EXERCISE 19.2

1. Find:
(i) 10^{th} term of the A.P. 1, 4, 7, 10, ...
(ii) 18^{th} term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$
(iii) nth term of the A.P 13, 8, 3, -2, ….
Solution:
(i) 10^{th} term of the A.P. 1, 4, 7, 10, ...
Arithmetic Progression (AP) whose common difference is $= a_n - a_{n-1}$ where $n > 0$
Let us consider, $a = a_1 = 1$, $a_2 = 4$ …
So, Common difference, $d = a_2 - a_1 = 4 - 1 = 3$
To find the 10^{th} term of A.P, firstly find a_n
By using the formula,
$a_n = a + (n-1) d$
$= 1 + (n-1) 3$
$= 1 + 3n - 3$
$= 3n - 2$
When n = 10:
$a_{10} = 3(10) - 2$
$= 30 - 2$
$= 28$
Hence, 10^{th} term is 28.

(ii) 18^{th} term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$
Arithmetic Progression (AP) whose common difference is $= a_n - a_{n-1}$ where $n > 0$
Let us consider, $a = a_1 = \sqrt{2}$, $a_2 = 3\sqrt{2}$ …
So, Common difference, $d = a_2 - a_1 = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$
To find the 18^{th} term of A.P, firstly find a_n
By using the formula,
$a_n = a + (n-1) d$
$= \sqrt{2} + (n - 1) 2\sqrt{2}$
$= \sqrt{2} + 2\sqrt{2}n - 2\sqrt{2}$
$= 2\sqrt{2}n - \sqrt{2}$
When n = 18:
$a_{18} = 2\sqrt{2}(18) - \sqrt{2}$
$= 36\sqrt{2} - \sqrt{2}$
$= 35\sqrt{2}$
Hence, 18^{th} term is $35\sqrt{2}$

(iii) nth term of the A.P 13, 8, 3, -2, ….
Arithmetic Progression (AP) whose common difference is = $a_n - a_{n-1}$ where n > 0
Let us consider, $a = a_1 = 13$, $a_2 = 8$ …
So, Common difference, $d = a_2 - a_1 = 8 - 13 = -5$
To find the n^{th} term of A.P, firstly find a_n
By using the formula,
$a_n = a + (n-1) d$
$= 13 + (n-1) (-5)$
$= 13 - 5n + 5$
$= 18 - 5n$
Hence, n^{th} term is 18 – 5n

2. In an A.P., show that $a_{m+n} + a_{m-n} = 2a_m$.
Solution:
We know the first term is 'a' and the common difference of an A.P is d.
Given:
$a_{m+n} + a_{m-n} = 2a_m$
By using the formula,
$a_n = a + (n - 1)d$
Now, let us take LHS: $a_{m+n} + a_{m-n}$
$a_{m+n} + a_{m-n} = a + (m + n - 1)d + a + (m - n - 1)d$
$= a + md + nd - d + a + md - nd - d$
$= 2a + 2md - 2d$
$= 2(a + md - d)$
$= 2[a + d(m - 1)]$ $\{\because a_n = a + (n - 1)d\}$
$a_{m+n} + a_{m-n} = 2a_m$
Hence Proved.

3. (i) Which term of the A.P. 3, 8, 13,… is 248 ?
(ii) Which term of the A.P. 84, 80, 76,… is 0 ?
(iii) Which term of the A.P. 4, 9, 14,… is 254 ?
Solution:
(i) Which term of the A.P. 3, 8, 13,… is 248 ?
Given A.P is 3, 8, 13,…
Here, $a_1 = a = 3$, $a_2 = 8$
Common difference, $d = a_2 - a_1 = 8 - 3 = 5$
We know, $a_n = a + (n - 1)d$
$a_n = 3 + (n - 1)5$
$= 3 + 5n - 5$

$= 5n - 2$

Now, to find which term of A.P is 248

Put $a_n = 248$

$\therefore 5n - 2 = 248$

$= 248 + 2$

$= 250$

$= 250/5$

$= 50$

Hence, 50th term of given A.P is 248.

(ii) Which term of the A.P. 84, 80, 76,… is 0 ?

Given A.P is 84, 80, 76,…

Here, $a_1 = a = 84$, $a_2 = 88$

Common difference, $d = a_2 - a_1 = 80 - 84 = -4$

We know, $a_n = a + (n - 1)d$

$a_n = 84 + (n - 1)-4$

$= 84 - 4n + 4$

$= 88 - 4n$

Now, to find which term of A.P is 0

Put $a_n = 0$

$88 - 4n = 0$

$-4n = -88$

$n = 88/4$

$= 22$

Hence, 22nd term of given A.P is 0.

(iii) Which term of the A.P. 4, 9, 14,… is 254 ?

Given A.P is 4, 9, 14,…

Here, $a_1 = a = 4$, $a_2 = 9$

Common difference, $d = a_2 - a_1 = 9 - 4 = 5$

We know, $a_n = a + (n - 1)d$

$a_n = 4 + (n - 1)5$

$= 4 + 5n - 5$

$= 5n - 1$

Now, to find which term of A.P is 254

Put $a_n = 254$

$5n - 1 = 254$

$5n = 254 + 1$

5n = 255
n = 255/5
= 51
Hence, 51^{st} term of given A.P is 254.

4. (i) Is 68 a term of the A.P. 7, 10, 13,…?
(ii) Is 302 a term of the A.P. 3, 8, 13,…?
Solution:
(i) Is 68 a term of the A.P. 7, 10, 13,…?
Given A.P is 7, 10, 13,…
Here, $a_1 = a = 7$, $a_2 = 10$
Common difference, $d = a_2 - a_1 = 10 - 7 = 3$
We know, $a_n = a + (n - 1)d$ [where, a is first term or a_1 and d is common difference and n is any natural number]
$a_n = 7 + (n - 1)3$
$= 7 + 3n - 3$
$= 3n + 4$
Now, to find whether 68 is a term of this A.P. or not
Put $a_n = 68$
3n + 4 = 68
3n = 68 – 4
3n = 64
n = 64/3
64/3 is not a natural number
Hence, 68 is not a term of given A.P.

(ii) Is 302 a term of the A.P. 3, 8, 13,…?
Given A.P is 3, 8, 13,…
Here, $a_1 = a = 3$, $a_2 = 8$
Common difference, $d = a_2 - a_1 = 8 - 3 = 5$
We know, $a_n = a + (n - 1)d$
$a_n = 3 + (n - 1)5$
$= 3 + 5n - 5$
$= 5n - 2$
To find whether 302 is a term of this A.P. or not
Put $a_n = 302$
5n – 2 = 302
5n = 302 + 2
5n = 304

n = 304/5
304/5 is not a natural number
Hence, 304 is not a term of given A.P.

5. (i) Which term of the sequence 24, 23 ¼, 22 ½, 21 ¾ is the first negative term?
Solution:
Given:
AP: 24, 23 ¼, 22 ½, 21 ¾, … = 24, 93/4, 45/2, 87/4, …
Here, $a_1 = a = 24$, $a_2 = 93/4$
Common difference, $d = a_2 – a_1 = 93/4 – 24$
$= (93 - 96)/4$
$= - 3/4$
We know, $a_n = a + (n – 1) d$ [where a is first term or a_1 and d is common difference and n is any natural number]
We know, $a_n = a + (n – 1) d$
$a_n = 24 + (n - 1) (-3/4)$
$= 24 – 3/4n + ¾$
$= (96+3)/4 – 3/4n$
$= 99/4 – 3/4n$
Now we need to find, first negative term.
Put $a_n < 0$
$a_n = 99/4 – 3/4n < 0$
$99/4 < 3/4n$
$3n > 99$
$n > 99/3$
$n > 33$
Hence, 34^{th} term is the first negative term of given AP.

(ii) Which term of the sequence 12 + 8i, 11 + 6i, 10 + 4i, … is (a) purely real (b) purely imaginary ?
Solution:
Given:
AP: 12 + 8i, 11 + 6i, 10 + 4i, …
Here, $a_1 = a = 12 + 8i$, $a_2 = 11 + 6i$
Common difference, $d = a_2 – a_1$
$= 11 + 6i – (12 + 8i)$
$= 11 – 12 + 6i – 8i$
$= -1 – 2i$
We know, $a_n = a + (n – 1) d$ [where a is first term or a_1 and d is common difference and n

is any natural number]

$a_n = 12 + 8i + (n – 1) -1 – 2i$
$= 12 + 8i – n – 2ni + 1 + 2i$
$= 13 + 10i – n – 2ni$
$= (13 – n) + (10 – 2n) i$

To find purely real term of this A.P., imaginary part have to be zero

$10 – 2n = 0$
$2n = 10$
$n = 10/2$
$= 5$

Hence, 5^{th} term is purely real.

To find purely imaginary term of this A.P., real part have to be zero

$\therefore 13 – n = 0$
$n = 13$

Hence, 13^{th} term is purely imaginary.

6. (i) How many terms are in A.P. 7, 10, 13,…43?

Solution:

Given:

AP: 7, 10, 13,…

Here, $a_1 = a = 7$, $a_2 = 10$

Common difference, $d = a_2 – a_1 = 10 – 7 = 3$

We know, $a_n = a + (n – 1) d$ [where a is first term or a_1 and d is common difference and n is any natural number]

$a_n = 7 + (n – 1)3$
$= 7 + 3n – 3$
$= 3n + 4$

To find total terms of the A.P., put $a_n = 43$ as 43 is last term of A.P.

$3n + 4 = 43$
$3n = 43 – 4$
$3n = 39$
$n = 39/3$
$= 13$

Hence, total 13 terms exists in the given A.P.

(ii) How many terms are there in the A.P. -1, -5/6, -2/3, -1/2, …, 10/3 ?

Solution:

Given:

AP: -1, -5/6, -2/3, -1/2, …

Here, $a_1 = a = -1$, $a_2 = -5/6$
Common difference, $d = a_2 - a_1$
$= -5/6 - (-1)$
$= -5/6 + 1$
$= (-5+6)/6$
$= 1/6$
We know, $a_n = a + (n - 1) d$ [where a is first term or a_1 and d is common difference and n is any natural number]
$a_n = -1 + (n - 1) 1/6$
$= -1 + 1/6n - 1/6$
$= (-6-1)/6 + 1/6n$
$= -7/6 + 1/6n$
To find total terms of the AP,
Put $a_n = 10/3$ [Since, 10/3 is the last term of AP]
$a_n = -7/6 + 1/6n = 10/3$
$1/6n = 10/3 + 7/6$
$1/6n = (20+7)/6$
$1/6n = 27/6$
$n = 27$
Hence, total 27 terms exists in the given A.P.

7. The first term of an A.P. is 5, the common difference is 3, and the last term is 80; find the number of terms.
Solution:
Given:
First term, $a = 5$; last term, $l = a_n = 80$
Common difference, $d = 3$
We know, $a_n = a + (n - 1) d$ [where a is first term or a_1 and d is common difference and n is any natural number]
$a_n = 5 + (n - 1)3$
$= 5 + 3n - 3$
$= 3n + 2$
To find total terms of the A.P., put $a_n = 80$ as 80 is last term of A.P.
$3n + 2 = 80$
$3n = 80 - 2$
$3n = 78$
$n = 78/3$
$= 26$
Hence, total 26 terms exists in the given A.P.

8. The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively. Find the 40^{th} term.

Solution:

Given:

6^{th} term of an A.P is 19 and 17^{th} terms of an A.P. is 41

So, $a_6 = 19$ and $a_{17} = 41$

We know, $a_n = a + (n – 1) d$ [where a is first term or a_1 and d is common difference and n is any natural number]

When n = 6:

$a_6 = a + (6 – 1) d$

$= a + 5d$

Similarly, When n = 17:

$a_{17} = a + (17 – 1)d$

$= a + 16d$

According to question:

$a_6 = 19$ and $a_{17} = 41$

a + 5d = 19..........................(i)

And a + 16d = 41…............. (ii)

Let us subtract equation (i) from (ii) we get,

$a + 16d – (a + 5d) = 41 – 19$

$a + 16d – a – 5d = 22$

$11d = 22$

$d = 22/11$

$= 2$

put the value of d in equation (i):

$a + 5(2) = 19$

$a + 10 = 19$

$a = 19 – 10$

$= 9$

As, $a_n = a + (n – 1)d$

$a_{40} = a + (40 – 1)d$

$= a + 39d$

Now put the value of a = 9 and d = 2 in a_{40} we get,

$a_{40} = 9 + 39(2)$

$= 9 + 78$

$= 87$

Hence, 40^{th} term of the given A.P. is 87.

9. If 9^{th} term of an A.P. is Zero, prove that its 29^{th} term is double the 19^{th} term.

Solution:

Given:

9^{th} term of an A.P is 0

So, $a_9 = 0$

We need to prove: $a_{29} = 2a_{19}$

We know, $a_n = a + (n – 1) d$ [where a is first term or a_1 and d is common difference and n is any natural number]

When n = 9:

$a_9 = a + (9 – 1)d$

$= a + 8d$

According to question:

$a_9 = 0$

$a + 8d = 0$

$a = -8d$

When n = 19:

$a_{19} = a + (19 – 1)d$

$= a + 18d$

$= -8d + 18d$

$= 10d$

When n = 29:

$a_{29} = a + (29 – 1)d$

$= a + 28d$

$= -8d + 28d$ [Since, a = -8d]

$= 20d$

$= 2×10d$

$a_{29} = 2a_{19}$ [Since, $a_{19} = 10d$]

Hence Proved.

10. If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that the 25^{th} term of the A.P. is Zero.

Solution:

Given:

10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term

So, $10a_{10} = 15a_{15}$

We need to prove: $a_{25} = 0$
We know, $a_n = a + (n – 1) d$ [where a is first term or a_1 and d is common difference and n is any natural number]
When n = 10:
$a_{10} = a + (10 – 1)d$
$= a + 9d$

When n = 15:
$a_{15} = a + (15 – 1)d$
$= a + 14d$

When n = 25:
$a_{25} = a + (25 – 1)d$
$= a + 24d$ ………(i)
According to question:
$10a_{10} = 15a_{15}$
$10(a + 9d) = 15(a + 14d)$
$10a + 90d = 15a + 210d$
$10a – 15a + 90d – 210d = 0$
$-5a – 120d = 0$
$-5(a + 24d) = 0$
$a + 24d = 0$
$a_{25} = 0$ [From (i)]
Hence Proved.

11. The 10^{th} and 18^{th} term of an A.P. are 41 and 73 respectively, find 26^{th} term.
Solution:
Given:
10^{th} term of an A.P is 41, and 18^{th} terms of an A.P. is 73
So, $a_{10} = 41$ and $a_{18} = 73$
We know, $a_n = a + (n – 1) d$ [where a is first term or a_1 and d is the common difference and n is any natural number]
When n = 10:
$a_{10} = a + (10 – 1)d$
$= a + 9d$

When n = 18:
$a_{18} = a + (18 – 1)d$
$= a + 17d$

According to question:
$a_{10} = 41$ and $a_{18} = 73$
$a + 9d = 41$ ....(i)
And $a + 17d = 73$… .... (ii)
Let us subtract equation (i) from (ii) we get,
$a + 17d – (a + 9d) = 73 – 41$
$a + 17d – a – 9d = 32$
$8d = 32$
$d = 32/8$
$d = 4$

Put the value of d in equation (i) we get,
$a + 9(4) = 41$
$a + 36 = 41$
$a = 41 – 36$
$a = 5$
we know, $a_n = a + (n – 1)d$
$a_{26} = a + (26 – 1)d$
$= a + 25d$
Now put the value of $a = 5$ and $d = 4$ in a_{26}
$a_{26} = 5 + 25(4)$
$= 5 + 100$
$= 105$
Hence, 26^{th} term of the given A.P. is 105.

12. In a certain A.P. the 24^{th} term is twice the 10^{th} term. Prove that the 72^{nd} term is twice the 34^{th} term.
Solution:
Given:
24^{th} term is twice the 10^{th} term
So, $a_{24} = 2a_{10}$
We need to prove: $a_{72} = 2a_{34}$
We know, $a_n = a + (n – 1)$ d [where a is first term or a_1 and d is common difference and n is any natural number]
When n = 10:
$a_{10} = a + (10 – 1)d$
$= a + 9d$

When n = 24:

$a_{24} = a + (24 – 1)d$
$= a + 23d$

When n = 34:
$a_{34} = a + (34 – 1)d$
$= a + 33d$ ………(i)

When n = 72:
$a_{72} = a + (72 – 1)d$
$= a + 71d$
According to question:
$a_{24} = 2a_{10}$
$a + 23d = 2(a + 9d)$
$a + 23d = 2a + 18d$
$a – 2a + 23d – 18d = 0$
$-a + 5d = 0$
$a = 5d$

Now, $a_{72} = a + 71d$
$a_{72} = 5d + 71d$
$= 76d$
$= 10d + 66d$
$= 2(5d + 33d)$
$= 2(a + 33d)$ [since, a = 5d]
$a_{72} = 2a_{34}$ (From (i))
Hence Proved.