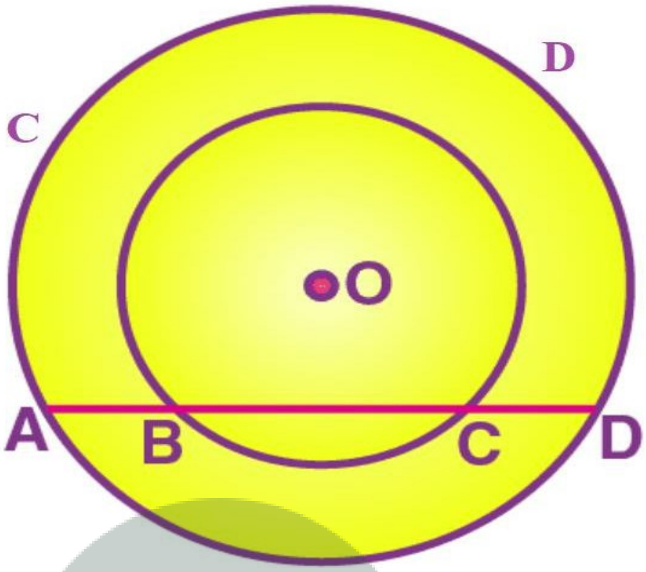
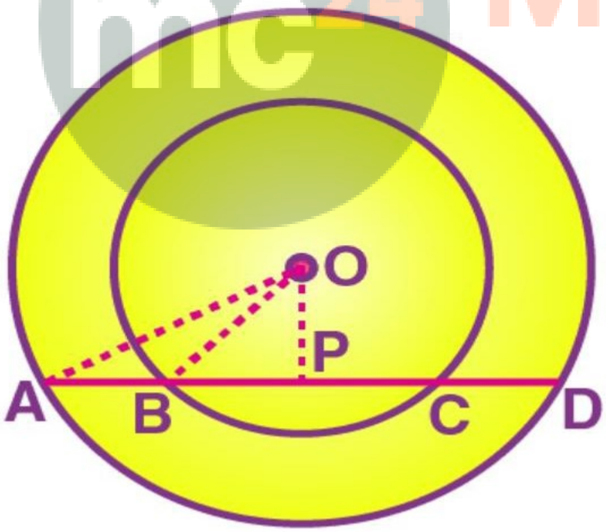
EXERCISE 17B

1. The figure shows two concentric circles and AD is a chord of larger circle. Prove that: AB = CD.



Solution:

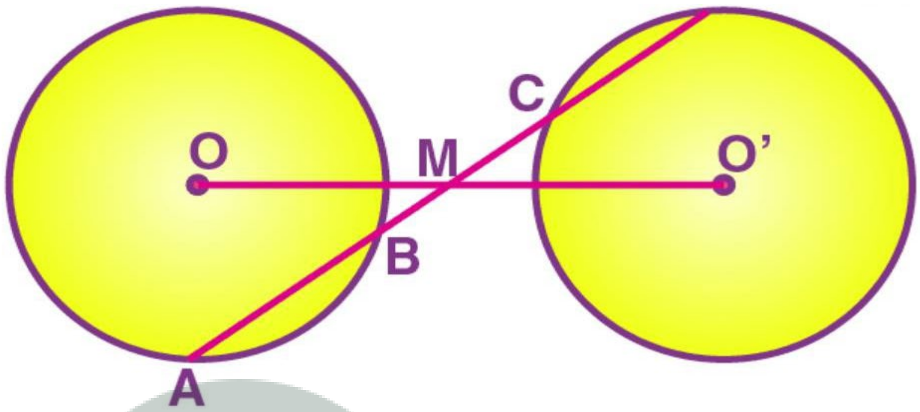


Draw OP ⊥ AD
So OP bisects AD
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
AP = PD ……… (i)

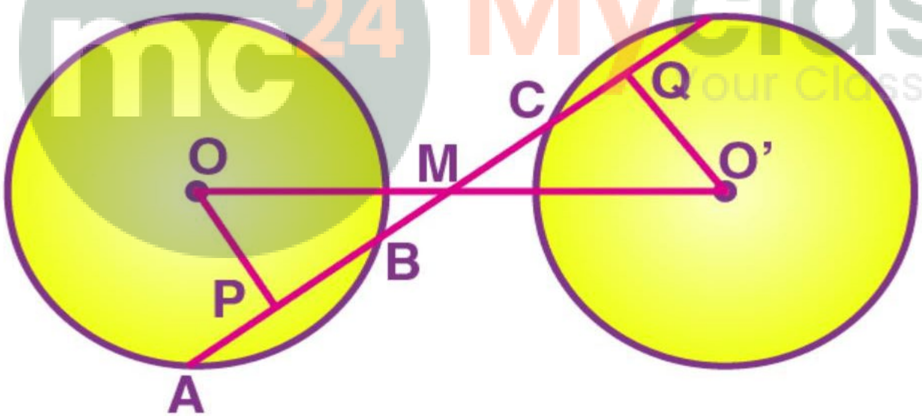
BC is a chord for the inner circle and OP ⊥ BC
So OP bisects BC
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
BP = PC ....(ii)

By subtracting equation (ii) from (i),
AP – BP = PD – PC
AB = CD

2. A straight line is drawn cutting two equal circles and passing through the midpoint M of the line joining their centres O and O'.
Prove that chords AB and CD, which are intercepted by the two circles are equal.



Solution:



Given –
A straight line AD intersects two circles of equal radii at A, B, C and D.
Line joining the centres OO' intersect AD at M
M is the midpoint of OO'

To prove – AB = CD.

Construction – From the centre O, draw OP ⊥ AB and from O' draw O'Q ⊥ CD.

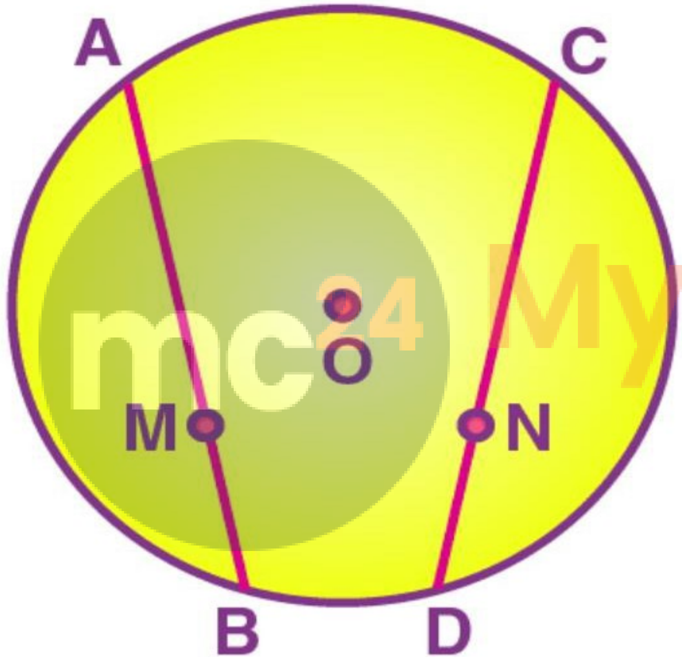
Proof –
In △ OMP and △ O'MQ,
∠ OMP = ∠ O'MQ [vertically opposite angles]
∠ OPM = ∠ O'QM [each = 90^0]

OM = O'M [given]

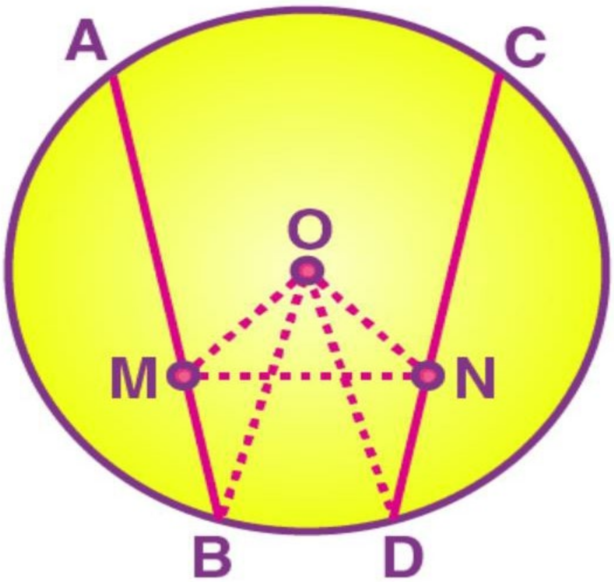
By AAS criterion of congruence,
△ OMP ≅ △ O'MP
OP = O'Q [c.p.c.t]

Here, two chords of a circle or equal circles which are equidistant from the centre are equal.
AB = CD.

3. M and N are the mid-points of two equal chords AB and CD respectively of a circle with centre O. Prove that:
(i) ∠ BMN = ∠ DNM,
(ii) ∠ AMN = ∠ CNM.



Solution:



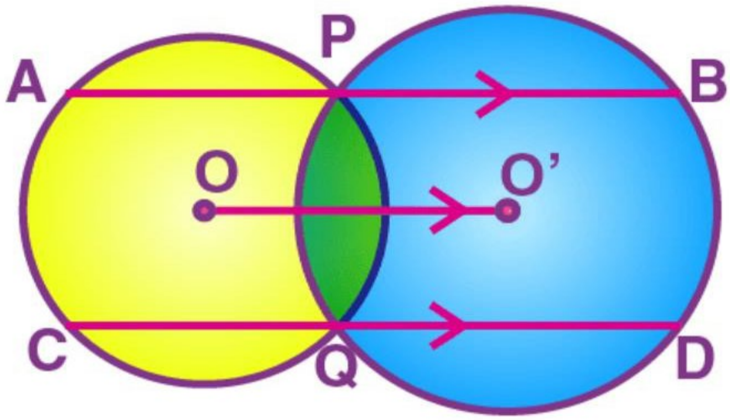
Draw OM ⊥ AB and ON ⊥ CD
So OM bisects AB and ON bisects CD
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
BM = ½ AB = ½ CD = DN.......... (1)

In △ OMB,
$OM^2 = OB^2 + BM^2$ [Using Pythagoras Theorem]
We can write it as
$OM^2 = OD^2 – DN^2$ [using equation (1)]
$OM^2 = ON^2$
OM = ON
So we get
∠ OMN = ∠ ONM ....(2) [Angles opposite to the equal sides are equal]

(i) ∠ OMB = ∠ OND [both 90^0]
By subtracting (2) from above
∠ BMN = ∠ DNM

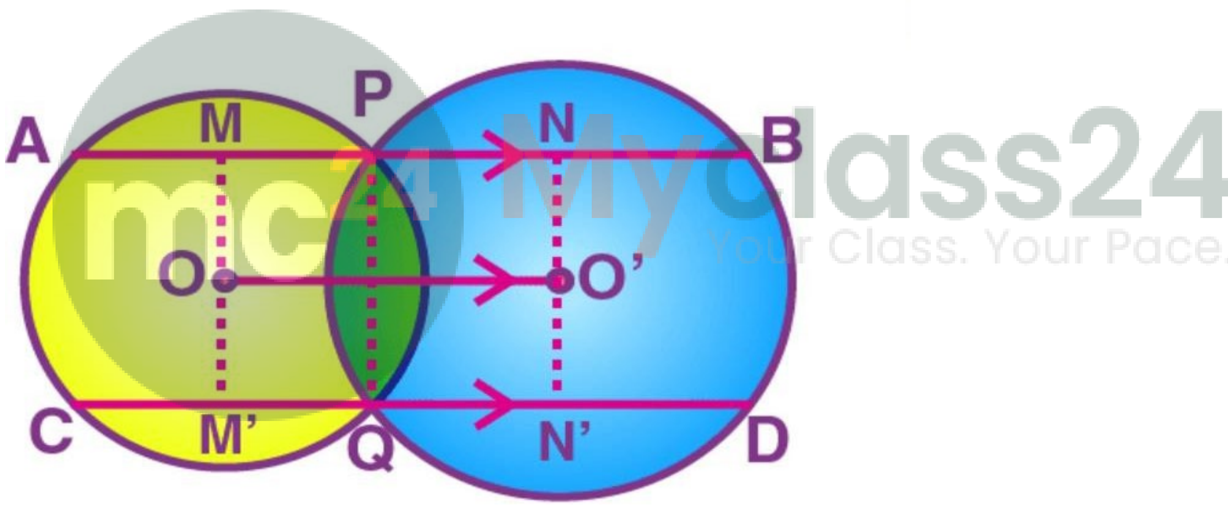
(ii) ∠ OMA = ∠ ONC [both 90^0]
By adding (2) to above
∠ AMN = ∠ CNM

4. In the following figure: P and Q are the points of intersection of two circles with centres O and O’. If straight lines APB and CQD are parallel to OO’. Prove that
(i) OO’ = ½ AB
(ii) AB = CD



Solution:

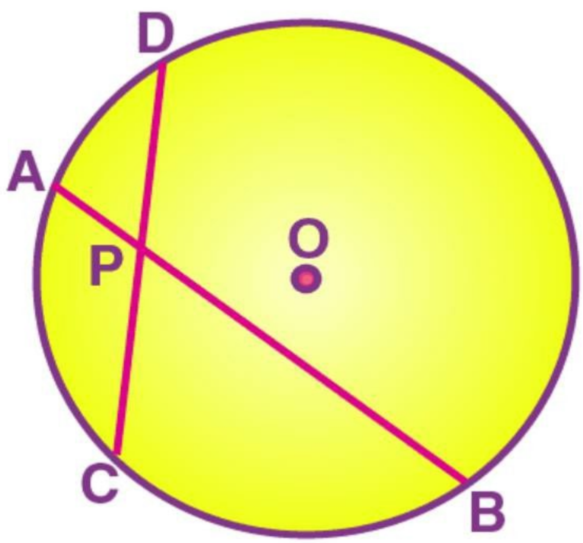
Draw OM and ON perpendicular on AB and OM' and O'N' perpendicular on CD.



So OM, O'N, OM' and O'N' bisect AP, PB, CQ and QD respectively
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
MP = ½ AP, PN = ½ BP, M'Q = ½ CQ, QN' = ½ QD

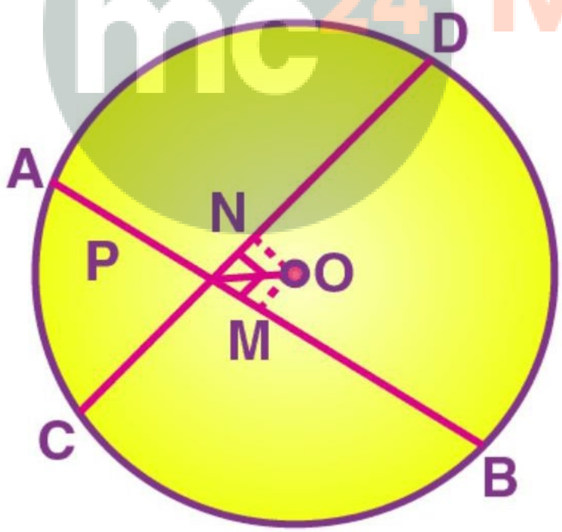
We know that
OO' = MN = MP + PN = ½ (AP + BP) = ½ AB ……. (i)
OO' = M'N' = M'Q + QN' = ½ (CQ + QD) = ½ CD …… (ii)
Equating (i) and (ii)
AB = CD

5. Two equal chords AB and CD of a circle with centre O, intersect each other at a point P inside the circle. Prove that:
(i) AP = CP
(ii) BP = DP



Solution:

Draw OM and ON perpendicular on AB and CD.
Join OP, OB and OD.



So OM and ON bisect AB and CD respectively.
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
MB = ½ AB = ½ CD = ND ……. (i)

In right triangle △ OMB,
$OM^2 = OB^2 – MB^2$ ……. (ii)
In right triangle △ OND,
$ON^2 = OD^2 – ND^2$ ……. (iii)
From equation (i), (ii) and (iii)

OM = ON

In △ OPM and △ OPN,
∠ OMP = ∠ ONP [both 90^0]
OP = OP [Common]
OM = ON [Proved]

Using RHS criterion of congruence,
△ OPM ≅ △ OPN
PM = PN [c.p.c.t]

By adding (i) both sides
MB + PM = ND + PN
BP = DP

We know that
AB = CD
AB – BP = CD – DP [BP = DP]
AP = CP