EXERCISE 12.3

1. Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the cube so formed.

Solution:

We know that,

Volume of cube = a^3, where a = side of cube

According to the question,

Side of first cube, a_1 = 3 cm

Side of second cube, a_2 = 4 cm

Side of third cube, a_3 = 5 cm

Let us assume that the side of cube recast from melting these cubes = a

We know that the total volume of the 3 cubes will be the same as the volume of the newly formed cube,

Volume of new cube = (volume of 1^{st} + 2^{nd} + 3^{rd} cube)

$\Rightarrow a^3 = a_1^3 + a_2^3 + a^3$

$\Rightarrow a^3 = (3)^3 + (4)^3 + (5)^3$

$\Rightarrow a^3 = 27 + 64 + 125 = 216$

$\Rightarrow$ a = 6 cm

Therefore, side of cube so formed is 6 cm.

2. How many shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9cm × 11cm × 12cm?

Solution:

Volume of cuboid = lbh, where, l = length, b = breadth and h = height

Cuboidal lead:

Length, l = 9 cm

Breadth, b = 11 cm

Height, h = 12 cm

Volume of lead = 9(11)(12) = 1188 cm^3

Volume of sphere = $4/3\pi r^3$, where r = radius of sphere

Spherical shots,

Diameter = 3 cm

Radius, r = 1.5 cm

Volume of one shot = $4/3 \times 22/7 \times (1.5)^3 = 99/7$ cm^3

$$\text{No. of shots can be made} = \frac{\text{Volume of lead}}{\text{Volume of one shot}} = \frac{1188}{\frac{99}{7}} = \frac{1188\times7}{99} = 84$$

Hence, the number of bullets that can be made from lead = 84.

3. A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm, respectively. Find the height of the bucket.

Solution:

According to the question,

The bucket is in the form of frustum of a cone.

We know that,

Volume of frustum of a cone = $1/3\ \pi h(r_1^2 + r_2^2 + r_1r_2)$, where, h = height, r_1 and r_2 are the radii($r_1 > r_2$)

For bucket,

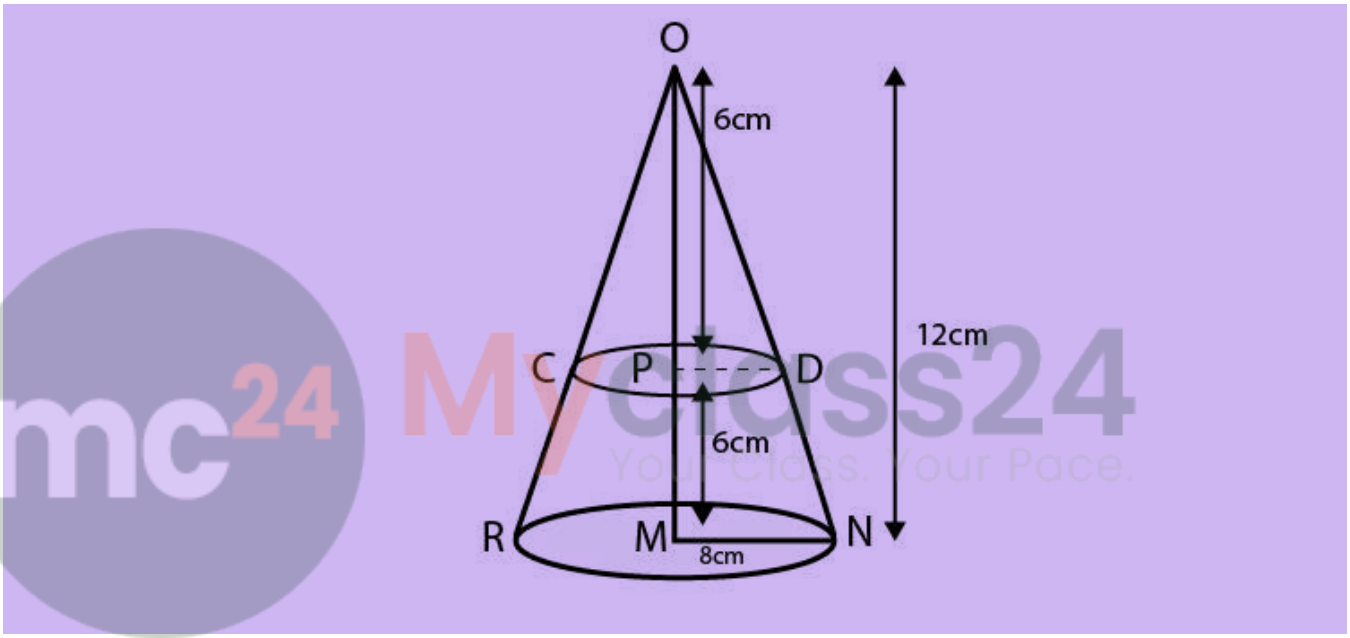
Volume of bucket = 28.490 L

1 L = 1000 cm^3

Volume of bucket = 28490 cm^3

Radius of top, r_1 = 28 cm
Radius of bottom, r_2 = 21 cm
Let the height = h.
Substituting these values in the equation to find the volume of bucket,
We have.
Volume of bucket = 1/3 πh[28^2 + 21^2 + 28(21)]
28490 = 1/3 × 22/7 × h (784 + 441 + 588) = 22/7 × h × 1813
⇒ h = (28490 × 21) / (22 × 1813)
⇒ h = 15

4. A cone of radius 8 cm and height 12 cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.
Solution:

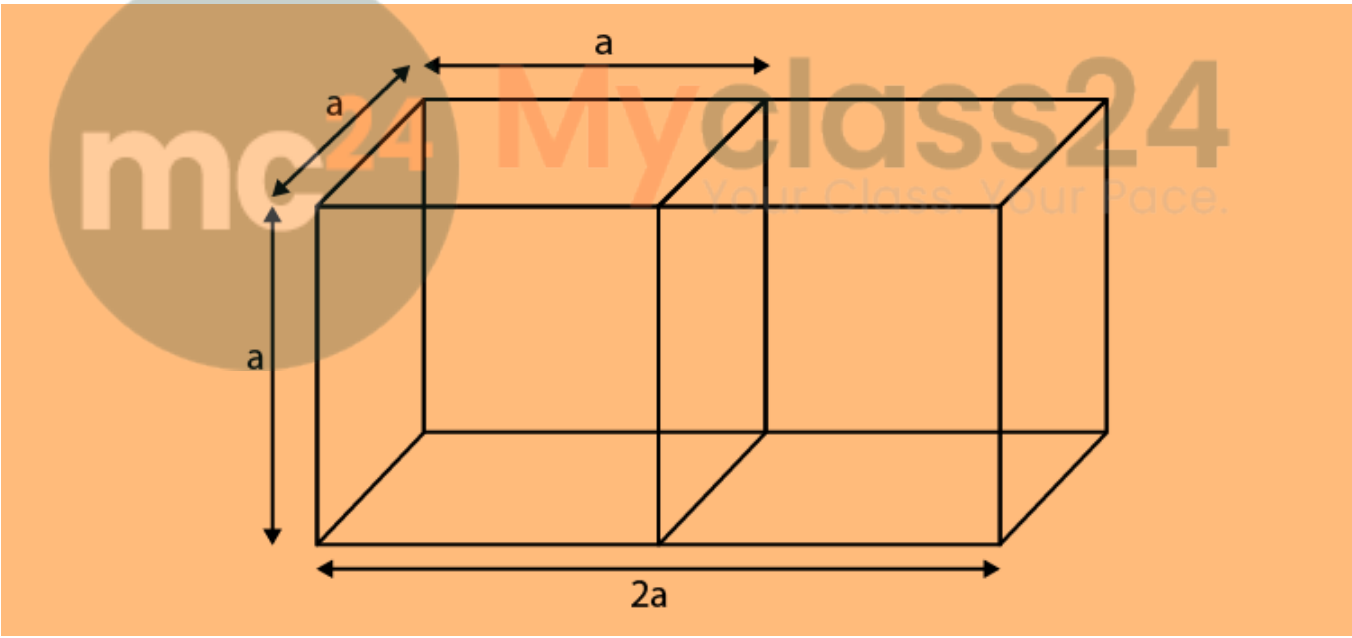


According to the question,
Height of cone = OM = 12 cm
The cone is divided from mid-point.
Hence, let the mid-point of cone = P
OP = PM = 6 cm
From △OPD and △OMN
∠POD = ∠POD [Common]
∠OPD = ∠OMN [Both 90°]
Hence, by the Angle-Angle similarity criterion
We have,
△OPD ~ △OMN
And
Similar triangles have corresponding sides in equal ratio,
So, we have,
PD/MN = OP/OM
PD/8 = 6/12
PD = 4cm
[MN = 8 cm = radius of base of cone]

For First part i.e. cone
Base Radius, r = PD = 4 cm
Height, h = OP = 6 cm
We know that,
Volume of cone for radius r and height h, $V = 1/3\ \pi r^2 h$
Volume of first part = $1/3\ \pi(4)^2 6 = 32\pi$

For second part, i.e. Frustum
Bottom radius, r_1 = MN = 8 cm
Top Radius, r_2 = PD = 4 cm
Height, h = PM = 6 cm
We know that,
Volume of frustum of a cone = $1/3\ \pi h(r_1^2 + r_2^2 + r_1 r_2)$, where, h = height, r_1 and r_2 are radii, ($r_1 > r$
Volume of second part = $1/3\ \pi(6)[8^2 + 4^2 + 8(4)]$
= $2\pi(112) = 224\pi$
Therefore, we get the ratio,
Volume of first part : Volume of second part = $32\pi : 224\pi = 1 : 7$

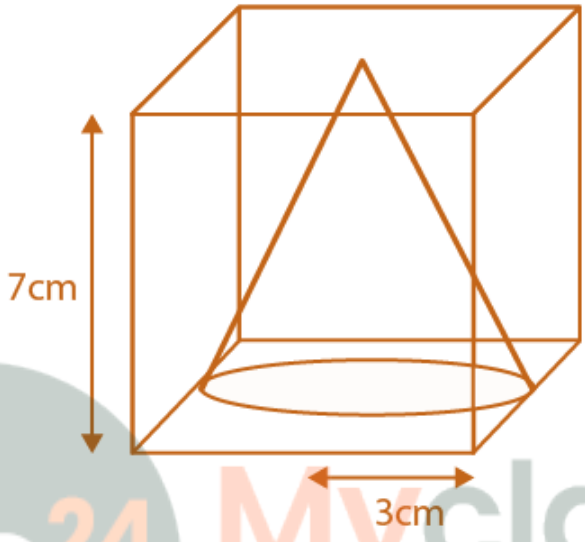
5. Two identical cubes each of volume 64 cm3 are joined together end to end. What is the surface area of the resulting cuboid?
Solution:



Let the side of one cube = a
Surfaces area of resulting cuboid = 2(Total surface area of a cube) - 2(area of single surface)
We know that,
Total surface area of cube = $6a^2$, where a = side of cube
⇒ Surfaces area of resulting cuboid = $2(6a^2) - 2(a^2) = 10a^2$
Also,
According to the question,
Volume of cube = 64 cm^3
Volume of cube = a^3
$64 = a^3$

a = 4 cm
Therefore,
Surface area of resulting cuboid = $10a^2 = 10(4)^2 = 160\ cm^2$

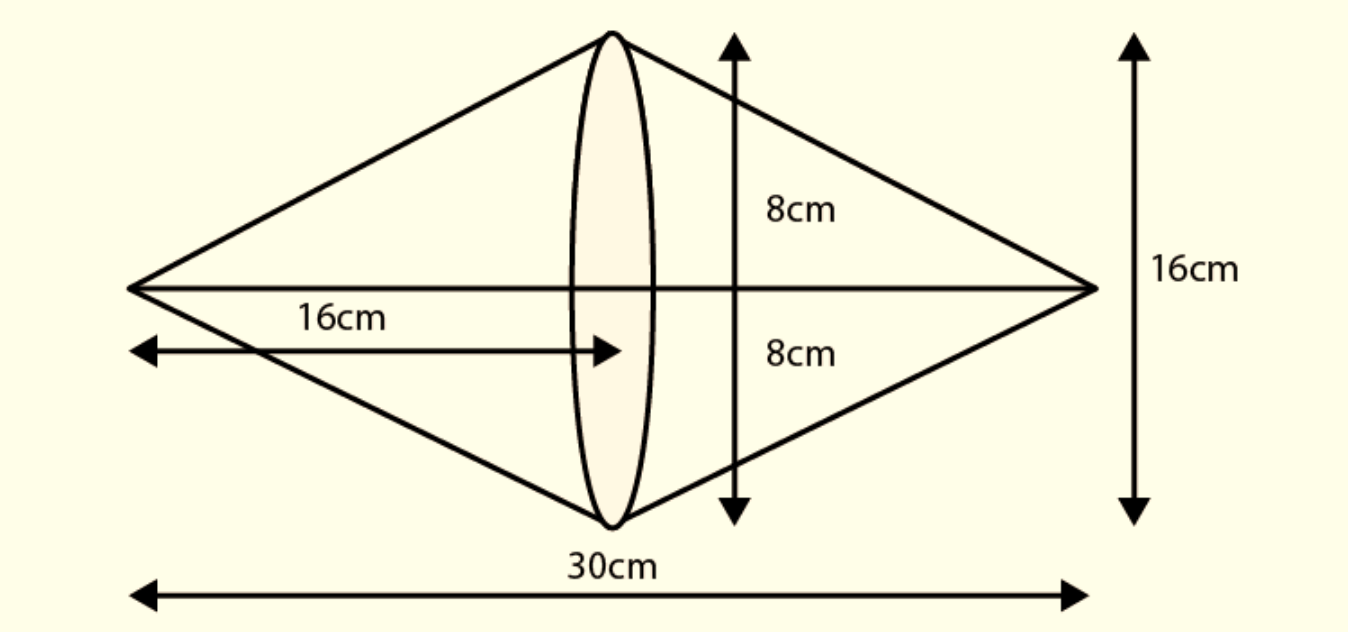
6. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid.
Solution:



From the figure, we get,
Volume of remaining solid = volume of cube - volume of cone
For Cube
Side, a = 7 cm
We know that,
Volume of cube = a^3, where a = side of cube
Volume of cube = $(7)^3 = 343\ cm^3$
For cone
Radius, r = 3 cm
Height, h = 7 cm
Volume of cone = $1/3\ \pi r^2h = 1/3\ \pi(3)^27 = 3 \times (22/7) \times 7 = 66\ cm^3$
Volume of remaining solid = volume of cube - volume of cone
= 343 – 66
= $277\ cm^3$

7. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.
Solution:

According to the question,
We get the figure given below,



We know that,
Total surface area of shape formed = Curved area of first cone + Curved surface area of second cone
Since, both cones are identical,
We have,
Total surface area of shape formed = Curved area of first cone + Curved surface area of second cone
= 2(Surface area of cone)
We also know that,
Surface area of cone = πrl, where r = radius and l = slant height
And, the total Surface area of shape so formed = 2πrl
Given in the question that,
Radius, r = 8 cm
Height, h = 15 cm
Therefore,
Area = 2(3.14)(8)(15) = 753.6 cm^2