, we get

6561 = 3 × 3 × 3 × 3 × 3 × 3 × 3 × 3

3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

6561 = (3 × 3 × 3) × (3 × 3 × 3) × 3 × 3

After grouping the equal factors in 3's, it's seen that 3 × 3 is left ungrouped in 3's.

In order to complete it in triplet, we should multiply it by 3.

Hence, the required smallest number = 3

And cube root of the product = 3 × 3 × 3 = 27.

8. Divide the number 8748 by the smallest number so that the quotient is a perfect cube. Also, find the cube root of the quotient.
Solution:

The given number is 8748

On prime factorising, we get

8748 = 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3 × 3

3	8748
3	2916
3	972
3	324
3	108
3	36
3	12
2	4
2	2
	1

Grouping of the equal factor in 3's, it's seen that 2 × 2 × 3 is left without grouping.

8748 = 2 × 2 × 3 × (3 × 3 × 3) × (3 × 3 × 3)

Hence, on dividing the number 8748 by 12, we get 729

And, the cube root of 729 is 3 × 3 = 9.

9. The volume of a cubical box is 21952 m^3. Find the length of the side of the box.
Solution:

Given, the volume of a cubical box is 21952 m^2.

We know that,

It's edge = $\sqrt[3]{21952}$ m

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}$ m

= 2 × 2 × 7 = 28 m

Thus, the length of the side of the box is 28 m.

10. Three numbers are in the ratio 3 : 4 : 5. If their product is 480, find the numbers.
Solution:

Given,

Three numbers are in the ratio 3:4:5, and their product = 480

Let's assume the numbers to be 3x, 4x and 5x, then we have

3x × 4x × 5x = 480

⇒ 60 x^3 = 480

⇒ x^3 = 480/60 = 8 = $(2)^3$

∴ x = 2

Thus, the number are 2 × 3, 2 × 4 and 2 × 5 = 6, 8 and 10

11. Two numbers are in the ratio 4 : 5. If the difference of their cubes is 61, find the numbers.
Solution:

Given,

Two numbers are in the ratio = 4 : 5

Difference between their cubes = 61

Let's assume the numbers to be 4x and 5x

So, we have

$(5x)^3 - (4x)^3 = 61$

$125x^3 - 64x^3 = 61$

$61x^3 = 61$

$\Rightarrow x^3 = 1 = (1)^3$

$\therefore x = 1$

Hence, $4x = 4 \times 1 = 4$ and $5x = 5 \times 1 = 5$

Therefore, the numbers are 4 and 5.

12. Difference of two perfect cubes is 387. If the cube root of the greater of two numbers is 8, find the cube root of the smaller number.
Solution:

Given,

The difference in two cubes = 387

And, the cube root of the greater number = 8

So, the greater number = $(8)^3 = 8 \times 8 \times 8 = 512$

Hence, the second number = 512 – 387 = 125

Thus,

The cube root of 125 is

$$= \sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$$

Check Your Progress

1. Show that each of the following numbers is a perfect cube. Also, find the number whose cube is the given number:
(i) 74088
(ii) 15625
Solution:

(i) The prime factorisation of 74088 is given by,

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

= 2 × 2 × 2 × 3 × 3 × 3 × 7 × 7 × 7

And, after grouping the same kind of prime factors in 3's, its seen that no factor has been left ungrouped.

74088 = (2 × 2 × 2) × (3 × 3 × 3) × (7 × 7 × 7)

Thus, 74088 is a perfect cube, and its cube root is 2 × 3 × 7 = 42.

(ii) The prime factorisation of 15625 is given by,

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

= 5 × 5 × 5 × 5 × 5 × 5

And, after grouping the same kind of prime factors, it's seen that no factor is left ungrouped.

15625 = (5 × 5 × 5) × (5 × 5 × 5)

Thus, 15625 is a perfect cube, and its cube root is 5 × 5 = 25.

2. Find the cube of the following numbers:
(i) -17

(ii) $-3\frac{4}{9}$

Solution:

(i) Cube of -17 = (-17) × (-17) × (-17)

= -4913

(ii) Cube of $-3\frac{4}{9}$
= -31/9 is

= (-31/9) x (-31/9) x (-31/9)

= -29791/729

=

$-40\frac{631}{729}$

3. Find the cube root of each of the following numbers by prime factorisation:
(i) 59319
(ii) 21952
Solution:

(i) $\sqrt[3]{59319}$

3	59319
3	19773
3	6591
13	2197
13	169
13	13
	1

$= \sqrt[3]{3\times3\times3\times13\times13\times13}$

$= 3 \times 13 = 39$

Thus, the cube root of 59319 is 39.

(ii) $\sqrt[3]{21952}$

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$= \sqrt[3]{2\times2\times2\times2\times2\times2\times7\times7\times7}$

$= 2 \times 2 \times 7 = 28$

Thus, the cube root of 21952 is 28.

4. Find the cube root of each of the following numbers:
(i) -9261
(ii) $2\frac{43}{343}$
(iii) 0.216
Solution:

(i) $\sqrt[3]{-9261} = -\sqrt[3]{9261}$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$= -\sqrt[3]{3\times3\times3\times7\times7\times7}$

$= -3 \times 7 = -21$

(ii) $2\frac{43}{343} = \frac{686+43}{343} = \frac{729}{343}$

$\therefore \sqrt[3]{\frac{729}{343}} = \frac{\sqrt[3]{729}}{\sqrt[3]{343}}$

3	729
3	243
3	81
3	27
3	9
3	3
	1

7	343
7	49
7	7
	1

$= \frac{\sqrt[3]{3\times3\times3\times3\times3\times3}}{\sqrt[3]{7\times7\times7}}$

$= \frac{3\times3}{7} = \frac{9}{7} = 1\frac{2}{7}$

(iii) $0.216 = \frac{216}{1000}$

(*i*) $\sqrt[3]{-9261} = -\sqrt[3]{9261}$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$= -\sqrt[3]{3\times3\times3\times7\times7\times7}$

$= -3 \times 7 = -21$

(*ii*) $2\frac{43}{343} = \frac{686+43}{343} = \frac{729}{343}$

$\therefore \sqrt[3]{\frac{729}{343}} = \frac{\sqrt[3]{729}}{\sqrt[3]{343}}$

3	729
3	243
3	81
3	27
3	9
3	3
	1

7	343
7	49
7	7
	1

$= \frac{\sqrt[3]{3\times3\times3\times3\times3\times3}}{\sqrt[3]{7\times7\times7}}$

$= \frac{3\times3}{7} = \frac{9}{7} = 1\frac{2}{7}$

(*iii*) $0.216 = \frac{216}{1000}$

5. Find the smallest number by which 5184 should be multiplied so that the product is a perfect cube. Also, find the cube root of the product.
Solution:

The given number is 5184

On prime factorising of 5184, we get

2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

5184 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 3

After grouping, the same kind of prime factor is 3’s, it’s seen that one factor 3 is left ungroup.

5184 = (2 × 2 × 2) × (2 × 2 × 2) × (3 × 3 × 3) × 3

So, in order to complete it in 3’s, we must multiply the factors 3 × 3 i.e. equal to 9.

Thus, the required smallest number that should be multiplied by 5184 so that the product is a perfect cube is 9.

And,

The cube root of 5184 × 9 = 46656 is

= 2 × 2 × 3 × 3 = 36.

6. Find the smallest number by which 8788 should be divided so that the quotient is a perfect cube. Also, find the cube root of the quotient.
Solution:

On prime factorising the given number 8788, we have

2	8788
2	4394
13	2197
13	169
13	13
	1

8788 = 2 × 2 × 13 × 13 × 13

On grouping the same kind of factors, it's seen that 2 × 2 has been left ungrouping.

8788 = 2 × 2 × (13 × 13 × 13)

So, 2 × 2 = 4 is the least number by which 8788 should be divided so that the quotient is a perfect cube.

Thus, 8788 ÷ 4 = 2197 and its cube root = 13.

7. Find the side of a cube whose volume is 4096 m³.
Solution:

Given,

Volume of a cube = 4096 m^3

We know that,

$\therefore$ Its side = $\sqrt[3]{4096}$ m

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$= \sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2}$$

$$= 2 \times 2 \times 2 \times 2 = 16 \text{ m}$$

Thus, the side of the cube is 16 m.