$$2I = \int_0^{\frac{\pi}{2}} \log\frac{2\sin x\cos x}{2}dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\sin 2x - \log 2\, dx$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2I = \frac{1}{2}\int_0^{\pi} \log\sin t\, dt - \frac{\pi}{2}\log 2$$

$$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log\sin x\, dx - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin x\, dx \ = -\frac{\pi}{2}\log 2$$

Similarly, $\int_0^{\frac{\pi}{2}} \log\cos x\, dx = -\frac{\pi}{2}\log 2$

$$y = \int_0^{\frac{\pi}{2}} \log 2\, dx + \int_0^{\frac{\pi}{2}} \log\sin x\, dx + \int_0^{\frac{\pi}{2}} \log\cos x\, dx$$

$$y = \frac{\pi}{2}\log 2 - \frac{\pi}{2}\log 2 - \frac{\pi}{2}\log 2$$

$$y = -\frac{\pi}{2}\log 2$$

35. Question

Prove that

$$\int_0^{\pi} x\log(\sin x)dx = -\frac{\pi^2}{2}(\log\ 2)$$

Answer

$y = \int_0^{\pi} x\log\sin x\, dx$ …(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_0^{\pi} (\pi - x)\log\sin(\pi - x)\, dx$$

$y = \int_0^{\pi} \pi \log \sin x - x \log \sin x \, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} x \log \sin x \, dx + \int_0^{\pi} \pi \log \sin x - x \log \sin x \, dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \log \sin x \, dx$$

$y = \frac{2\pi}{2} \int_0^{\frac{\pi}{2}} \log \sin x \, dx$ ...(3)

Use King theorem of definite integral

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

$$y = \pi \int_0^{\frac{\pi}{2}} \log \sin \left(\frac{\pi}{2} - x\right) dx$$

$y = \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx$ ...(4)

Adding eq.(3) and eq.(4)

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \sin x \, dx + \int_0^{\frac{\pi}{2}} \log \cos x \, dx \right)$$

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} dx \right)$$

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2 \, dx \right)$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2y = \frac{\pi}{2} \int_0^{\pi} \log \sin t \, dt - \frac{\pi^2}{2} \log 2$$

$$2y = \frac{2\pi}{2} \int_0^{\frac{\pi}{2}} \log \sin x \, dx - \frac{\pi^2}{2} \log 2$$

$$2y = y - \frac{\pi^2}{2} \log 2$$

$$y = -\frac{\pi^2}{2} \log 2$$

36. Question

Prove that

$$\int_0^{\pi} \log(1+\cos x)\,dx = -\pi(\log 2)$$

Answer

$y = \int_0^{\pi} \log(1+\cos x)\,dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_0^{\pi} \log(1+\cos(\pi - x))\,dx$$

$y = \int_0^{\pi} \log(1-\cos x)\,dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \log(1+\cos x)\,dx + \int_0^{\pi} \log(1-\cos x)\,dx$$

$$2y = \int_0^{\pi} \log sin^2 x\,dx$$

$y = 2\int_0^{\frac{\pi}{2}} \log \sin x\,dx$ ...(3)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = 2\int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right)dx$$

$y = 2\int_0^{\frac{\pi}{2}} \log \cos x\,dx$ ...(4)

Adding eq.(3) and eq.(4)

$$2y = 2\left(\int_0^{\frac{\pi}{2}} \log \sin x\,dx + \int_0^{\frac{\pi}{2}} \log \cos x\,dx\right)$$

$$2y = 2\left(\int_0^{\frac{\pi}{2}} \log \frac{2\sin x \cos x}{2}\,dx\right)$$

$$2y = 2\left(\int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2\,dx\right)$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2y = \frac{2}{2}\int_0^{\pi} \log\sin t\,dt - \frac{2\pi}{2}\log 2$$

$$2y = \frac{4}{2}\int_0^{\frac{\pi}{2}} \log\sin x\,dx - \frac{2\pi}{2}\log 2$$

$$2y = y - \pi\log 2$$

$$y = -\pi\log 2$$

37. Question

Prove that

$$\int_0^{\pi/2} \log(\tan x + \cot x)\,dx = \pi(\log 2)$$

Answer

$$y = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) dx$$

$$y = \int_0^{\frac{\pi}{2}} \log\frac{1}{\sin x\cos x}\,dx$$

$$y = -\left(\int_0^{\frac{\pi}{2}} \log\sin x\,dx + \int_0^{\frac{\pi}{2}} \log\cos x\,dx\right)$$

Let, $I = \int_0^{\frac{\pi}{2}} \log\sin x\,dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2} - x\right) dx$$

$I = \int_0^{\frac{\pi}{2}} \log\cos x\,dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log\sin x\,dx + \int_0^{\frac{\pi}{2}} \log\cos x\,dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\frac{2\sin x\cos x}{2}\,dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\sin 2x - \log 2\,dx$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2I = \frac{1}{2}\int_0^{\pi} \log\sin t\,dt - \frac{\pi}{2}\log 2$$

$$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log\sin x\,dx - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin x\,dx = -\frac{\pi}{2}\log 2$$

Similarly, $\int_0^{\frac{\pi}{2}} \log\cos x\,dx = -\frac{\pi}{2}\log 2$

$$y = -\left(\int_0^{\frac{\pi}{2}} \log\sin x\,dx + \int_0^{\frac{\pi}{2}} \log\cos x\,dx\right)$$

$$y = \frac{\pi}{2}\log 2 + \frac{\pi}{2}\log 2$$

$$y = \pi\log 2$$

38. Question

Prove that

$$\int_{\pi/8}^{3\pi/8} \frac{\cos x}{(\cos x + \sin x)}\,dx = \frac{\pi}{8}$$

Answer

$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos x}{\cos x + \sin x}\,dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos\left(\frac{3\pi}{8} + \frac{\pi}{8} - x\right)}{\sin\left(\frac{3\pi}{8} + \frac{\pi}{8} - x\right) + \cos\left(\frac{3\pi}{8} + \frac{\pi}{8} - x\right)} dx$$

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x}{\sin x + \cos x} dx \quad \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos x}{\sin x + \cos x} dx + \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x}{\sin x + \cos x} dx$$

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} 1\, dx$$

$$2y = (x)_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$$

$$2y = \frac{3\pi}{8} - \frac{\pi}{8}$$

$$y = \frac{\pi}{8}$$

39. Question

Prove that

$$\int_{\pi/6}^{\pi/3} \frac{1}{\left(1 + \sqrt{\tan x}\right)} dx = \frac{\pi}{12}$$

Answer

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} + \sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}\right)} dx$$

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\left(\sqrt{\cos x} + \sqrt{\sin x}\right)} dx \quad \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\left(\sqrt{\sin x} + \sqrt{\cos x}\right)} dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\left(\sqrt{\cos x} + \sqrt{\sin x}\right)} dx$$

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\left(\sqrt{\sin x} + \sqrt{\cos x}\right)} dx$$

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\,dx$$

$$2y = (x)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$y = \frac{\pi}{12}$$

40. Question

Prove that

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{(1+\cos x)} = 2$$

Answer

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2\cos^2\frac{x}{2}} dx$$

$$y = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} sec^2\frac{x}{2} dx$$

$$y = \frac{1}{2}\left(\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right)_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$y = \tan\frac{3\pi}{8} - \tan\frac{\pi}{8}$$

$$y = (\sqrt{2}+1) - (\sqrt{2}-1) = 2$$

41. Question

Prove that

$$\int_{\pi/4}^{3\pi/4} \frac{x}{(1+\sin x)} dx = \pi\left(\sqrt{2}-1\right)$$

Answer

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx \quad ...(1)$$

Use King theorem of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}{1+\sin\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)} dx$$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1+\sin x} dx \quad ...(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1+\sin x} dx$$

$$y = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+\sin x} dx$$

$$y = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$$

$$y = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{cos^2 x} dx$$

$$y = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} sec^2 x - \frac{\sin x}{cos^2 x} dx$$

Let, cos x = t

⇒ -sin x dx = dt

At x = π/4, t = $\frac{1}{\sqrt{2}}$

At x = 3π/4, t = $\frac{-1}{\sqrt{2}}$

$$y = \frac{\pi}{2} \left((\tan x)_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + \int_{\frac{1}{\sqrt{2}}}^{\frac{-1}{\sqrt{2}}} \frac{1}{t^2} dt \right)$$

$$y = \frac{\pi}{2} (\tan \frac{3\pi}{4} - \tan \frac{\pi}{4} + \left(\frac{-1}{t}\right)_{\frac{1}{\sqrt{2}}}^{\frac{-1}{\sqrt{2}}}$$

$$y = \frac{\pi}{2} \left(-1 - 1 + \sqrt{2} + \sqrt{2}\right) = \pi\left(\sqrt{2} - 1\right)$$

42. Question

Prove that

$$\int_{\alpha/4}^{3\alpha/4} \frac{\sqrt{x}}{\left(\sqrt{a-x} + \sqrt{x}\right)} dx = \frac{a}{4}$$

Answer

$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{\frac{3a}{4} + \frac{a}{4} - x}}{\sqrt{\frac{3a}{4} + \frac{a}{4} - x} + \sqrt{x}} dx$$

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}} dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}} dx + \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}} dx$$

$$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x} + \sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}} dx$$

$$y = \frac{1}{2} \int_{\frac{a}{4}}^{\frac{3a}{4}} 1\, dx$$

$$y = \frac{1}{2} (x)_{\frac{a}{4}}^{\frac{3a}{4}}$$

$$y = \frac{a}{4}$$

43. Question

Prove that

$$\int_1^4 \frac{\sqrt{x}}{\left(\sqrt{5 - x} + \sqrt{x}\right)} dx = \frac{3}{2}$$

Answer

$$y = \int_1^4 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$$

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

$$y = \int_1^4 \frac{\sqrt{4 + 1 - x}}{\sqrt{4 + 1 - x} + \sqrt{x}} dx$$

$$y = \int_1^4 \frac{\sqrt{5 - x}}{\sqrt{5 - x} + \sqrt{x}} dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_1^4 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5 - x}} dx + \int_1^4 \frac{\sqrt{5 - x}}{\sqrt{5 - x} + \sqrt{x}} dx$$

$$2y = \int_1^4 \frac{\sqrt{x} + \sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx$$

$$y = \frac{1}{2}\int_1^4 1\, dx$$

$$y = \frac{1}{2}(x)_1^4$$

$$y = \frac{3}{2}$$

44. Question

Prove that

$$\int_0^{\pi/2} x \cot x\, dx = \frac{\pi}{4}(\log 2)$$

Answer

Use integration by parts

$$\int I \times II\, dx = I \int II\, dx - \int \frac{d}{dx} I \left(\int II\, dx\right) dx$$

$$y = x \int \cot x\, dx - \int \frac{d}{dx} x \left(\int \cot x\, dx\right) dx$$

$$y = (x \log \sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin x\, dx$$

Let, $I = \int_0^{\frac{\pi}{2}} \log \sin x\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$I = \int_0^{\frac{\pi}{2}} \log \cos x\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin x\, dx + \int_0^{\frac{\pi}{2}} \log \cos x\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\sin 2x - \log 2\, dx$$

Let, 2x = t

⇒ 2 dx = dt

At x = 0, t = 0

At x = π/2, t = π

$$2I = \frac{1}{2}\int_0^{\pi} \log\sin t\, dt - \frac{\pi}{2}\log 2$$

$$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log\sin x\, dx - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin x\, dx \;=\; -\frac{\pi}{2}\log 2$$

$$y = (x\log\sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log\sin x\, dx$$

$$y = \frac{\pi}{2}\log\sin\frac{\pi}{2} - \left(-\frac{\pi}{2}\log 2\right)$$

$$y = \frac{\pi}{2}\log 2$$

45. Question

Prove that

$$\int_0^1 \left(\frac{\sin^{-1} x}{x}\right) dx = \frac{\pi}{2}(\log 2)$$

Answer

Let, x = sin t

⇒ dx = cos t dt

At x = 0, t = 0

At x = 1, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin^{-1}\sin t}{\sin t}\cos t\, dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{t\cos t}{\sin t}\, dt$$

$$y = \int_0^{\frac{\pi}{2}} t \cot t \, dt$$

Use integration by parts

$$\int I \times II \, dt = I \int II \, dt - \int \frac{d}{dt} I \left(\int II \, dt \right) dt$$

$$y = t \int \cot t \, dt - \int \frac{d}{dt} t \left(\int \cot t \, dt \right) dt$$

$$y = (t \log \sin t)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin t \, dt$$

Let, $I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t) \, dt = \int_a^b f(a + b - t) \, dt$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin \left(\frac{\pi}{2} - t \right) dt$$

$I = \int_0^{\frac{\pi}{2}} \log \cos t \, dt$ ...(2)

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt + \int_0^{\frac{\pi}{2}} \log \cos t \, dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin t \cos t}{2} dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2t - \log 2 \, dt$$

Let, 2t = z

⇒ 2 dt = dz

At t = 0, z = 0

At t = π/2, z = π

$$2I = \frac{1}{2} \int_0^{\pi} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin z\, dz = -\frac{\pi}{2}\log 2$$

$$y = (t\log\sin t)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log t\, dt$$

$$y = \frac{\pi}{2}\log\sin\frac{\pi}{2} - \left(-\frac{\pi}{2}\log 2\right)$$

$$y = \frac{\pi}{2}\log 2$$

46. Question

Prove that

$$\int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx = -\frac{\pi}{2}(\log 2)$$

Answer

Use integration by parts

$$\int I \times II\, dx = I\int II\, dx - \int \frac{d}{dx}I\left(\int II\, dx\right) dx$$

$$y = \log x \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{d}{dx}\log x\left(\int \frac{1}{\sqrt{1-x^2}} dx\right) dx$$

$$y = (\log x \sin^{-1} x)_0^1 - \int_0^1 \frac{\sin^{-1} x}{x} dx$$

$$y = -\int_0^1 \frac{\sin^{-1} x}{x} dx$$

Let, x = sin t

⇒ dx = cos t dt

At x = 0, t = 0

At x = 1, t = π/2

$$y = -\int_0^{\frac{\pi}{2}} \frac{\sin^{-1}\sin t}{\sin t}\cos t\, dt$$

$$y = -\int_0^{\frac{\pi}{2}} \frac{t\cos t}{\sin t} dt$$

$$y = -\int_0^{\frac{\pi}{2}} t\cot t\, dt$$

Use integration by parts

$$\int I \times II\,dt = I\int II\,dt - \int \frac{d}{dt}I\left(\int II\,dt\right)dt$$

$$y = -\left(t\int \cot t\,dt - \int \frac{d}{dt}t\left(\int \cot t\,dt\right)dt\right)$$

$$y = -\left((t\log\sin t)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log\sin t\,dt\right)$$

Let, $I = \int_0^{\frac{\pi}{2}} \log\sin t\,dt$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\,dt = \int_a^b f(a+b-t)\,dt$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2} - t\right)dt$$

$I = \int_0^{\frac{\pi}{2}} \log\cos t\,dt$ ...(2)

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log\sin t\,dt + \int_0^{\frac{\pi}{2}} \log\cos t\,dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log\frac{2\sin t\cos t}{2}\,dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log\sin 2t - \log 2\,dt$$

Let, 2t = z

⇒ 2 dt = dz

At t = 0, z = 0

At t = π/2, z = π

$$2I = \frac{1}{2}\int_0^{\pi} \log\sin z\,dz - \frac{\pi}{2}\log 2$$

$$2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log\sin z\,dz - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin z\,dz = -\frac{\pi}{2}\log 2$$

$$y=-\left((t\log\sin t)_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\log t\,dt\right)$$

$$y=\frac{-\pi}{2}\log\sin\frac{\pi}{2}+\left(-\frac{\pi}{2}\log 2\right)$$

$$y=\frac{-\pi}{2}\log 2$$

47. Question

Prove that

$$\int_0^1\frac{\log(1+x)}{(1+x^2)}dx=\frac{\pi}{8}(\log\ 2)$$

Answer

Let x = tan t

⇒ dx = $\sec^2 t$ dt

At x = 0, t = 0

At x = 1, t = π/4

$$y=\int_0^{\frac{\pi}{4}}\frac{\log(1+\tan t)}{1+tan^2 t}sec^2 t\,dt$$

$$y=\int_0^{\frac{\pi}{4}}\log(1+\tan t)\,dt\ \ldots(1)$$

Use King theorem of definite integral

$$\int_a^b f(t)\,dt=\int_a^b f(a+b-t)\,dt$$

$$y=\int_0^{\frac{\pi}{4}}\log\left(1+\tan\left(\frac{\pi}{4}-t\right)\right)dt$$

$$y=\int_0^{\frac{\pi}{4}}\log\left(1+\frac{1-\tan t}{1+\tan t}\right)dt$$

$$y=\int_0^{\frac{\pi}{4}}\log\left(\frac{2}{1+\tan t}\right)dt\ \ldots(2)$$

Adding eq.(1) and eq.(2)

$$2y=\int_0^{\frac{\pi}{4}}\log(1+\tan t)\,dt+\int_0^{\frac{\pi}{4}}\log\left(\frac{2}{1+\tan t}\right)dt$$

$$2y=\int_0^{\frac{\pi}{4}}\log(1+\tan t)\left(\frac{2}{1+\tan t}\right)dt$$

$$2y = \int_0^{\frac{\pi}{4}} \log 2\, dt$$

$$y = \frac{\pi}{8}\log 2$$

48. Question

Prove that

$$\int_{-a}^{a} x^3\sqrt{a^2 - x^2}\, dx = 0$$

Answer

$y = \int_{-a}^{a} x^3\sqrt{a^2 - x^2}\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\, dt = \int_a^b f(a+b-t)\, dt$$

$$y = \int_{-a}^{a} (a-a-x)^3\sqrt{a^2-(a-a-x)^2}\, dx$$

$y = \int_{-a}^{a} -x^3\sqrt{a^2 - x^2}\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_{-a}^{a} x^3\sqrt{a^2-x^2}\, dx + \left(-\int_{-a}^{a} x^3\sqrt{a^2-x^2}\, dx\right)$$

y = 0

49. Question

Prove that

$$\int_{-\pi}^{\pi}\left(\sin^{75} x + x^{125}\right)dx = 0$$

Answer

$y = \int_{-\pi}^{\pi} sin^{75} x + x^{125}\, dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\, dt = \int_a^b f(a+b-t)\, dt$$

$$y = \int_{-\pi}^{\pi} sin^{75}(\pi-\pi-x) + (\pi-\pi-x)^{125}\, dx$$

$y = \int_{-\pi}^{\pi} -sin^{75} x - x^{125}\, dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_{-\pi}^{\pi} sin^{75}x + x^{125}\,dx + \left(-\int_{-\pi}^{\pi} sin^{75}x + x^{125}\,dx\right)$$

y = 0

50. Question

Prove that

$$\int_{-\pi}^{\pi} x^{12}\sin^{9}x\,dx = 0$$

Answer

$y = \int_{-\pi}^{\pi} x^{12}sin^9x\,dx$ ...(1)

Use King theorem of definite integral

$$\int_a^b f(t)\,dt = \int_a^b f(a+b-t)\,dt$$

$$y = \int_{-\pi}^{\pi} (\pi - \pi - x)^{12} sin^9(\pi - \pi - x)\,dx$$

$y = \int_{-\pi}^{\pi} -x^{12}sin^9x\,dx$ ...(2)

Adding eq.(1) and eq.(2)

$$2y = \int_{-\pi}^{\pi} x^{12}sin^9x\,dx + \left(-\int_{-\pi}^{\pi} x^{12}sin^9x\,dx\right)$$

y = 0

51. Question

Prove that

$$\int_{-1}^{1} e^{|x|}dx = 2(e-1)$$

Answer

We know that

|x| = -x in [-1, 0)

|x| = x in [0, 1]

$$y = \int_{-1}^{0} e^{|x|}\,dx + \int_{0}^{1} e^{|x|}\,dx$$

$$y = \int_{-1}^{0} e^{-x}\,dx + \int_{0}^{1} e^{x}\,dx$$

$y = (-e^{-x})_{-1}^{0} + (e^x)_0^1$

y= -(1-e)+(e-1)

y = 2(e – 1)

52. Question

$$\int_{-2}^{2} |x+1| dx = 6$$

Answer

We know that

|x+1| = -(x+1) in [-2, -1)

|x+1| = (x+1) in [-1, 2]

$$y = \int_{-2}^{-1} |x+1|\, dx + \int_{-1}^{2} |x+1|\, dx$$

$$= -\int_{-2}^{-1} (x+1)\, dx + \int_{-1}^{2} (x+1)\, dx$$

$$= -\left(\frac{x^2}{2} + x\right)_{-2}^{-1} + \left(\frac{x^2}{2} + x\right)_{-1}^{2}$$

$$= -\left(\frac{1}{2} - 1 - 2 + 2\right) + \left(2 + 2 - \frac{1}{2} + 1\right)$$

=5

53. Question

Prove that

$$\int_{0}^{8} |x-5| dx = 17$$

Answer

We know that

|x – 5| = -(x – 5) in [0, 5)

|x – 5| = (x – 5) in [5, 8]

$$y = \int_{0}^{5} |x-5|\, dx + \int_{5}^{8} |x-5|\, dx$$

$$y = -\int_{0}^{5} (x-5)\, dx + \int_{5}^{8} (x-5)\, dx$$

$$y = -\left(\frac{x^2}{2} - 5x\right)_{0}^{5} + \left(\frac{x^2}{2} - 5x\right)_{5}^{8}$$

$$y = -\left(\frac{25}{2} - 25\right) + \left(32 - 40 - \frac{25}{2} + 25\right)$$

=17

54. Question

Prove that

$$\int_0^{2\pi} |\cos x| dx = 4$$

Answer

We know that

|cos x| = cos x in [0, π/2)

|cos x| = -cos x in [π/2, 3π/2)

|cos x| = cos x in [3π/2, 2π]

$$y = \int_0^{\frac{\pi}{2}} |\cos x| dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |\cos x| dx + \int_{\frac{3\pi}{2}}^{2\pi} |\cos x| dx$$

$$y = \int_0^{\frac{\pi}{2}} \cos x\, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x\, dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x\, dx$$

$$y = (\sin x)_0^{\frac{\pi}{2}} - (\sin x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + (\sin x)_{\frac{3\pi}{2}}^{2\pi}$$

y=(1-0)—1-1+(0+1)

=4

55. Question

Prove that

$$\int_{-\pi/4}^{\pi/4} |\sin x| dx = \left(2 - \sqrt{2}\right)$$

Answer

We know that

|sin x| = -sin x in [-π/4, 0)

|sin x| = sin x in [0, π/4]

$$y = \int_{\frac{-\pi}{4}}^{0} |\sin x| dx + \int_0^{\frac{\pi}{4}} |\sin x| dx$$

$$y = -\int_{\frac{-\pi}{4}}^{0} \sin x\, dx + \int_0^{\frac{\pi}{4}} \sin x\, dx$$

$$y = -(-\cos x)_{\frac{-\pi}{4}}^{0} + (-\cos x)_0^{\frac{\pi}{4}}$$

$$y = \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= 2 - \frac{1}{\sqrt{2}}$$

56. Question

Prove that

Let $f(x)=\begin{cases}2x+1, \text{ when } 1\le x\le 2\\ x^2+1, \text{ when } 2\le x\le 3\end{cases}$

Show that $\int_1^3 f(x)dx=\frac{34}{3}.$

Answer

$y = \int_1^3 f(x)\,dx$

$$y=\int_1^2 f(x)\,dx+\int_2^3 f(x)\,dx$$

$$y=\int_1^2 2x+1\,dx+\int_2^3 x^2+1\,dx$$

$$y=(x^2+x)_1^2+\left(\frac{x^3}{3}+x\right)_2^3$$

$$y=(4+2-1-1)+\left(9+3-\frac{8}{3}-2\right)$$

$$=\frac{34}{3}$$

57. Question

Prove that

Let $f(x)=\begin{cases}3x^2+4, \text{ when } 0\le x\le 2\\ 9x-2, \quad \text{when } 2\le x\le 4\end{cases}$

Show that $\int_0^4 f(x)dx=66$

Answer

$y = \int_0^4 f(x)\,dx$

$$y=\int_0^2 f(x)\,dx+\int_2^4 f(x)\,dx$$

$$y=\int_0^2 3x^2+4\,dx+\int_2^4 9x-2\,dx$$

$$y=(x^3+4x)_0^2+\left(\frac{9x^2}{2}-2x\right)_2^4$$

y=(8+8)+(72-8-18+4)

=66

58. Question

Prove that

$$\int_0^4 \{|x|+|x-2|+|x-4|dx\} = 20$$

Answer

$$y = \int_0^4 |x| + |x-2| + |x-4|\, dx$$

$$y = \int_0^2 |x| + |x-2| + |x-4|\, dx + \int_2^4 |x| + |x-2| + |x-4|\, dx$$

$$y = \int_0^2 x - (x-2) - (x-4)\, dx + \int_2^4 x + (x-2) - (x-4)\, dx$$

$$y = \left(-\frac{x^2}{2} + 6x\right)_0^2 + \left(\frac{x^2}{2} + 2x\right)_2^4$$

y=(-2+12)+(8+8-2-4)

=20

Exercise 16D

1. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_0^2 (x+4)dx$$

Answer

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_0^2 (x+4)dx = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f(2r/n)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} \left(\frac{2r}{n}\right) + 4$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \left(\frac{(n-1)(n)}{n} + 4(n-1)\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \frac{n^2 - n + 4n^2 - 4n}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n} \frac{5n^2 - 5n}{n}$$

$$= \lim_{n\to\infty} \frac{10n^2 - 10n}{n^2}$$

$$= \lim_{n\to\infty} 10 - (10/n)$$

=10

2. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_1^2 (3x - 2)dx$$

Answer

f(x) is continuous in [1,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a + rh), where\ h = (b - a)/n$$

here h=1/n

$$\int_1^2 (3x - 2)dx = \lim_{n\to\infty} \left(\frac{1}{n}\right) \sum_{r=0}^{n-1} f\left(1 + \left(\frac{r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right) \sum_{r=0}^{n-1} (3 + 3\frac{r}{n} - 2)$$

$$\lim_{n\to\infty} \left(\frac{1}{n}\right) \left(n + \frac{3(n-1)(n)}{2n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right) \left(\frac{2n^2 + 3n^2 - 3n}{2n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{5n^2 - 3n}{2n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{5}{2}\right) - \left(\frac{3}{2n}\right)$$

=5/2

3. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_1^3 x^2 dx$$

Answer

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_1^3 (x^2)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(1+\left(\frac{2r}{n}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(1+\left(\frac{2r}{n}\right)\right)^2$$

$$=\lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{4r^2}{n^2}+1+\frac{4r}{n}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n-1)(n)(2n-1)}{6n^2}+n+\frac{4(n-1)(n)}{2n}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\left(\frac{4(2n^3-2n^2-n^2+n)}{6n^2}+n+\frac{2(n^2-n)}{n}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\left(\frac{(8n^3-12n^2+4n)+(6n^3)+(12n^3-12n^2)}{6n^2}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\left(\frac{26n^3-24n^2+4n}{6n^2}\right)$$

$$=\lim_{n\to\infty}\left(\frac{52n^3-48n^2+8n}{6n^3}\right)$$

$$=\lim_{n\to\infty}\left(\frac{52}{6}\right)-\left(\frac{26}{6n}\right)+\left(\frac{8}{6n^2}\right)$$

=26/3

4. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_0^3 \left(x^2+1\right)dx$$

Answer

f(x) is continuous in [0,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_0^3 (x^2+1)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\left(\frac{3r}{n}\right)^2+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\frac{9r^2}{n^2}+1\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n-1)(n)(2n-1)}{6n^2}+n\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n^2-n)(2n-1)}{6n^2}+n\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{9(2n^3-2n^2-n^2+n)}{6n^2}+n)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(18n^3-27n^2+9n)+(6n^3)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{24n^3-27n^2+9n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{72n^3-81n^2+27n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{72}{6}\right)-\left(\frac{81}{6n}\right)+\left(\frac{27}{6n^2}\right)$$

=12

5. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_2^5 \left(3x^2-5\right)dx$$

Answer

f(x) is continuous in [2,5]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_{2}^{5}(3x^2-5)dx=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}f\left(\left(2+\frac{3r}{n}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(3\left(2+\frac{3r}{n}\right)^2-5)$$

$$=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}3\left(\frac{9r^2}{n^2}+4+\frac{12r}{n}\right)-5$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n-1)(n)(2n-1)}{6n^2}+12n+\frac{18n(n-1)}{n}-5n\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n^2-n)(2n-1)}{6n^2}+12n+\frac{18n(n-1)}{n}-5n\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(2n^3-2n^2-n^2+n)}{6n^2}+12n+\frac{18n(n-1)}{n}-5n\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{(54n^3-81n^2+27n)+(42n^3)+(108n^3-108n^2)}{6n^2}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{204n^3-189n^2+27n}{6n^2}\right)$$

$$=\lim_{n\to\infty}\left(\frac{612n^3-567n^2+27n}{6n^3}\right)$$

$$=\lim_{n\to\infty}\left(\frac{612}{6}\right)-\left(\frac{567}{6n}\right)+\left(\frac{27}{6n^2}\right)$$

=102

6. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_{0}^{3}\left(x^2+2x\right)dx$$

Answer

f(x) is continuous in [2,5]

$$\int_a^b f(x)dx=\lim_{n\to\infty}h\sum_{r=0}^{n-1}f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_0^3 (x^2+2x)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(\left(\frac{3r}{n}\right)^2 + \frac{6r}{n})$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\frac{9r^2}{n^2} + \frac{6r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n-1)(n)(2n-1)}{6n^2} + \frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n^2-n)(2n-1)}{6n^2} + \frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{9(2n^3-2n^2-n^2+n)}{6n^2} + \frac{3n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(18n^3-27n^2+9n)+(18n^3-18n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{36n^3-45n^2+9n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{108n^3-135n^2+27n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{108}{6}\right) - \left(\frac{135}{6n}\right) + \left(\frac{27}{6n^2}\right)$$

=18

7. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_1^4 \left(3x^2 + 2x\right)dx$$

Answer

f(x) is continuous in [1,4]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_1^4 (3x^2+2x)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{3r}{n}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(3\left(1+\frac{3r}{n}\right)^2+2\left(1+\frac{3r}{n}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}3\left(\frac{9r^2}{n^2}+1+\frac{6r}{n}\right)+2\left(1+\frac{3r}{n}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n-1)(n)(2n-1)}{6n^2}+3n+\frac{9n(n-1)}{n}+2n+\frac{3n(n-1)}{n}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n^2-n)(2n-1)}{6n^2}+5n+\frac{12n(n-1)}{n}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{27(2n^3-2n^2-n^2+n)}{6n^2}+5n+\frac{12n(n-1)}{n}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{(54n^3-81n^2+27n)+(30n^3)+(72n^3-72n^2)}{6n^2}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\frac{156n^3-153n^2+27n}{6n^2}\right)$$

$$=\lim_{n\to\infty}\left(\frac{468n^3-459n^2+81n}{6n^3}\right)$$

$$=\lim_{n\to\infty}\left(\frac{468}{6}\right)-\left(\frac{459}{6n}\right)+\left(\frac{81}{6n^2}\right)$$

=78

8. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_1^3 \left(x^2+5x\right)dx$$

Answer

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_1^3 (x^2+5x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(\left(1+\frac{2r}{n}\right)^2 + 5\left(1+\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(1+\frac{4r^2}{n^2}+\frac{4r}{n}+5+\frac{10r}{n})$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(1+\frac{4r^2}{n^2}+\frac{4r}{n}+5+\frac{10r}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{4(n-1)(n)(2n-1)}{6n^2}+6n+\frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{4(n^2-n)(2n-1)}{6n^2}+6n+\frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{4(2n^3-2n^2-n^2+n)}{6n^2}+6n+\frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{(8n^3-12n^2+4n)+(42n^3-42n^2)+(36n^3)}{6n^2})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{86n^3-54n^2+4n}{6n^2})$$

$$= \lim_{n\to\infty}(\frac{172n^3-108n^2+8n}{6n^3})$$

$$= \lim_{n\to\infty}\left(\frac{172}{6}\right)-\left(\frac{108}{6n}\right)+\left(\frac{8}{6n^2}\right)$$

=86/3

9. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_1^3 \left(2x^2+5x\right)dx$$

Answer

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_1^3 (2x^2+5x)dx = \lim_{n\to\infty}(\frac{2}{n})\sum_{r=0}^{n-1} f\left((1+\frac{2r}{n})\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(2\left(1+\frac{2r}{n}\right)^2 + 5\left(1+\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}(2+\frac{8r^2}{n^2}+\frac{8r}{n}+5+\frac{10r}{n})$$

$$= \lim_{n\to\infty}(\frac{2}{n})\sum_{r=0}^{n-1}(7+\frac{8r^2}{n^2}+\frac{18r}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{8(n-1)(n)(2n-1)}{6n^2}+7n+\frac{9n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{8(n^2-n)(2n-1)}{6n^2}+7n+\frac{9n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{8(2n^3-2n^2-n^2+n)}{6n^2}+7n+\frac{9n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{(16n^3-24n^2+8n)+(54n^3-54n^2)+(42n^3)}{6n^2})$$

$$= \lim_{n\to\infty}\frac{2}{n}(\frac{112n^3-78n^2+8n}{6n^2})$$

$$= \lim_{n\to\infty}(\frac{224n^3-156n^2+8n}{6n^3})$$

$$= \lim_{n\to\infty}\left(\frac{224}{6}\right)-\left(\frac{156}{6n}\right)+\left(\frac{8}{6n^2}\right)$$

=112/3

10. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_0^2 x^3\, dx$$

Answer

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_0^2 (x^3)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{2r}{n}\right)^3$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{8r^3}{n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n-1)^2(n)^2}{4n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n^2-2n+1)(n^2)}{4n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n^4-2n^3+n^2)}{4n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{16n^4-32n^3+16n^2}{4n^4}\right)$$

$$= \lim_{n\to\infty}\left(\frac{16}{4}\right)-\left(\frac{32}{4n}\right)+\left(\frac{16}{4n^2}\right)$$

=4

11. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_2^4 \left(x^2-3x+2\right)dx$$

Answer

f(x) is continuous in [2,4]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=3/n

$$\int_2^4 (x^2-3x+2)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\left(2+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\left(2+\frac{2r}{n}\right)^2-3\left(2+\frac{2r}{n}\right)+2\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{4r^2}{n^2}+\frac{8r}{n}+4-6-\frac{6r}{n}+2\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n-1)(n)(2n-1)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(n^2-n)(2n-1)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{4(2n^3-2n^2-n^2+n)}{6n^2}+\frac{n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{(8n^3-12n^2+4n)+(6n^3-6n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{14n^3-18n^2+4n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{28n^3-36n^2+8n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{28}{6}\right)-\left(\frac{36}{6n}\right)+\left(\frac{8}{6n^2}\right)$$

=14/3

12. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_0^2 \left(x^2+x\right)dx$$

Answer

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_0^2 (x^2+x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\left(\frac{2r}{n}\right)^2+\left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{4r^2}{n^2}+\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(n-1)(n)(2n-1)}{6n^2} + \frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(n^2-n)(2n-1)}{6n^2} + \frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(2n^3-2n^2-n^2+n)}{6n^2} + \frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{(8n^3-12n^2+4n)+(6n^3-6n^2)}{6n^2})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{14n^3-18n^2+4n}{6n^2})$$

$$= \lim_{n\to\infty} (\frac{28n^3-36n^2+8n}{6n^3})$$

$$= \lim_{n\to\infty} \left(\frac{28}{6}\right) - \left(\frac{36}{6n}\right) + \left(\frac{8}{6n^2}\right)$$

=14/3

13. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_0^3 \left(2x^2+3x+5\right)dx$$

Answer

f(x) is continuous in [0,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_0^3 (2x^2+3x+5)dx = \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{3r}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} (2\left(\frac{3r}{n}\right)^2 + 3\left(\frac{3r}{n}\right) + 5)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} (\frac{18r^2}{n^2} + \frac{9r}{n} + 5)$$

$$= \lim_{n\to\infty} \frac{3}{n} (\frac{18(n-1)(n)(2n-1)}{6n^2} + \frac{9n(n-1)}{2n} + 5n)$$

$$=\lim_{n\to\infty}\frac{3}{n}(\frac{18(n^2-n)(2n-1)}{6n^2}+\frac{9n(n-1)}{2n}+5n)$$

$$=\lim_{n\to\infty}\frac{3}{n}(\frac{18(2n^3-2n^2-n^2+n)}{6n^2}+\frac{9n(n-1)}{2n}+5n)$$

$$=\lim_{n\to\infty}\frac{3}{n}(\frac{(36n^3-54n^2+18n)+(27n^3-27n^2)+30n^3}{6n^2})$$

$$=\lim_{n\to\infty}\frac{3}{n}(\frac{93n^3-81n^2+18n}{6n^2})$$

$$=\lim_{n\to\infty}(\frac{279n^3-243n^2+54n}{6n^3})$$

$$=\lim_{n\to\infty}\left(\frac{279}{6}\right)-\left(\frac{243}{6n}\right)+\left(\frac{54}{6n^2}\right)$$

=93/2

14. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_0^1 |3x-1|dx$$

Answer

Since it is modulus function so we need to break the function and then solve it

$$f(x)=\int_0^{\frac{1}{3}}(1-3x)dx+\int_{\frac{1}{3}}^{1}(3x-1)dx$$

it is continuous in [0,1]

let $g(x)=\int_0^{\frac{1}{3}}(1-3x)dx$ and $h(x)=\int_{\frac{1}{3}}^{1}(3x-1)dx$

$$g(x)=\int_0^{\frac{1}{3}}(1-3x)dx$$

here h=1/3n

$$\int_0^{\frac{1}{3}}(1-3x)dx=\lim_{n\to\infty}(\frac{1}{3n})\sum_{r=0}^{n-1}f(r/3n)$$

$$=\lim_{n\to\infty}\left(\frac{1}{3n}\right)\sum_{r=0}^{n-1}\left(1-3\left(\frac{r}{3n}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{3n}\right)\left(n-\frac{3(n-1)(n)}{6n}\right)$$

$$=\lim_{n\to\infty}\frac{1}{3n}\frac{6n^2-3n^2+3n}{3n}$$

$$=\lim_{n\to\infty}\frac{1}{3n}\frac{3n^2+3n}{3n}$$

$$=\lim_{n\to\infty}\frac{3n^2+3n}{9n^2}$$

$$=\lim_{n\to\infty}\frac{1}{3}+\left(\frac{3}{9n}\right)$$

=1/3

$$h(x)=\int_{\frac{1}{3}}^{1}(3x-1)dx$$

here h=2/3n

$$\int_{\frac{1}{3}}^{1}(3x-1)dx=\lim_{n\to\infty}\left(\frac{2}{3n}\right)\sum_{r=0}^{n-1}f\left(\left(\frac{1}{3}\right)+\left(\frac{2r}{3n}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{2}{3n}\right)\sum_{r=0}^{n-1}(3\left(\frac{1}{3}+\frac{2r}{3n}\right)-1)$$

$$=\lim_{n\to\infty}\left(\frac{2}{3n}\right)\left(\frac{(n-1)(n)}{n}\right)$$

$$=\lim_{n\to\infty}\frac{2}{3n}.\frac{n^2-n}{n}$$

$$=\lim_{n\to\infty}\frac{2}{3n}.\frac{n^2-n}{n}$$

$$=\lim_{n\to\infty}\frac{2n^2-2n}{3n^2}$$

$$=\lim_{n\to\infty}\frac{2}{3}-\left(\frac{2}{3n}\right)$$

=2/3

f(x)=g(x)+h(x)

=(1/3)+(2/3)

=3/3

=1

15. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_0^2 e^x dx$$

Answer

f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=2/n

$$\int_0^2 (e^x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{\frac{2r}{n}}$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)(e^0+e^h+e^{2h}+\cdots\ldots\ldots\ldots\ldots.+e^{nh}$$

$sum\ of e^0+e^h+e^{2h}+\cdots\ldots\ldots\ldots\ldots.+e^{nh}$

Which is g.p with common ratio $e^{1/n}$

Whose sum is $= \frac{e^h(1-e^{nh})}{1-e^h}$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{e^h(1-e^{nh})}{1-e^h}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{e^h(1-e^{nh})}{\frac{1-e^h.h}{h}}\right)$$

$$\lim_{h\to 0}\frac{1-e^h}{h} = -1$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right).\frac{e^h(1-e^{nh})}{-h}$$

As h=2/n

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right).\frac{e^{(\frac{2}{n})}(1-e^{n*(2/n)})}{-2/n}$$

$=e^2-1$

16. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_1^3 e^{-x} dx$$

Answer

f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_1^3 (e^{-x})dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(1+\left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{-(1+\frac{2r}{n})}$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{-1}.e^{-\frac{2r}{n}}$$

Common ratio is $h = -2/n$

$$sum = e^{-1}(e^0 + e^h + e^{2h} + \cdots \dots\dots\dots\dots. + e^{nh})$$

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)(e^0 + e^h + e^{2h} + \cdots \dots\dots\dots\dots. + e^{nh}$$

$$sum\ of = e^0 + e^h + e^{2h} + \cdots \dots\dots\dots\dots. + e^{nh}$$

Which is g.p. with common ratio $e^{1/n}$

Whose sum is $= \frac{e^h(1-e^{nh})}{1-e^h}$

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{1-e^h}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{\frac{1-e^h.h}{h}}\right.$$

$$\lim_{h\to 0}\frac{1-e^h}{h} = -1$$

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{-h}\right.$$

As h=-2/n

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^{(-\frac{2}{n})}(1-e^{n*(-2/n)})}{2/n}\right.$$

$$= \frac{(1-e^{-2})}{e}$$

$$= \frac{(e^2-1)}{e^3}$$

17. Question

Evaluate each of the following integrals as the limit of sums:

$$\int_a^b \cos x\, dx$$

Answer

f(x) is continuous in [a,b]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=(b-a)/n

$$\int_a^b (cosx)dx = \lim_{n\to\infty} \left(\frac{b-a}{n}\right) \sum_{r=0}^{n-1} f(a+rh)$$

$$= \lim_{n\to\infty} \left(\frac{b-a}{n}\right) \sum_{r=0}^{n-1} \cos(a+rh)$$

S=cos(a)+ cos(a+h)+ cos(a+2h)+ cos(a+3h)+......................+ cos(a+(n-1)h)= $\frac{\sin\left(\frac{nh}{2}\right)\cos\left(a+\frac{(n-1)h}{2}\right)}{\sin\left(\frac{h}{2}\right)}$

Putting h=(b-a)/n

$$= \lim_{n\to\infty} \left(\frac{b-a}{n}\right) \frac{\sin\left(\frac{n(b-a)}{2n}\right)\cos\left(a+\frac{(n-1)(b-a)}{2n}\right)}{\frac{\sin\left(\frac{b-a}{2n}\right)}{\frac{b-a}{2n}} \cdot \frac{b-a}{2n}}$$

As we know

$$\lim_{h\to 0} \left(\frac{sinh}{h}\right) = 1$$

$$= \lim_{n\to\infty} 2\sin\left(\frac{(b-a)}{2}\right)\cos(a+\left(\frac{1}{2}-\frac{1}{2n}\right)(b-a)$$

$$= 2\sin\left(\frac{b-a}{2}\right)\cos\left(\frac{b+a}{2}\right)$$

Which is trigonometry formula of sin(b)-sin(a)

Final answer is sin(b)-sin(a)

Objective Questions

1. Question

Mark (√) against the correct answer in the following:

$$\int_1^4 x\sqrt{x}\, dx = ?$$

A. 12.8

B. 12.4

C. 7

D. none of these

Answer

$y = \int_1^4 x\sqrt{x}\,dx$

$= \int_1^4 x^{\frac{3}{2}}\,dx$

$= \left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right)_1^4$

$= \frac{2}{5}\left(4^{\frac{5}{2}} - 1^{\frac{5}{2}}\right)$

$= \frac{2}{5}(32 - 1)$

$= \frac{62}{5}$

=12.4

2. Question

Mark (√) against the correct answer in the following:

$\int_0^2 \sqrt{6x+4}\,dx = ?$

A. $\frac{64}{9}$

B. 7

C. $\frac{56}{9}$

D. $\frac{60}{9}$

Answer

$y = \int_0^2 \sqrt{6x+4}\,dx$

$= \left(\frac{(6x+4)^{\frac{1}{2}+1}}{6\left(\frac{1}{2}+1\right)}\right)_0^2$

$= \frac{2}{6\times 3}\left(16^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$

$= \frac{2}{6\times 3}(64 - 8)$

$= \frac{56}{9}$

3. Question

Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{dx}{\sqrt{5x+3}} = ?$$

A. $\frac{2}{5}\left(\sqrt{8}-\sqrt{3}\right)$

B. $\frac{2}{5}\left(\sqrt{8}+\sqrt{3}\right)$

C. $\frac{2}{5}\sqrt{8}$

D. none of these

Answer

$y = \int_0^1 \frac{dx}{\sqrt{5x+3}}$

$= \left(\frac{(5x+3)^{\frac{-1}{2}+1}}{5\left(\frac{-1}{2}+1\right)}\right)_0^1$

$= \frac{2}{5}\left(8^{\frac{1}{2}} - 3^{\frac{1}{2}}\right)$

$= \frac{2}{5}(\sqrt{8}-\sqrt{3})$

4. Question

Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{1}{\left(1+x^2\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. none of these

Answer

$y = \int_0^1 \frac{1}{1+x^2} dx$

$= (\tan^{-1} x)_0^1$

$= \tan^{-1} 1 - \tan^{-1} 0$

$= \frac{\pi}{4} - 0$

$= \frac{\pi}{4}$

5. Question

Mark (√) against the correct answer in the following:

$$\int_0^2 \frac{dx}{\sqrt{4-x^2}} = ?$$

A. 1

B. $\sin^{-1}\frac{1}{2}$

C. $\frac{\pi}{4}$

D. none of these

Answer

$y = \int_0^2 \frac{dx}{\sqrt{4-x^2}}$

Use formula $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$

$y = \left(\sin^{-1}\frac{x}{2}\right)_0^2$

$= \sin^{-1} 1 - \sin^{-1} 0$

$= \frac{\pi}{2}$

6. Question

Mark (√) against the correct answer in the following:

$$\int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1+x^2}\,dx = ?$$

A. $\frac{19}{3}$

B. $\frac{19}{6}$

C. $\frac{38}{3}$

D. $\frac{9}{4}$

Answer

$y = \int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1+x^2}\,dx$

Let, $x^2 = t$

Differentiating both side with respect to t

$2x\frac{dx}{dt} = 1$

$\Rightarrow xdx = \frac{1}{2}dt$

At $x = \sqrt{3}$, $t = 3$

At $x = \sqrt{8}$, $t = 8$

$$y = \frac{1}{2}\int_{3}^{8} \sqrt{1+t}\,dt$$

$$= \frac{1}{2}\left(\frac{(1+t)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)}\right)_{3}^{8}$$

$$= \frac{1}{3}\left(9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$$

$$= \frac{1}{3}(27-8)$$

$$= \frac{19}{3}$$

7. Question

Mark (√) against the correct answer in the following:

$$\int_{0}^{1} \frac{x^3}{\left(1+x^8\right)}dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{8}$

D. $\frac{\pi}{16}$

Answer

Let, $x^4 = t$

Differentiating both side with respect to t

$$4x^3\frac{dx}{dt} = 1$$

$$\Rightarrow x^3 dx = \frac{1}{4}dt$$

At x = 0, t = 0

At x = 1, t = 1

$$y = \frac{1}{4}\int_{0}^{1} \frac{1}{1+t^2}dt$$

$$= \frac{1}{4}(\tan^{-1} t)_0^1$$

$= \frac{1}{4}(\tan^{-1} 1 - \tan^{-1} 0)$

$= \frac{\pi}{16}$

8. Question

Mark (√) against the correct answer in the following:

$\int_1^e \frac{(\log x)^2}{x} dx = ?$

A. $\frac{1}{3}$

B. $\frac{1}{3}e^3$

C. $\frac{1}{3}\left(e^3 - 1\right)$

D. none of these

Answer

Let, log x = t

Differentiating both side with respect to t

$\frac{1}{x}\frac{dx}{dt} = 1$

$\Rightarrow \frac{1}{x} dx = dt$

At x = 1, t = 0

At x = e, t = 1

$y = \int_0^1 t^2 dt$

$= \left(\frac{t^3}{3}\right)_0^1$

$= \frac{1}{3}$

9. Question

Mark (√) against the correct answer in the following:

$\int_{\pi/4}^{\pi/2} \cot x \, dx = ?$

A. log 2

B. 2 log 2

C. $\frac{1}{2}\log 2$

D. none of these

Answer

$y = (\ln(\sin x))_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \ln(\sin\frac{\pi}{2}) - \ln(\sin\frac{\pi}{4})$

$= \ln 1 - \ln\frac{1}{\sqrt{2}}$

$= \frac{1}{2}\ln 2$

10. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/4} \tan^2 x\, dx = ?$$

A. $\left(1 - \frac{\pi}{4}\right)$

B. $\left(1 + \frac{\pi}{4}\right)$

C. $\left(1 - \frac{\pi}{2}\right)$

D. $\left(1 + \frac{\pi}{2}\right)$

Answer

$y = \int_0^{\frac{\pi}{4}}(\sec^2 x - 1)\, dx$

$= (\tan x - x)_0^{\frac{\pi}{4}}$

$= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)$

$= 1 - \frac{\pi}{4}$

11. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \cos^2 x\, dx = ?$$

A. $\frac{\pi}{2}$

B. π

C. $\frac{\pi}{4}$

D. 1

Answer

$y = \int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx$

$= \left(\frac{x}{2} + \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$

$= \left(\frac{\frac{\pi}{2}}{2} + \frac{\sin \pi}{4}\right) - \left(\frac{0}{2} + \frac{\sin 0}{4}\right)$

$= \frac{\pi}{4}$

12. Question

Mark (√) against the correct answer in the following:

$\int_{\pi/3}^{\pi/2} \text{cosec } x \, dx = ?$

A. $\frac{1}{2}\log 2$

B. $\frac{1}{2}\log 3$

C. - log 2

D. none of these

Answer

$y = (\ln(\text{cosec } x - \cot x))_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= \ln\left(\text{cosec } \frac{\pi}{2} - \cot\frac{\pi}{2}\right) - \ln\left(\text{cosec } \frac{\pi}{3} - \cot\frac{\pi}{3}\right)$

$= \ln(1-0) - \ln\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)$

$= \frac{1}{2}\log 3$

13. Question

Mark (√) against the correct answer in the following:

$\int_0^{\pi/2} \cos^3 x \, dx = ?$

A. 1

B. $\frac{3}{4}$

C. $\frac{2}{3}$

D. none of these

Answer

$y = \int_0^{\frac{\pi}{2}} \cos x \,(1 - \sin^2 x)\, dx$

Let, sin x = t

Differentiating both side with respect to t

$\text{Cos}\, x \frac{dx}{dt} = 1$

$\Rightarrow \cos x\, dx = dt$

At x = 0, t = 0

At x = $\frac{\pi}{2}$, t = 1

$y = \int_0^1 1 - t^2\ dt$

$= \left(t - \frac{t^3}{3}\right)_0^1$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

14. Question

Mark (√) against the correct answer in the following:

$\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x} dx = ?$

A. (e – 1)

B. (e + 1)

C. $\left(\frac{1}{e} + 1\right)$

D. $\left(\frac{1}{e} - 1\right)$

Answer

$y = \int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x\, dx$

Let, tan x = t

Differentiating both side with respect to t

$\sec^2 x \frac{dx}{dt} = 1$