EXERCISE 5.1

1. In each of the following, determine whether the given numbers are roots of the given equations or not;
(i) $x^2 - 5x + 6 = 0$; 2, -3
(ii) $3x^2 - 13x - 10 = 0$; 5, -2/3
Solution:

(i) $x^2 - 5x + 6 = 0$; 2, -3
Let us substitute the given values in the expression and check,
When, x = 2
$x^2 - 5x + 6 = 0$
$(2)^2 - 5(2) + 6 = 0$
$4 - 10 + 6 = 0$
$0 = 0$
$\therefore x = 0$

When, x = -3
$x^2 - 5x + 6 = 0$
$(-3)^2 - 5(-3) + 6 = 0$
$9 + 15 + 6 = 0$
$30 = 0$
$\therefore x \neq 0$
Hence, the value x = 2 is the root of the equation.
And value x = -3 is not a root of the equation.

(ii) $3x^2 - 13x - 10 = 0$; 5, -2/3
Let us substitute the given values in the expression and check,
When, x = 5
$3x^2 - 13x - 10 = 0$
$3(5)^2 - 13(5) - 10 = 0$
$3(25) - 65 - 10 = 0$
$75 - 75 = 0$
$0 = 0$
$\therefore x = 0$

When, x = -2/3
$3x^2 - 13x - 10 = 0$
$3(-2/3)^2 - 13(-2/3) - 10 = 0$

$4/9 + 26/3 - 10 = 0$
$4/3 + 26/3 - 10 = 0$
$30/3 - 10 = 0$
$10 - 10 = 0$
$\therefore x = 0$
Hence, the value $x = 5, -2/3$ are the roots of the equation.

2. In each of the following, determine whether the given numbers are solutions of the given equation or not:
(i) $x^2 - 3\sqrt{3}x + 6 = 0$; $x = \sqrt{3}, -2\sqrt{3}$
(ii) $x^2 - \sqrt{2}x - 4 = 0$; $x = -\sqrt{2}, 2\sqrt{2}$
Solution:

(i) $x^2 - 3\sqrt{3}x + 6 = 0$; $x = \sqrt{3}, -2\sqrt{3}$
Let us substitute the given values in the expression and check,
When, $x = \sqrt{3}$
$x^2 - 3\sqrt{3}x + 6 = 0$
$(\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6 = 0$
$3 - 9 + 6 = 0$
$-9 + 9 = 0$
$0 = 0$
$\therefore \sqrt{3}$ is the solution of the equation.

When, $x = -2\sqrt{3}$
$x^2 - 3\sqrt{3}x + 6 = 0$
$(-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6 = 0$
$4(3) + 18 + 6 = 0$
$12 + 18 + 6 = 0$
$36 = 0$
$\therefore -2\sqrt{3}$ is not the solution of the equation.

(ii) $x^2 - \sqrt{2}x - 4 = 0$; $x = -\sqrt{2}, 2\sqrt{2}$
Let us substitute the given values in the expression and check,
When, $x = -\sqrt{2}$
$x^2 - \sqrt{2}x - 4 = 0$
$(-\sqrt{2})^2 - \sqrt{2}(-\sqrt{2}) - 4 = 0$
$2 + 2 - 4 = 0$
$4 - 4 = 0$
$0 = 0$

∴ $-\sqrt{2}$ is the solution of the equation.

When, $x = 2\sqrt{2}$
$x^2 - \sqrt{2}x - 4 = 0$
$(2\sqrt{2})^2 - \sqrt{2}(2\sqrt{2}) - 4 = 0$
$4(2) - 4 - 4 = 0$
$4 - 4 = 0$
$0 = 0$
∴ $2\sqrt{2}$ is the solution of the equation.

3. (i) If –1/2 is a solution of the equation $3x^2 + 2kx - 3 = 0$, find the value of k.
(ii) If 2/3 is a solution of the equation $7x^2 + kx - 3 = 0$, find the value of k.
Solution:

(i) If –1/2 is a solution of the equation $3x^2 + 2kx - 3 = 0$, find the value of k.
Let us substitute the given value x = -1/2 in the expression, we get
$3x^2 + 2kx - 3 = 0$
$3(-1/2)^2 + 2k(-1/2) - 3 = 0$
$3/4 - k - 3 = 0$
$\frac{3}{4} - 3 = k$
By taking LCM
$k = (3-12)/4$
$= -9/4$
∴ Value of k = -9/4.

(ii) If 2/3 is a solution of the equation $7x^2 + kx - 3 = 0$, find the value of k.
Let us substitute the given value x = 2/3 in the expression, we get
$7x^2 + kx - 3 = 0$
$7(2/3)^2 + k(2/3) - 3 = 0$
$7(4/9) + 2k/3 - 3 = 0$
$28/9 - 3 + 2k/3 = 0$
$2k/3 = 3 - 28/9$
By taking LCM on the RHS
$2k/3 = (27 - 28)/9$
$= -1/9$
$k = -1/9 \times (3/2)$
$= -1/6$
∴ Value of k = -1/6.

4. (i) If $\sqrt{2}$ is a root of the equation $kx^2 + \sqrt{2}x - 4 = 0$, find the value of k.
(ii) If a is a root of the equation $x^2 - (a + b)x + k = 0$, find the value of k.
Solution:

(i) If $\sqrt{2}$ is a root of the equation $kx^2 + \sqrt{2}x - 4 = 0$, find the value of k.
Let us substitute the given value $x = \sqrt{2}$ in the expression, we get
$kx^2 + \sqrt{2}x - 4 = 0$
$k(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 0$
$2k + 2 - 4 = 0$
$2k - 2 = 0$
$k = 2/2$
$= 1$
∴ Value of k = 1.

(ii) If a is a root of the equation $x^2 - (a + b)x + k = 0$, find the value of k.
Let us substitute the given value x = a in the expression, we get
$x^2 - (a + b)x + k = 0$
$a^2 - (a + b)a + k = 0$
$a^2 - a^2 - ab + k = 0$
$-ab + k = 0$
$k = ab$
∴ Value of k = ab.

5. If 2/3 and -3 are the roots of the equation $px^2 + 7x + q = 0$, find the values of p and q.
Solution:

Let us substitute the given value x = 2/3 in the expression, we get
$px^2 + 7x + q = 0$
$p(2/3)^2 + 7(2/3) + q = 0$
$4p/9 + 14/3 + q = 0$
By taking LCM
$4p + 42 + 9q = 0$
$4p + 9q = - 42$ … (1)

Now, substitute the value x = -3 in the expression, we get
$px^2 + 7x + q = 0$
$p(-3)^2 + 7(-3) + q = 0$
$9p + q - 21 = 0$

9p + q = 21
q = 21 – 9p…. (2)

By substituting the value of q in equation (1), we get
4p + 9q = - 42
4p + 9(21 – 9p) = -42
4p + 189 – 81p = -42
189 – 77p = -42
189 + 42 = 77p
231 = 77p
p = 231/77
p = 3

Now, substitute the value of p in equation (2), we get
q = 21 – 9p
= 21 – 9(3)
= 21 – 27
= -6
∴ Value of p is 3 and q is -6.

EXERCISE 5.2

Solve the following equations (1 to 24) by factorization:
1. (i) $x^2 - 3x - 10 = 0$
(ii) $x(2x + 5) = 3$
Solution:

(i) $x^2 - 3x - 10 = 0$
Let us simplify the given expression,
$x^2 - 5x + 2x - 10 = 0$
$x(x - 5) + 2(x - 5) = 0$
$(x + 2)(x - 5) = 0$
So now,
$(x + 2) = 0$ or $(x - 5) = 0$
$x = -2$ or $x = 5$
∴ Value of $x = -2, 5$

(ii) $x(2x + 5) = 3$
Let us simplify the given expression,
$2x^2 + 5x - 3 = 0$
Now, let us factorize
$2x^2 + 6x - x - 3 = 0$
$2x(x + 3) - 1(x + 3) = 0$
$(2x - 1)(x + 3) = 0$
So now,
$(2x - 1) = 0$ or $(x + 3) = 0$
$2x = 1$ or $x = -3$
$x = ½$ or $x = -3$
∴ Value of $x = ½, -3$

2. (i) $3x^2 - 5x - 12 = 0$
(ii) $21x^2 - 8x - 4 = 0$
Solution:

(i) $3x^2 - 5x - 12 = 0$
Let us simplify the given expression,
$3x^2 - 9x + 4x - 12 = 0$
$3x(x - 3) + 4(x - 3) = 0$
$(3x + 4)(x - 3) = 0$

So now,
$(3x + 4) = 0$ or $(x - 3) = 0$
$3x = -4$ or $x = 3$
$x = -4/3$ or $x = 3$
∴ Value of $x = -4/3, 3$

(ii) $21x^2 – 8x – 4 = 0$
Let us simplify the given expression,
$21x^2 – 14x + 6x – 4 = 0$
$7x(3x - 2) + 2(3x - 2) = 0$
$(7x + 2) (3x - 2) = 0$
So now,
$(7x + 2) = 0$ or $(3x - 2) = 0$
$7x = -2$ or $3x = 2$
$x = -2/7$ or $x = 2/3$
∴ Value of $x = -2/7, 2/3$

3. (i) $3x^2 = x + 4$
(ii) $x(6x – 1) = 35$
Solution:

(i) $3x^2 = x + 4$
Let us simplify the given expression,
$3x^2 – x – 4 = 0$
Now, let us factorize
$3x^2 – 4x + 3x – 4 = 0$
$x(3x - 4) + 1(3x – 4) = 0$
$(x + 1) (3x – 4) = 0$
So now,
$(x + 1) = 0$ or $(3x – 4) = 0$
$x = -1$ or $3x = 4$
$x = -1$ or $x = 4/3$
∴ Value of $x = -1, 4/3$

(ii) $x(6x – 1) = 35$
Let us simplify the given expression,
$6x^2 – x – 35 = 0$
Now, let us factorize
$6x^2 – 15x + 14x – 35 = 0$

$3x(2x - 5) + 7(2x - 5) = 0$
$(3x + 7)(2x - 5) = 0$
So now,
$(3x + 7) = 0$ or $(2x - 5) = 0$
$3x = -7$ or $2x = 5$
$x = -7/3$ or $x = 5/2$
∴ Value of $x = -7/3, 5/2$

4. (i) $6p^2 + 11p - 10 = 0$
(ii) $2/3x^2 - 1/3x = 1$
Solution:

(i) $6p^2 + 11p - 10 = 0$
Let us factorize the given expression,
$6p^2 + 15p - 4p - 10 = 0$
$3p(2p + 5) - 2(2p + 5) = 0$
$(3p - 2)(2p + 5) = 0$
So now,
$(3p - 2) = 0$ or $(2p + 5) = 0$
$3p = 2$ or $2p = -5$
$p = 2/3$ or $p = -5/2$
∴ Value of $p = 2/3, -5/2$

(ii) $2/3x^2 - 1/3x = 1$
Let us simplify the given expression,
$2x^2 - x = 3$
$2x^2 - x - 3 = 0$
Let us factorize the given expression,
$2x^2 - 3x + 2x - 3 = 0$
$x(2x - 3) + 1(2x - 3) = 0$
$(x + 1)(2x - 3) = 0$
So now,
$(x + 1) = 0$ or $(2x - 3) = 0$
$x = -1$ or $2x = 3$
$x = -1$ or $x = 3/2$
∴ Value of $x = -1, 3/2$

5. (i) $3(x - 2)^2 = 147$
(ii) $1/7(3x - 5)^2 = 28$

Solution:

(i) $3(x-2)^2 = 147$
Firstly let us expand the given expression,
$3(x^2 - 4x + 4) = 147$
$3x^2 - 12x + 12 = 147$
$3x^2 - 12x + 12 - 147 = 0$
$3x^2 - 12x - 135 = 0$
Divide by 3, we get
$x^2 - 4x - 45 = 0$
Let us factorize the expression,
$x^2 - 9x + 5x - 45 = 0$
$x(x - 9) + 5(x - 9) = 0$
$(x + 5)(x - 9) = 0$
So now,
$(x + 5) = 0$ or $(x - 9) = 0$
$x = -5$ or $x = 9$
∴ Value of $x = -5, 9$

(ii) $1/7(3x-5)^2 = 28$
Let us simplify the expression,
$(3x - 5)^2 = 28 \times 7$
$(3x - 5)^2 = 196$
Now let us expand,
$9x^2 - 30x + 25 = 196$
$9x^2 - 30x + 25 - 196 = 0$
$9x^2 - 30x - 171 = 0$
Divide by 3, we get
$3x^2 - 10x - 57 = 0$
Let us factorize the expression,
$3x^2 - 19x + 9x - 57 = 0$
$x(3x - 19) + 3(3x - 19) = 0$
$(x + 3)(3x - 19) = 0$
So now,
$(x + 3) = 0$ or $(3x - 19) = 0$
$x = -3$ or $3x = 19$
$x = -3$ or $x = 19/3$
∴ Value of $x = -3, 19/3$

6. $x^2 - 4x - 12 = 0$, when $x \in N$
Solution:

Let us factorize the expression,
$x^2 - 4x - 12 = 0$
$x^2 - 6x + 2x - 12 = 0$
$x(x - 6) + 2(x - 6) = 0$
$(x + 2)(x - 6) = 0$
So now,
$(x + 2) = 0$ or $(x - 6) = 0$
$x = -2$ or $x = 6$
∴ Value of $x = 6$ (Since, -2 is not a natural number).

7. $2x^2 - 9x + 10 = 0$, when
(i) $x \in N$
(ii) $x \in Q$
Solution:

Let us factorize the expression,
$2x^2 - 9x + 10 = 0$
$2x^2 - 4x - 5x + 10 = 0$
$2x(x - 2) - 5(x - 2) = 0$
$(2x - 5)(x - 2) = 0$
So now,
$(2x - 5) = 0$ or $(x - 2) = 0$
$2x = 5$ or $x = 2$
$x = 5/2$ or $x = 2$

(i) When, $x \in N$ then, $x = 2$
(ii) When, $x \in Q$ then, $x = 2, 5/2$

8. (i) $a^2x^2 + 2ax + 1 = 0, a \neq 0$
(ii) $x^2 - (p + q)x + pq = 0$
Solution:

(i) $a^2x^2 + 2ax + 1 = 0, a \neq 0$
Let us factorize the expression,
$a^2x^2 + 2ax + 1 = 0$
$a^2x^2 + ax + ax + 1 = 0$

$ax(ax + 1) + 1(ax + 1) = 0$
$(ax + 1)(ax + 1) = 0$
So now,
$(ax + 1) = 0$ or $(ax + 1) = 0$
$ax = -1$ or $ax = -1$
$x = -1/a$ or $x = -1/a$
∴ Value of $x = -1/a, -1/a$

(ii) $x^2 – (p + q)x + pq = 0$
Let us simplify the expression,
$x^2 – (p + q)x + pq = 0$
$x^2 – px – qx + pq = 0$
$x(x - p) – q(x - p) = 0$
$(x - q)(x - p) = 0$
So now,
$(x - q) = 0$ or $(x - p) = 0$
$x = q$ or $x = p$
∴ Value of $x = q, p$

9. $a^2x^2 + (a^2 + b^2)x + b^2 = 0, a\neq0$
Solution:

Let us simplify the expression,
$a^2x^2 + (a^2 + b^2)x + b^2 = 0$
$a^2x^2 + a^2x + b^2x + b^2 = 0$
$a^2x(x + 1) + b^2(x + 1) = 0$
$(a^2x + b^2)(x + 1) = 0$
So now,
$(a^2x + b^2) = 0$ or $(x + 1) = 0$
$a^2x = -b^2$ or $x = -1$
$x = -b^2/a^2$ or $x = -1$
∴ Value of $x = -b^2/a^2, -1$

10. (i) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$ (ii) $4\sqrt{3}x^2 + 5x – 2\sqrt{3} = 0$.
Solution:

(i) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$
Let us factorize the given expression,
$\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$ [As $\sqrt{3} \times 7\sqrt{3} = 3 \times 7 = 21$ and $3 + 7 = 10$]

$\sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$
$(\sqrt{3}x + 7)(x + \sqrt{3}) = 0$
So now,
$(\sqrt{3}x + 7) = 0$ or $(x + \sqrt{3}) = 0$
$\sqrt{3}x = -7$ or $x = -\sqrt{3}$
$x = -7/\sqrt{3}$ or $x = -\sqrt{3}$
∴ Value of $x = -7/\sqrt{3}, -\sqrt{3}$

(ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$
Let us factorize the given expression,
$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$ [As, $4\sqrt{3} \times (-2\sqrt{3}) = -8 \times 3 = -24$ and $8 \times (-3) = -24$]
$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$
$(4x - \sqrt{3})(\sqrt{3}x + 2) = 0$
So now,
$(4x - \sqrt{3}) = 0$ or $(\sqrt{3}x + 2) = 0$
$4x = \sqrt{3}$ or $\sqrt{3}x = -2$
$x = \sqrt{3}/4$ or $x = -2/\sqrt{3}$
∴ Value of $x = \sqrt{3}/4, -2/\sqrt{3}$

11. (i) $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$ (ii) $x + 1/x = 2(1/20)$.
Solution:

(i) $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$
Let us expand the given expression,
$x^2 - x - \sqrt{2}x + \sqrt{2} = 0$
Taking common, we have
$x(x - 1) - \sqrt{2}(x - 1) = 0$
$(x - 1)(x - \sqrt{2}) = 0$
So now,
$(x - 1) = 0$ or $(x + \sqrt{2}) = 0$
$x = 1$ or $x = -\sqrt{2}$
∴ Value of $x = 1, -\sqrt{2}$

(ii) $x + 1/x = 2(1/20)$
Rewriting the given expression, we have
$(x^2 + 1)/x = 41/20$
On cross multiplication, we get
$20(x^2 + 1) = 41x$
$20x^2 + 20 = 41x$

$20x^2 – 41x + 20 = 0$
Let us factorize the expression now,
$20x^2 – 25x – 16x + 20 = 0$
$5x(4x – 5) – 4(4x – 5) = 0$
$(5x – 4) (4x – 5) = 0$
So,
$(5x – 4) = 0$ or $(4x – 5) = 0$
$5x = 4$ or $4x = 5$
$x = 4/5$ or $x = 5/4$
∴ Value of $x = 4/5, 5/4$

12. (i) $2/x^2 – 5/x + 2 = 0, x \neq 0$ (ii) $x^2/15 – x/3 – 10 = 0$.
Solution:

(i) $2/x^2 – 5/x + 2 = 0$
Taking L.C.M for the given expression,
$(2 – 5x + 2x^2)/x^2 = 0$
$2x^2 – 5x + 2 = 0$
Now, on factorizing the above expression we get
$2x^2 – 4x – x + 2 = 0$
$2x(x – 2) – 1(x – 2) = 0$
$(2x – 1) (x – 2) = 0$
So,
$(2x – 1) = 0$ or $(x – 2) = 0$
$2x = 1$ or $x = 2$
$x = ½$ or $x = 2$
∴ Value of $x = ½, 2$

(ii) $x^2/15 – x/3 – 10 = 0$
Taking L.C.M for the given expression,
$(x^2 – 5x – 150)/15 = 0$
$x^2 – 5x – 150 = 0$
Now, on factorizing the above expression we get
$x^2 – 15x + 10x – 150 = 0$
$x(x – 15) + 10(x – 15) = 0$
$(x – 15) (x + 10) = 0$
So,
$(x – 15) = 0$ or $(x + 10) = 0$
$x = 15$ or $x = -10$

∴ Value of $x = 15, -10$

13. (i) $3x - 8/x = 2$ (ii) $(x + 2)/(x + 3) = (2x - 3)/(3x - 7)$.
Solution:

(i) $3x - 8/x = 2$
Taking L.C.M, we have
$(3x^2 - 8)/x = 2$
$3x^2 - 8 = 2x$
$3x^2 - 2x - 8 = 0$
On factorizing the above expression, we get
$3x^2 - 6x + 4x - 8 = 0$
$3x(x - 2) + 4(x - 2) = 0$
$(3x + 4)(x - 2) = 0$
So,
$(3x - 4) = 0$ or $(x - 2) = 0$
$3x = 4$ or $x = 2$
$x = 4/3$ or $x = 2$
∴ Value of $x = 4/3, 2$

(ii) $(x + 2)/(x + 3) = (2x - 3)/(3x - 7)$
Upon cross multiplication, we get
$(x + 2)(3x - 7) = (2x - 3)(x + 3)$
$3x^2 - 7x + 6x - 14 = 2x^2 + 6x - 3x - 9$
$3x^2 - x - 14 = 2x^2 + 3x - 9$
$3x^2 - 2x^2 - x - 3x - 14 + 9 = 0$
$x^2 - 4x - 5 = 0$
Factorizing the above expression, we get
$x^2 - 5x + x - 5 = 0$
$x(x - 5) + 1(x - 5) = 0$
$(x + 1)(x - 5) = 0$
So,
$x + 1 = 0$ or $x - 5 = 0$
$x = -1$ or $x = 5$
∴ Value of $x = -1, 5$

14. (i) $8/(x + 3) - 3/(2 - x) = 2$ (ii) $x/(x - 1) + (x - 1)/x = 2\frac{1}{2}$
Solution:

(i) $8/(x + 3) - 3/(2 - x) = 2$
Taking L.C.M, we have
$[8(2 - x) - 3(x + 3)]/[(x + 3)(2 - x)] = 2$
Upon cross-multiplication,
$16 - 8x - 3x - 9 = 2 (x + 3) (2 - x)$
$7 - 11x = 2 (2x + 6 - x^2 - 3x)$
$7 - 11x = 2 (6 - x^2 - x)$
$7 - 11x = 12 - 2x^2 - 2x$
$2x^2 - 11x + 2x + 7 - 12 = 0$
$2x^2 - 9x - 5 = 0$
Now, let's factorize the above equation to find x
$2x^2 - 10x + x - 5 = 0$
$2x(x - 5) + 1(x - 5) = 0$
$(2x + 1) (x - 5) = 0$
So,
$2x + 1 = 0$ or $x - 5 = 0$
$x = -1/2$ or $x = 5$
∴ Value of $x = -1/2, 5$

(ii) $x/(x - 1) + (x - 1)/x = 2½$
Taking L.C.M, we have
$[x^2 + (x - 1)^2] / x(x- 1) = 5/2$
$(x^2 + x^2 - 2x + 1)/ (x^2 - x) = 5/2$
$(2x^2 - 2x + 1)/ (x^2 - x) = 5/2$
Upon cross-multiplication, we get
$2 (2x^2 - 2x + 1) = 5 (x^2 - x)$
$4x^2 - 4x + 2 = 5x^2 - 5x$
$5x^2 - 4x^2 - 5x + 4x - 2 = 0$
$x^2 - x - 2 = 0$
Now, let's factorize the above equation to find x
$x^2 - 2x + x - 2 = 0$
$x(x - 2) + 1(x - 2) = 0$
$(x + 1) (x - 2) = 0$
So,
$x + 1 = 0$ or $x - 2 = 0$
$x = -1$ or $x = 2$
∴ Value of $x = -1, 2$

15. (i) $(x + 1)/(x - 1) + (x - 2)/(x + 2) = 3$

(ii) 1/(x – 3) – 1/(x + 5) = 1/6.
Solution:

(i) $(x + 1)/(x – 1) + (x – 2)/(x + 2) = 3$
$[(x + 1)(x + 2) + (x - 2)(x - 1)]/[(x – 1)(x + 2)] = 3$ [Taking L.C.M]
On expanding, we get
$x^2 + 3x + 2 + x^2 – 3x + 2 = 3(x - 1)(x + 2)$
$2x^2 + 4 = 3(x^2 + x - 2)$
$2x^2 + 4 = 3x^2 + 3x – 6$
$3x^2 – 2x^2 + 3x – 6 – 4 = 0$
$x^2 + 3x – 10 = 0$
Now, let's factorize the above equation to find x
$x^2 + 5x – 2x – 10 = 0$
$x(x + 5) – 2(x – 5) = 0$
$(x + 5)(x - 5) = 0$
So,
$x + 5 = 0$ or $x – 5 = 0$
$x = -5$ or $x = 5$
∴ Value of x = -5, 5

(ii) $1/(x – 3) – 1/(x + 5) = 1/6$
Taking L.C.M, we have
$[x + 5 – (x – 3)] / [(x – 3)(x + 5)] = 1/6$
$(x + 5 – x + 3) / [(x – 3)(x + 5)] = 1/6$
$8/ [(x – 3)(x + 5)] = 1/6$
Upon cross-multiplying, we have
$8 \times 6 = (x – 3)(x + 5)$
$48 = x^2 + 5x – 3x – 15$
$x^2 + 2x – 15 – 48 = 0$
$x^2 + 2x – 63 = 0$
Now, let's factorize the above equation to find x
$x^2 + 9x – 7x – 63 = 0$
$x(x + 9) – 7(x + 9) = 0$
$(x - 7)(x + 9) = 0$
So,
$x – 7 = 0$ or $x + 9 = 0$
$x = 7$ or $x = -9$
∴ Value of x = 7, -9

16. (i) a/(ax – 1) + b/(bx – 1) = a + b, a + b ≠ 0, ab ≠ 0
(ii) 1/(2a + b + 2x) = 1/2a + 1/b + 1/2x
Solution:

(i) a/(ax – 1) + b/(bx – 1) = a + b, a + b ≠ 0, ab ≠ 0
Let's rearrange the equation for simple solving,
[a/(ax – 1) – b] + [b/(bx – 1) – a] = 0
[a – b(ax - 1)]/(ax - 1) + [b – a(bx - 1)]/(bx - 1) = 0
(a – abx + b)/(ax - 1) + (b – abx + a)/(bx - 1) = 0
(a – abx + b) [1/(ax – 1) + 1/(bx – 1)] = 0 {Taking common terms out}
(a – abx + b) [(bx – 1 + ax - 1)/(ax - 1)(bx - 1)] = 0
(a – abx + b) [(ax + bx - 2)/ (ax - 1)(bx - 1)] = 0
So,
(a – abx + b) = 0 or (ax + bx - 2)/ [(ax - 1) (bx - 1)] = 0
If (a – abx + b) = 0,
a + b = abx
x = (a + b)/ab
And,
if (ax + bx - 2)/ [(ax - 1) (bx - 1)] = 0
ax + bx – 2 = 0
(a + b)x = 2
x = 2/(a + b)
∴ Value of x = (a + b)/ab, 2/(a + b)

(ii)

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$

Taking L.C.M on both sides, we have

$$\frac{2x-2a-b-2x}{2x(2a+b+2x)} = \frac{(2a+b)}{2ab}$$

$$\frac{-2a-b}{2x(2a+b+2x)} = \frac{(2a+b)}{2ab}$$

$$\frac{-(2a+b)}{2x(2a+b+2x)} = \frac{(2a+b)}{2ab}$$

$$\frac{-1}{x(2a+b+2x)} = \frac{1}{ab}$$

$-ab = x(2a+b) + 2x^2$ [After cross-multiplication]

$0 = 2x^2 + 2ax + bx + ab$

$2x(x+a)+b(x+a)=0$

$(x+a)(2x+b)=0$

$(x+a)=0\ Or\ 2x+b=0$

$x=-a\ Or\ 2x=-b$

$\Rightarrow\ x=-a\ Or\ x=\frac{-b}{2}$

∴ Value of x = -a, -b/2

17. 1/(x + 6) + 1/(x – 10) = 3/(x – 4).
Solution:

Given equation,
1/(x + 6) + 1/(x – 10) = 3/(x – 4)
Taking L.C.M for the R.H.S of the equation,
[(x - 10) + (x + 6)]/ [(x + 6) (x - 10)] = 3/(x- 4)
(2x - 4)/ (x^2 – 4x – 60) = 3/(x- 4)
On cross-multiplying, we get
(2x - 4) (x - 4) = 3(x^2 – 4x – 60)
$2x^2$ – 8x – 4x + 16 = $3x^2$ – 12x – 180
$2x^2$ – 12x + 16 = $3x^2$ – 12x – 180
$3x^2$ – $2x^2$ – 12x + 12x – 180 – 16 = 0
x^2 – 196 = 0
x^2 = 196
x = √196
∴ x = ± 14

18. (i) √(3x + 4) = x (ii) √[x(x – 7)] = 3√2
Solution:

(i) √(3x + 4) = x
On squaring on both sides, we get
3x + 4 = x^2
x^2 – 3x – 4 = 0
Let us factorize the above expression,
x^2 – 4x + x – 4 = 0
x(x - 4) + 1(x - 4) = 0
(x - 4) (x + 1) = 0
So,

$x - 4 = 0$ or $x + 1 = 0$
$x = 4$ or $x = -1$
∴ Value of $x = 4, -1$

(ii) $\sqrt{[x(x - 7)]} = 3\sqrt{2}$
On squaring on both sides, we get
$x(x - 7) = (3\sqrt{2})^2$
$x^2 - 7x = 9 \times 2$
$x^2 - 7x - 18 = 0$
Let us factorize the above expression,
$x^2 - 9x + 2x - 18 = 0$
$x(x - 9) + 2(x - 9) = 0$
$(x - 9)(x + 2) = 0$
So,
$x - 9 = 0$ or $x + 2 = 0$
$x = 9$ or $x = -2$
∴ Value of $x = 9, -2$

19. Use the substitution $y = 3x + 1$ to solve for x:
$5(3x + 1)^2 + 6(3x + 1) - 8 = 0$.
Solution:

Given equation,
$5(3x + 1)^2 + 6(3x + 1) - 8 = 0$
Upon substituting $y = 3x + 1$,
$5y^2 + 6y - 8 = 0$
We get a quadratic equation in y
Now, solving for y by factorization, we get
$5y^2 + 10y - 4y - 8 = 0$
$5y(y + 2) - 4(y + 2) = 0$
$(5y - 4)(y + 2) = 0$
So,
$5y - 4 = 0$ or $y + 2 = 0$
$5y = 4$ or $y = -2$
$y = 4/5$ or $y = -2$
Now, to find the value of x let's back substitute y
$3x + 1 = 4/5$ or $3x + 1 = -2$
$3x = 4/5 - 1$ or $3x = -2 - 1$

$3x = (4 - 5)/5$ or $3x = -3$
$3x = -1/5$ or $x = -3/3$
$x = -1/15$ or $x = -1$
∴ Value of $x = -1, -1/15$

20. Find the values of x if $p + 1 = 0$ and $x^2 + px – 6 = 0$.
Solution:

Given quadratic equation: $x^2 + px – 6 = 0$
And, $p + 1 = 0$
So,
$p = -1$
Substituting the value of p in the given quadratic equation, we get
$x^2 + (-1)x – 6 = 0$
$x^2 – x – 6 = 0$
Solving for x by factorization, we have
$x^2 – 3x + 2x – 6 = 0$
$x(x - 3) + 2(x - 3) = 0$
$(x + 2) (x - 3) = 0$
So,
$x + 2 = 0$ or $x – 3 = 0$
$x = -2$ or $x = 3$
∴ Value of $x = -2, 3$

21. Find the values of x if $p + 7 = 0$, $q – 12 = 0$ and $x^2 + px + q = 0$.
Solution:

Given quadratic equation: $x^2 + px + q = 0$
And, $p + 7 = 0$ and $q – 12 = 0$
So,
$p = -7$ and $q = 12$
Substituting the value of p and q in the given quadratic equation, we get
$x^2 + (-7)x + 12 = 0$
$x^2 – 7x + 12 = 0$
Solving for x by factorization, we have
$x^2 – 4x – 3x + 12 = 0$
$x(x - 4) – 3(x - 4) = 0$
$(x - 3) (x - 4) = 0$
So,

$x - 3 = 0$ or $x - 4 = 0$
$x = 3$ or $x = 4$
∴ Value of $x = 3, 4$

22. If x = p is a solution of the equation x(2x + 5) = 3, then find the value of p.
Solution:

Given that, $x = p$ is a solution of the equation $x(2x + 5) = 3$
Then, upon substituting $x = p$ in must satisfy the equation
$p(2p + 5) = 3$
$2p^2 + 5p = 3$
$2p^2 + 5p - 3 = 0$
Factorizing the above expression, we get
$2p^2 + 6p - p - 3 = 0$
$2p(p + 3) - 1(p + 3) = 0$
$(2p - 1)(p + 3) = 0$
So,
$2p - 1 = 0$ or $p + 3 = 0$
$2p = 1$ or $p = -3$
$p = \frac{1}{2}$ or $p = -3$
∴ Value of $p = \frac{1}{2}, -3$

23. If x = 3 is a solution of the equation $(k + 2)x^2 - kx + 6 = 0$, find the value of k. Hence, find the other root of the equation.
Solution:

Given equation: $(k + 2)x^2 - kx + 6 = 0$
And $x = 3$ is a solution of the equation
So, upon substituting $x = 3$ it must satisfy the equation
$(k + 2)(3)^2 - k(3) + 6 = 0$
$(k + 2)(9) - 3k + 6 = 0$
$9k + 18 - 3k + 6 = 0$
$6k + 24 = 0$
$6(k + 4) = 0$
So,
$k + 4 = 0$
$k = -4$
Now, putting $k = -4$ in the given equation, we have
$(-4 + 2)x^2 - (-4)x + 6 = 0$

$-2x^2 + 4x + 6 = 0$
$x^2 - 2x - 3 = 0$ [Dividing by -2 on both sides]
Factorizing the above expression, we get
$x^2 - 3x + x - 3 = 0$
$x(x - 3) + 1(x - 3) = 0$
$(x + 1)(x - 3) = 0$
So,
$x + 1 = 0$ or $x - 3 = 0$
$x = -1$ or $x = 3$
Hence, the other root of the given equation is -1.

EXERCISE 5.3

Solve the following (1 to 8) equations by using the formula:
1. (i) $2x^2 - 7x + 6 = 0$
(ii) $2x^2 - 6x + 3 = 0$
Solution:

(i) $2x^2 - 7x + 6 = 0$
Let us consider,
a = 2, b = -7, c = 6
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-7)^2 - 4(2)(6)$
$= 49 - 48$
$= 1$
So,
$x = [-(-7) \pm \sqrt{1}] / 2(2)$
$= [7 + 1]/4$ or $[7 - 1]/4$
$= 8/4$ or $6/4$
$= 2$ or $3/2$
∴ Value of x = 2, 3/2

(ii) $2x^2 - 6x + 3 = 0$
Let us consider,
a = 2, b = -6, c = 3
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-6)^2 - 4(2)(3)$

= 36 – 24
= 12
So,
$x = [-(-6) \pm \sqrt{12}] / 2(2)$
$= [6 \pm 2\sqrt{3}] / 4$
$= [6 + 2\sqrt{3}]/ 4$ or $[6 - 2\sqrt{3}]/4$
$= 2(3 + \sqrt{3})/4$ or $2(3 - \sqrt{3})/4$
$= (3 + \sqrt{3})/2$ or $(3 - \sqrt{3})/2$
∴ Value of $x = (3 + \sqrt{3})/2, (3 - \sqrt{3})/2$

2. (i) $256x^2 - 32x + 1 = 0$
(ii) $25x^2 + 30x + 7 = 0$
Solution:

(i) $256x^2 - 32x + 1 = 0$
Let us consider,
a = 256, b = -32, c = 1
So, by using the formula,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
So let, $b^2 - 4ac = D$
$$x = \frac{-b \pm \sqrt{D}}{2a}$$
$D = b^2 - 4ac$
$= (-32)^2 - 4(256)(1)$
$= 1024 - 1024$
$= 0$
So,
$x = [-(-32) \pm \sqrt{0}] / 2(256)$
$= [32] / 512$
$= 1/16$
∴ Value of x = 1/16

(ii) $25x^2 + 30x + 7 = 0$
Let us consider,
a = 25, b = 30, c = 7
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (30)^2 - 4(25)(7)$
$= 900 - 700$
$= 200$

So,
$x = [-(30) \pm \sqrt{200}] / 2(25)$
$= [-30 \pm \sqrt{(100\times2)}]/ 50$
$= [-30 \pm 10\sqrt{2}]/ 50$
$= [-3 \pm \sqrt{2}]/ 5$
$= [-3 + \sqrt{2})]/ 5$ or $[-3 - \sqrt{2}]/ 5$
∴ Value of $x = [-3 + \sqrt{2})]/ 5, [-3 - \sqrt{2}]/ 5$

3. (i) $2x^2 + \sqrt{5}x - 5 = 0$
(ii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$
Solution:

(i) $2x^2 + \sqrt{5}x - 5 = 0$
Let us consider,
$a = 2, b = \sqrt{5}, c = -5$
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (\sqrt{5})^2 - 4(2)(-5)$
$= 5 + 40$
$= 45$

So,
$x = [-(\sqrt{5}) \pm \sqrt{45}] / 2(2)$
$= [-\sqrt{5} \pm 3\sqrt{5})]/ 4$
$= [-\sqrt{5} + 3\sqrt{5})]/ 4$ or $[-\sqrt{5} - 3\sqrt{5}]/ 4$
$= 2\sqrt{5}/4$ or $-4\sqrt{5}/4$

= √5/2 or -√5
∴ Value of x = √5/2, -√5

(ii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$
Let us consider,
a = √3, b = 10, c = -8√3
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (10)^2 - 4(\sqrt{3})(-8\sqrt{3})$
= 100 + 96
= 196
So,
x = [-(10) ± √196] / 2(√3)
= [-10 ± 14] / 2(√3)
= [-10 + 14)]/ 2√3 or [-10 - 14)]/ 2√3
= 4/2√3 or -24/2√3
∴ Value of x = 4/2√3, -24/2√3

4. $(i)\ \frac{x-2}{x+2} + \frac{x+2}{x-2} = 4$

$(ii)\ \frac{x+1}{x+3} = \frac{3x+2}{2x+3}$

Solution:

$$(i)\ \frac{x-2}{x+2} + \frac{x+2}{x-2} = 4$$

By taking LCM,

$$\frac{(x-2)^2 + (x+2)^2}{(x+2)(x-2)} = 4$$

$$\frac{x^2 - 4x + 4 + x^2 + 4x + 4}{x^2 - 4} = 4$$

By simplifying the equation, we get

$2x^2 + 8 = 4x^2 - 16$

$2x^2 + 8 - 4x^2 + 16 = 0$

$-2x^2 + 24 = 0$

$x^2 - 12 = 0$

Let us consider,

a = 1, b = 0, c = -12

So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$

$= (0)^2 - 4(1)(-12)$

$= 0 + 48$

$= 48$

So,

$x = [-(0) \pm \sqrt{48}] / 2(1)$

$= [\pm\sqrt{48}] / 2$

$= [\pm\sqrt{(16\times3)}]/2$

$= \pm4\sqrt{3}/2$

$= \pm2\sqrt{3}$

$= 2\sqrt{3}$ or $-2\sqrt{3}$

∴ Value of $x = 2\sqrt{3}, -2\sqrt{3}$

(ii) $\frac{x+1}{x+3} = \frac{3x+2}{2x+3}$

Let us cross multiply, we get

$(x + 1)(2x + 3) = (x + 3)(3x + 2)$

Now by simplifying we get

$2x^2 + 3x + 2x + 3 = 3x^2 + 9x + 2x + 6$

$2x^2 + 5x + 3 - 3x^2 - 11x - 6 = 0$

$-x^2 - 6x - 3 = 0$

$x^2 + 6x + 3 = 0$

Let us consider,

a = 1, b = 6, c = 3

So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (6)^2 - 4(1)(3)$
$= 36 - 12$
$= 24$

So,
$x = [-(6) \pm \sqrt{24}] / 2(1)$
$= [-6 \pm \sqrt{(4\times6)}] / 2$
$= [-6 \pm 2\sqrt{6}]/2$
$= -3 \pm \sqrt{6}$
$= -3 + \sqrt{6}$ or $-3 - \sqrt{6}$
∴ Value of $x = -3 + \sqrt{6}, -3 - \sqrt{6}$

5. (i) $a(x^2 + 1) = (a^2 + 1)x, a \neq 0$
(ii) $4x^2 - 4ax + (a^2 - b^2) = 0$
Solution:
(i) $a(x^2 + 1) = (a^2 + 1)x, a \neq 0$
Let us simplify the expression,
$ax^2 + a - a^2x + x = 0$
$ax^2 - (a^2 + 1)x + a = 0$
Let us consider,
$a = a, b = -(a^2 + 1), c = a$
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-(a^2 + 1))^2 - 4(a)(a)$
$= a^4 + 2a^2 + 1 - 4a^2$
$= a^4 - 2a^2 + 1$
$= (a^2 - 1)^2$

So,
$x = [-(-(a^2 + 1)) \pm \sqrt{(a^2 - 1)^2}] / 2(a)$

$= [(a^2 + 1) \pm (a^2 - 1)] / 2a$
$= [(a^2 + 1) + (a^2 - 1)] / 2a$ or $[(a^2 + 1) - (a^2 - 1)] / 2a$
$= [a^2 + 1 + a^2 – 1]/2a$ or $[a^2 + 1 – a^2 + 1]/2a$
$= 2a^2/2a$ or $2/2a$
$= a$ or $1/a$
∴ Value of $x = a, 1/a$

(ii) $4x^2 – 4ax + (a^2 – b^2) = 0$
Let us consider,
$a = 4, b = -4a, c = (a^2 – b^2)$
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 – 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 – 4ac$
$= (-4a)^2 – 4(4) (a^2 – b^2)$
$= 16a^2 – 16(a^2 – b^2)$
$= 16a^2 – 16a^2 + 16b^2$
$= 16b^2$
So,
$x = [-(-4a) \pm \sqrt{16b^2}] / 2(4)$
$= [4a \pm 4b] / 8$
$= 4[a \pm b] / 8$
$= [a \pm b] / 2$
$= [a + b] / 2$ or $[a - b] / 2$
∴ Value of $x = [a + b] / 2, [a - b] / 2$

6. (i) $x − 1/x = 3, x \neq 0$
(ii) $1/ x + 1/(x − 2) = 3, x \neq 0, 2$
Solution:
(i) $x−1/x = 3, x \neq 0$
Let us simplify the given expression,
By taking LCM
$x^2 – 1 = 3x$
$x^2 – 3x – 1 = 0$
Let us consider,
$a = 1, b = -3, c = -1$

So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-3)^2 - 4(1)(-1)$
$= 9 + 4$
$= 13$

So,
$x = [-(-3) \pm \sqrt{13}] / 2(1)$
$= [3 \pm \sqrt{13}] / 2$
$= [3 + \sqrt{13}] / 2$ or $[3 - \sqrt{13}] / 2$
∴ Value of $x = [3 + \sqrt{13}] / 2$ or $[3 - \sqrt{13}] / 2$

(ii) $1/x + 1/(x-2) = 3, x \neq 0, 2$
Let us simplify the given expression,
By taking LCM
$[(x - 2) + x] / [x(x - 2)] = 3$
$[x - 2 + x] / [x^2 - 2x] = 3$
$2x - 2 = 3(x^2 - 2x)$
$2x - 2 = 3x^2 - 6x$
$3x^2 - 6x - 2x + 2 = 0$
$3x^2 - 8x + 2$
Let us consider,
$a = 3, b = -8, c = 2$
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-8)^2 - 4(3)(2)$
$= 64 - 24$
$= 40$

So,

x = [-(-8) ± √40] / 2(3)
= [8 ± 2√10] / 6
= 2[4 ± √10] / 6
= [4 ± √10] / 3
= [4 + √10] / 3 or [4 - √10] / 3
∴ Value of x = [4 + √10] / 3 or [4 - √10] / 3

7. Solve for x:

$$2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5, x\neq -3, \frac{1}{2}$$

Solution:

Let us consider, $\left(\frac{2x-1}{x+3}\right)$ = x then, $\left(\frac{x+3}{2x-1}\right)$ = 1/x

So the equation becomes,
2x – 3/x = 5
By taking LCM
$2x^2 - 3 = 5x$
$2x^2 - 5x - 3 = 0$
Let us consider,
a = 2, b = -5, c = -3
So, by using the formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-5)^2 - 4(2)(-3)$
= 25 + 24
= 49
So,
x = [-(-5) ± √49] / 2(2)
= [5 ± 7] / 4
= [5 + 7] / 4 or [5 - 7] / 4
= [12]/4 or [-2]/4
= 3 or -1/2
So, x = 3 or -1/2
Now,

Let us substitute in the equations,
When x = 3, then

$$\left(\frac{2x-1}{x+3}\right) = 3$$

By cross multiplying,
2x – 1 = 3x + 9
3x + 9 – 2x + 1 = 0
x + 10 = 0
x = -10

When x = -1/2, then

$$\left(\frac{2x-1}{x+3}\right) = -1/2$$

By cross multiplying,
2(2x - 1) = -(x + 3)
4x - 2 = -x - 3
4x - 2 + x + 3 = 0
5x + 1 = 0
5x = -1
x = -1/5
∴ Value of x = -10, -1/5

8. Solve the following quadratic equations for x and give your answer correct to 2 decimal places:
(i) $x^2 - 5x - 10 = 0$
(ii) $x^2 + 7x = 7$
Solution:

(i) $x^2 - 5x - 10 = 0$
Let us consider,
a = 1, b = -5, c = -10
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$

$= (-5)^2 – 4(1)(-10)$
$= 25 + 40$
$= 65$

So,

$x = [-(-5) \pm \sqrt{65}] / 2(1)$
$= [5 \pm \sqrt{65}] / 2$
$= [5 \pm 8.06] / 2$
$= [5 + 8.06] / 2$ or $[5 - 8.06] / 2$
$= [13.06]/2$ or $[-3.06]/2$
$= 6.53$ or -1.53

∴ Value of x = 6.53 or -1.53

(ii) $x^2 + 7x = 7$

On rearranging the expression, we get

$x^2 + 7x – 7 = 0$

Let us consider,

a = 1, b = 7, c = -7

So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 – 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 – 4ac$
$= (7)^2 – 4(1)(-7)$
$= 49 + 28$
$= 77$

So,

$x = [-7 \pm \sqrt{77}] / 2(1)$
$= [-7 \pm 8.77] / 2$
$= [-7 + 8.77] / 2$ or $[-7 – 8.77] / 2$
$= 1.77/2$ or $-15.77/2$
$= 0.885$ or -7.885

∴ Value of x = 0.89 or -7.89

9. Solve the following equations by using quadratic formula and give your answer correct to 2 decimal places:

(i) $4x^2 – 5x – 3 = 0$

(ii) $2x – 1/x = 7$

Solution:

(i) $4x^2 - 5x - 3 = 0$

Let us consider,

a = 4, b = -5, c = -3

So, by using the formula,

$D = b^2 - 4ac$

$= (-5)^2 - 4(4)(-3)$

$= 25 + 48$

$= 73$

So,

$x = [-(-5) \pm \sqrt{73}] / 2(4)$

$= [5 \pm 8.54] / 8$

$= [5 + 8.54] / 8$ or $[5 - 8.54] / 8$

$= 13.54/8$ or $-3.54/8$

$= 1.6925$ or -0.4425

∴ Value of x = 1.69 or -0.44

(ii) $2x - 1/x = 7$

By taking LCM

$2x^2 - 1 = 7x$

$2x^2 - 7x - 1 = 0$

Let us consider,

a = 2, b = -7, c = -1

So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$

$= (-7)^2 - 4(2)(-1)$

$= 49 + 8$

$= 57$

So,

$x = [-(-7) \pm \sqrt{57}] / 2(2)$

$= [7 \pm 7.549] / 4$

$= [7 + 7.549] / 4$ or $[7 - 7.549] / 4$

$= 14.549/4$ or $-0.549/4$

= 3.637 or -0.137
= 3.64 or -0.14
∴ Value of x = 3.64 or -0.14

12. Solve the following equations and give your answer correct to two significant figures.
(i) $x^2 - 4x - 8 = 0$ (ii) $x - 18/x = 6$.
Solution:

(i) Given equation:
$x^2 - 4x - 8 = 0$
Let us consider,
a = 1, b = -4, c = -8
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-4)^2 - 4(1)(-8)$
$= 16 + 32$
$= 48$
So,
$x = [-(-4) \pm \sqrt{48}] / 2(1)$
$= [4 \pm 6.93]/2$
$= [4 + 6.93]/2$ or $[4 - 6.93]/2$
$= [10.93]/2$ or $-2.93/2$
= 5.465 or -1.465
∴ Value of x = 5.47 or -1.47

(ii) Given equation:
$x - 18/x = 6$
By taking LCM
$x^2 - 18 = 6x$
$x^2 - 6x - 18 = 0$
Let us consider,
a = 1, b = -6, c = -18

So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-6)^2 - 4(1)(-18)$
$= 36 + 72$
$= 108$

So,
$x = [-(-6) \pm \sqrt{108}] / 2(1)$
$= [6 \pm 10.39]/2$
$= [6 + 10.39]/2$ or $[6 - 10.39]/2$
$= [16.39]/2$ or $-4.39/2$
$= 8.19$ or -2.19
∴ Value of x = 8.19 or -2.19

11. Solve the equation $5x^2 - 3x - 4 = 0$ and give your answer correct to 3 significant figures:

Solution:

Given equation:
$5x^2 - 3x - 4 = 0$
Let us consider,
a = 5, b = -3, c = -4
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-3)^2 - 4(5)(-4)$
$= 9 + 80$
$= 89$

So,
$x = [-(-3) \pm \sqrt{89}] / 2(5)$
$= [3 \pm 9.43] / 10$
$= [3 + 9.43] / 10$ or $[3 \quad 9.43] / 10$

= 12.433/10 or -6.43/10
= 1.24 or -0.643
∴ Value of x = 1.24 or -0.643

EXERCISE 5.4

1. Find the discriminate of the following equations and hence find the nature of roots:
(i) $3x^2 - 5x - 2 = 0$
(ii) $2x^2 - 3x + 5 = 0$
(iii) $16x^2 - 40x + 25 = 0$
(iv) $2x^2 + 15x + 30 = 0$
Solution:
(i) $3x^2 - 5x - 2 = 0$
Let us consider,
$a = 3, b = -5, c = -2$
By using the formula,
$D = b^2 - 4ac$
$= (-5)^2 - 4(3)(-2)$
$= 25 + 24$
$= 49$
So,
Discriminate, $D = 49$
$D > 0$
∴ Roots are real and distinct.

(ii) $2x^2 - 3x + 5 = 0$
Let us consider,
$a = 2, b = -3, c = 5$
By using the formula,
$D = b^2 - 4ac$
$= (-3)^2 - 4(2)(5)$
$= 9 - 40$
$= -31$
So,
Discriminate, $D = -31$
$D < 0$
∴ Roots are not real.

(iii) $16x^2 - 40x + 25 = 0$
Let us consider,
$a = 16, b = -40, c = 25$
By using the formula,

$D = b^2 - 4ac$
$= (-40)^2 - 4(16)(25)$
$= 1600 - 1600$
$= 0$
So,
Discriminate, $D = 0$
$D = 0$
∴ Roots are real and equal.

(iv) $2x^2 + 15x + 30 = 0$
Let us consider,
$a = 2, b = 15, c = 30$
By using the formula,
$D = b^2 - 4ac$
$= (15)^2 - 4(2)(30)$
$= 225 - 240$
$= -15$
So,
Discriminate, $D = -15$
$D < 0$
∴ Roots are not real.

2. Discuss the nature of the roots of the following quadratic equations:
(i) $3x^2 - 4\sqrt{3}x + 4 = 0$
(ii) $x^2 - 1/2x + 4 = 0$
(iii) $-2x^2 + x + 1 = 0$
(iv) $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$
Solution:
(i) $3x^2 - 4\sqrt{3}x + 4 = 0$
Let us consider,
$a = 3, b = -4\sqrt{3}, c = 4$
By using the formula,
$D = b^2 - 4ac$
$= (-4\sqrt{3})^2 - 4(3)(4)$
$= 16(3) - 48$
$= 48 - 48$
$= 0$
So,
Discriminate, $D = 0$

D = 0
∴ Roots are real and equal.

(ii) $x^2 - 1/2x + 4 = 0$
Let us consider,
a = 1, b = -1/2, c = 4
By using the formula,
$D = b^2 - 4ac$
$= (-1/2)^2 - 4(1)(4)$
$= 1/4 - 16$
$= -63/4$
So,
Discriminate, D = -63/4
D < 0
∴ Roots are not real.

(iii) $-2x^2 + x + 1 = 0$
Let us consider,
a = -2, b = 1, c = 1
By using the formula,
$D = b^2 - 4ac$
$= (1)^2 - 4(-2)(1)$
$= 1 + 8$
$= 9$
So,
Discriminate, D = 9
D > 0
∴ Roots are real and distinct.

(iv) $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$
Let us consider,
a = 2√3, b = -5, c = √3
By using the formula,
$D = b^2 - 4ac$
$= (-5)^2 - 4(2\sqrt{3})(\sqrt{3})$
$= 25 - 24$
$= 1$
So,
Discriminate, D = 1

$D > 0$
∴ Roots are real and distinct.

3. Find the nature of the roots of the following quadratic equations:
(i) $x^2 - 1/2x - 1/2 = 0$
(ii) $x^2 - 2\sqrt{3}x - 1 = 0$
If real roots exist, find them.
Solution:
(i) $x^2 - 1/2x - 1/2 = 0$
Let us consider,
$a = 1, b = -1/2, c = -1/2$
By using the formula,
$D = b^2 - 4ac$
$= (-1/2)^2 - 4(1)(-1/2)$
$= 1/4 + 2$
$= (1+8)/4$
$= 9/4$
So,
Discriminate, $D = 9/4$
$D > 0$
∴ Roots are real and unequal.

(ii) $x^2 - 2\sqrt{3}x - 1 = 0$
Let us consider,
$a = 1, b = 2\sqrt{3}, c = -1$
By using the formula,
$D = b^2 - 4ac$
$= (2\sqrt{3})^2 - 4(1)(-1)$
$= 12 + 4$
$= 16$
So,
Discriminate, $D = 16$
$D > 0$
∴ Roots are real and unequal.

4. Without solving the following quadratic equation, find the value of 'p' for which the given equations have real and equal roots:
(i) $px^2 - 4x + 3 = 0$
(ii) $x^2 + (p - 3)x + p = 0$

Solution:

(i) $px^2 - 4x + 3 = 0$

Let us consider,

a = p, b = -4, c = 3

By using the formula,

$D = b^2 - 4ac$

$= (-4)^2 - 4(p)(3)$

$= 16 - 12p$

Since, roots are real.

$16 - 12p = 0$

$16 = 12p$

$p = 16/12$

$= 4/3$

$\therefore p = 4/3$

(ii) $x^2 + (p - 3)x + p = 0$

Let us consider,

a = 1, b = (p - 3), c = p

By using the formula,

$D = b^2 - 4ac$

$= (p - 3)^2 - 4(1)(p)$

$= p^2 - 3^2 - 2(3)(p) - 4p$

$= p^2 + 9 - 6p - 4p$

$= p^2 - 10p + 9$

Since, roots are real and have equal roots.

$p^2 - 10p + 9 = 0$

Now let us factorize,

$p^2 - 9p - p + 9 = 0$

$p(p - 9) - 1(p - 9) = 0$

$(p - 9)(p - 1) = 0$

So,

$(p - 9) = 0$ or $(p - 1) = 0$

$p = 9$ or $p = 1$

$\therefore p = 1, 9$

5. Find the value (s) of k for which each of the following quadratic equation has equal roots:

(i) $x^2 + 4kx + (k^2 - k + 2) = 0$

(ii) $(k - 4)x^2 + 2(k - 4)x + 4 = 0$

Solution:

(i) $x^2 + 4kx + (k^2 - k + 2) = 0$

Let us consider,

$a = 1, b = 4k, c = k^2 - k + 2$

By using the formula,

$D = b^2 - 4ac$

$= (4k)^2 - 4(1)(k^2 - k + 2)$

$= 16k^2 - 4k^2 + 4k - 8$

$= 12k^2 + 4k - 8$

As, roots are equal, D = 0

$12k^2 + 4k - 8 = 0$

Dividing by 4 on both sides, we get

$3k^2 + k - 2 = 0$

$3k^2 + 3k - k - 2 = 0$

$3k(k + 1) - 1(k + 2) = 0$

$(3k - 1)(k + 2) = 0$

So,

$(3k - 1) = 0$ or $(k + 2) = 0$

$k = 1/3$ or $k = -2$

$\therefore k = 1/3, -2$

(ii) $(k - 4)x^2 + 2(k - 4)x + 4 = 0$

Let us consider,

$a = (k - 4), b = 2(k - 4), c = 4$

By using the formula,

$D = b^2 - 4ac$

$= (2(k - 4))^2 - 4(k - 4)(4)$

$= (4(k^2 + 16 - 8k)) - 16(k - 4)$

$= 4(k^2 - 8k + 16) - 16k + 64$

$= 4[k^2 - 8k + 16 - 4k + 16]$

$= 4[k^2 - 12k + 32]$

Since, roots are equal.

$4[k^2 - 12k + 32] = 0$

$k^2 - 12k + 32 = 0$

Now let us factorize,

$k^2 - 8k - 4k + 32 = 0$

$k(k - 8) - 4(k - 8) = 0$

$(k - 8)(k - 4) = 0$

So,

$(k - 8) = 0$ or $(k - 4) \neq 0$
$k = 8$ or $k \neq 4$
$\therefore k = 8$

6. Find the value(s) of m for which each of the following quadratic equation has real and equal roots:
(i) $(3m + 1)x^2 + 2(m + 1)x + m = 0$
(ii) $x^2 + 2(m – 1) x + (m + 5) = 0$
Solution:
(i) $(3m + 1)x^2 + 2(m + 1)x + m = 0$
Let us consider,
$a = (3m + 1)$, $b = 2(m + 1)$, $c = m$
By using the formula,
$D = b^2 – 4ac$
$= (2(m + 1))^2 – 4 (3m + 1) (m)$
$= 4(m^2 + 1 + 2m) – 4m(3m + 1)$
$= 4(m^2 + 2m + 1) – 12m^2 – 4m$
$= 4m^2 + 8m + 4 – 12m^2 – 4m$
$= -8m^2 + 4m + 4$
Since, roots are equal.
$D = 0$
$-8m^2 + 4m + 4 = 0$
Divide by 4, we get
$-2m^2 + m + 1 = 0$
$2m^2 – m – 1 = 0$
Now let us factorize,
$2m^2 – 2m + m – 1 = 0$
$2m(m - 1) +1 (m - 1) = 0$
$(m – 1) (2m + 1) = 0$
So,
$(m – 1) = 0$ or $(2m + 1) = 0$
$m = 1$ or $2m = -1$
$m = 1$ or $m = -1/2$
$\therefore m = 1, -1/2$

(ii) $x^2 + 2(m – 1) x + (m + 5) = 0$
Let us consider,
$a = 1$, $b = 2(m - 1)$, $c = (m + 5)$
By using the formula,

$D = b^2 - 4ac$
$= (2(m - 1))^2 - 4 (1) (m + 5)$
$= [4(m^2 + 1 - 2m)] - 4m - 20$
$= 4m^2 - 8m + 4 - 4m - 20$
$= 4m^2 - 12m - 16$
Since, roots are equal.
$D = 0$
$4m^2 - 12m - 16 = 0$
Divide by 4, we get.
$m^2 - 3m - 4 = 0$
Now let us factorize,
$m^2 - 4m + m - 4 = 0$
$m(m - 4) + 1 (m - 4) = 0$
$(m - 4) (m + 1) = 0$
So,
$(m - 4) = 0$ or $(m + 1) = 0$
$m = 4$ or $m = -1$
$\therefore m = 4, -1$

7. Find the values of k for which each of the following quadratic equation has equal roots:
(i) $9x^2 + kx + 1 = 0$
(ii) $x^2 - 2kx + 7k - 12 = 0$
Also, find the roots for those values of k in each case.
Solution:
(i) $9x^2 + kx + 1 = 0$
Let us consider,
$a = 9, b = k, c = 1$
By using the formula,
$D = b^2 - 4ac$
$= (k)^2 - 4 (9) (1)$
$= k^2 - 36$
Since, roots are equal.
$D = 0$
$k^2 - 36 = 0$
$(k + 6) (k - 6) = 0$
So,
$(k + 6) = 0$ or $(k - 6) = 0$
$k = -6$ or $k = 6$

∴ $k = 6, -6$

Now, let us substitute in the equation

When $k = 6$, then

$9x^2 + kx + 1 = 0$

$9x^2 + 6x + 1 = 0$

$(3x)^2 + 2(3x)(1) + 1^2 = 0$

$(3x + 1)^2 = 0$

$3x + 1 = 0$

$3x = -1$

$x = -1/3, -1/3$

When $k = -6$, then

$9x^2 + kx + 1 = 0$

$9x^2 - 6x + 1 = 0$

$(3x)^2 - 2(3x)(1) + 1^2 = 0$

$(3x - 1)^2 = 0$

$3x - 1 = 0$

$3x = 1$

$x = 1/3, 1/3$

(ii) $x^2 - 2kx + 7k - 12 = 0$

Let us consider,

$a = 1, b = -2k, c = (7k - 12)$

By using the formula,

$D = b^2 - 4ac$

$= (-2k)^2 - 4 (1) (7k - 12)$

$= 4k^2 - 28k + 48$

Since, roots are equal.

$D = 0$

$4k^2 - 28k + 48 = 0$

Divide by 4, we get

$k^2 - 7k + 12 = 0$

Now let us factorize,

$k^2 - 3k - 4k + 12 = 0$

$k(k - 3) - 4 (k - 3) = 0$

$(k - 3) (k - 4) = 0$

So,

$(k - 3) = 0$ or $(k - 4) = 0$

$k = 3$ or $k = 4$

∴ k = 3, 4
Now, let us substitute in the equation
When k = 3, then
By using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

= [-(-2k) ± √0] / 2(1)
= [2(3)]/2
= 3
x = 3, 3

When k = 4, then
By using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

= [-(-2k) ± √0] / 2(1)
= [2(4)] / 2
= 8/2
= 4
x = 4, 4

8. Find the value(s) of p for which the quadratic equation $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also find these roots.

Solution:

Given:
$(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$
Let us compare with $ax^2 + bx + c = 0$
So we get,
a = (2p + 1), b = – (7p + 2), c = (7p – 3)
By using the formula,
$D = b^2 - 4ac$
$0 = (-(7p + 2))^2 - 4(2p + 1)(7p - 3)$
$= 49p^2 + 4 + 28p - 4[14p^2 - 6p + 7p - 3]$

$= 49p^2 + 4 + 28p - 56p^2 - 4p + 12$
$= -7p^2 + 24p + 16$
Let us factorize,
$-7p^2 + 28p - 4p + 16 = 0$
$-7p(p - 4) - 4 (p - 4) = 0$
$(p - 4) (-7p - 4) = 0$
So,
$(p - 4) = 0$ or $(-7p - 4) = 0$
$p = 4$ or $-7p = 4$
$p = 4$ or $p = -4/7$
∴ Value of $p = 4, -4/7$

9. Find the value(s) of p for which the equation $2x^2 + 3x + p = 0$ has real roots.
Solution:
Given:
$2x^2 + 3x + p = 0$
Let us consider,
$a = 2, b = 3, c = p$
By using the formula,
$D = b^2 - 4ac$
$= (3)^2 - 4 (2) (p)$
$= 9 - 8p$
Since, roots are real.
$9 - 8p \geq 0$
$9 \geq 8p$
$8p \leq 9$
$p \leq 9/8$

10. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.
Solution:
Given:
$x^2 + kx + 4 = 0$
Let us consider,
$a = 1, b = k, c = 4$
By using the formula,
$D = b^2 - 4ac$
$= (k)^2 - 4 (1) (4)$
$= k^2 - 16$

Since, roots are real and positive.
$k^2 - 16 \geq 0$
$k^2 \geq 16$
$k \geq 4$
$k = 4$
∴ Value of $k = 4$

11. Find the values of p for which the equation $3x^2 - px + 5 = 0$ has real roots.
Solution:
Given:
$3x^2 - px + 5 = 0$
Let us consider,
$a = 3, b = -p, c = 5$
By using the formula,
$D = b^2 - 4ac$
$= (-p)^2 - 4 (3) (5)$
$= p^2 - 60$
Since, roots are real.
$p^2 - 60 \geq 0$
$p^2 \geq 60$
$p \geq \pm \sqrt{60}$
$p \geq \pm 2\sqrt{15}$
$p \geq + 2\sqrt{15}$ or $p \leq -2\sqrt{15}$
∴ Value of $p = 2\sqrt{15}, -2\sqrt{15}$

EXERCISE 5.5

1. (i) Find two consecutive natural numbers such that the sum of their squares is 61.
(ii) Find two consecutive integers such that the sum of their squares is 61.
Solution:
(i) Find two consecutive natural numbers such that the sum of their squares is 61.
Let us consider first natural number be 'x'
Second natural number be 'x + 1'
So according to the question,
$x^2 + (x + 1)^2 = 61$
let us simplify the expression,
$x^2 + x^2 + 1^2 + 2x – 61 = 0$
$2x^2 + 2x – 60 = 0$
Divide by 2, we get
$x^2 + x – 30 = 0$
Let us factorize,
$x^2 + 6x – 5x – 30 = 0$
$x(x + 6) – 5 (x + 6) = 0$
$(x + 6) (x - 5) = 0$
So,
$(x + 6) = 0$ or $(x - 5) = 0$
$x = -6$ or $x = 5$
$\therefore$ $x = 5$ [Since -6 is not a positive number]
Hence the first natural number = 5
Second natural number = 5 + 1 = 6

(ii) Find two consecutive integers such that the sum of their squares is 61.
Let us consider first integer number be 'x'
Second integer number be 'x + 1'
So according to the question,
$x^2 + (x + 1)^2 = 61$
let us simplify the expression,
$x^2 + x^2 + 1^2 + 2x – 61 = 0$
$2x^2 + 2x – 60 = 0$
Divide by 2, we get
$x^2 + x – 30 = 0$
Let us factorize,
$x^2 + 6x – 5x – 30 = 0$
$x(x + 6) – 5 (x + 6) = 0$

(x + 6) (x - 5) = 0
So,
(x + 6) = 0 or (x - 5) = 0
x = -6 or x = 5
Now,
If x = -6, then
First integer number = -6
Second integer number = -6 + 1 = -5

If x = 5, then
First integer number = 5
Second integer number = 5 + 1 = 6

2. (i) If the product of two positive consecutive even integers is 288, find the integers.
(ii) If the product of two consecutive even integers is 224, find the integers.
(iii) Find two consecutive even natural numbers such that the sum of their squares is 340.
(iv) Find two consecutive odd integers such that the sum of their squares is 394.
Solution:
(i) If the product of two positive consecutive even integers is 288, find the integers.
Let us consider first positive even integer number be '2x'
Second even integer number be '2x + 2'
So according to the question,
2x × (2x + 2) = 288
$4x^2 + 4x – 288 = 0$
Divide by 4, we get
$x^2 + x – 72 = 0$
Let us factorize,
$x^2 + 9x – 8x – 72 = 0$
x(x + 9) – 8(x + 9) = 0
(x + 9) (x - 8) = 0
So,
(x + 9) = 0 or (x - 8) = 0
x = -9 or x = 8
∴ Value of x = 8 [since, -9 is not positive]
First even integer = 2x = 2 (8) = 16
Second even integer = 2x + 2 = 2(8) + 2 = 18

(ii) If the product of two consecutive even integers is 224, find the integers.
Let us consider first positive even integer number be '2x'
Second even integer number be '2x + 2'
So according to the question,
$2x \times (2x + 2) = 224$
$4x^2 + 4x - 224 = 0$
Divide by 4, we get
$x^2 + x - 56 = 0$
Let us factorize,
$x^2 + 8x - 7x - 56 = 0$
$x(x + 8) - 7(x + 8) = 0$
$(x + 8)(x - 7) = 0$
So,
$(x + 8) = 0$ or $(x - 7) = 0$
$x = -8$ or $x = 7$
∴ Value of x = 7 [since, -8 is not positive]
First even integer = 2x = 2 (7) = 14
Second even integer = 2x + 2 = 2(7) + 2 = 16

(iii) Find two consecutive even natural numbers such that the sum of their squares is 340.
Let us consider first positive even natural number be '2x'
Second even number be '2x + 2'
So according to the question,
$(2x)^2 + (2x + 2)^2 = 340$
$4x^2 + 4x^2 + 8x + 4 - 340 = 0$
$8x^2 + 8x - 336 = 0$
Divide by 8, we get
$x^2 + x - 42 = 0$
Let us factorize,
$x^2 + 7x - 6x - 56 = 0$
$x(x + 7) - 6(x + 7) = 0$
$(x + 7)(x - 6) = 0$
So,
$(x + 7) = 0$ or $(x - 6) = 0$
$x = -7$ or $x = 6$
∴ Value of x = 6 [since, -7 is not positive]
First even natural number = 2x = 2 (6) = 12
Second even natural number = 2x + 2 = 2(6) + 2 = 14

(iv) Find two consecutive odd integers such that the sum of their squares is 394.
Let us consider first odd integer number be ‘2x + 1’
Second odd integer number be ‘2x + 3’
So according to the question,
$(2x + 1)^2 + (2x + 3)^2 = 394$
$4x^2 + 4x + 1 + 4x^2 + 12x + 9 – 394 = 0$
$8x^2 + 16x – 384 = 0$
Divide by 8, we get
$x^2 + 2x – 48 = 0$
Let us factorize,
$x^2 + 8x – 6x – 48 = 0$
$x(x + 8) – 6(x + 8) = 0$
$(x + 8) (x - 6) = 0$
So,
$(x + 8) = 0$ or $(x - 6) = 0$
$x = -8$ or $x = 6$
When $x = -8$, then
First odd integer = $2x + 1 = 2 (-8) + 1 = -16 + 1 = -15$
Second odd integer = $2x + 3 = 2(-8) + 3 = -16 + 3 = -13$

When $x = 6$, then
First odd integer = $2x + 1 = 2 (6) + 1 = 12 + 1 = 13$
Second odd integer = $2x + 3 = 2(6) + 3 = 12 + 3 = 15$

∴ The required odd integers are -15, -13, 13, 15.

3. The sum of two numbers is 9 and the sum of their squares is 41. Taking one number as x, form ail equation in x and solve it to find the numbers.
Solution:
Given:
Sum of two number = 9
Let us consider first number be ‘x’
Second number be ‘9 – x’
So according to the question,
$(x)^2 + (9 – x)^2 = 41$
$x^2 + 81 - 18x + x^2 – 41 = 0$
$2x^2 - 18x + 40 = 0$
Divide by 2, we get
$x^2 - 9x + 20 = 0$

Let us factorize,
$x^2 - 4x - 5x + 20 = 0$
$x(x - 4) - 5(x - 4) = 0$
$(x - 4)(x - 5) = 0$
So,
$(x - 4) = 0$ or $(x - 5) = 0$
$x = 4$ or $x = 5$
When $x = 4$, then
First number = $x = 4$
Second number = $9 - x = 9 - 4 = 5$

When $x = 5$, then
First number = $x = 5$
Second number = $9 - x = 9 - 5 = 4$

∴ The required numbers are 4 and 5.

4. Five times a certain whole number is equal to three less than twice the square of the number. Find the number.
Solution:
Let us consider the number be 'x'
So according to the question,
$5x = 2x^2 - 3$
$2x^2 - 3 - 5x = 0$
$2x^2 - 5x - 3 = 0$
Let us factorize,
$2x^2 - 6x + x - 3 = 0$
$2x(x - 3) + 1(x - 3) = 0$
$(x - 3)(2x + 1) = 0$
So,
$(x - 3) = 0$ or $(2x + 1) = 0$
$x = 3$ or $2x = -1$
$x = 3$ or $x = -1/2$
∴ The required number is 3 [since, -1/2 cannot be a whole number].

5. Sum of two natural numbers is 8 and the difference of their reciprocals is 2/15. Find the numbers.
Solution:
Let us consider two numbers as 'x' and 'y'

So according to the question,
$1/x - 1/y = 2/15$ ….. (i)
It is given that, $x + y = 8$
So, $y = 8 - x$ … (ii)
Now, substitute the value of y in equation (i), we get
$1/x - 1/(8 - x)\ \ 2/15$
By taking LCM,
$[8 - x - x] / x(8 - x) = 2/15$
$(8 - 2x) / x(8 - x) = 2/15$
By cross multiplying,
$15(8 - 2x) = 2x(8 - x)$
$120 - 30x = 16x - 2x^2$
$120 - 30x - 16x + 2x^2 = 0$
$2x^2 - 46x + 120 = 0$
Divide by 2, we get
$x^2 - 23x + 60 = 0$
let us factorize,
$x^2 - 20x - 3x + 60 = 0$
$x(x - 20) - 3(x - 20) = 0$
$(x - 20)(x - 3) = 0$
So,
$(x - 20) = 0$ or $(x - 3) = 0$
$x = 20$ or $x = 3$
Now,
Sum of two natural numbers, $y = 8 - x = 8 - 20 = -12$, which is a negative value.
So value of $x = 3$, $y = 8 - x = 8 - 3 = 5$
∴ The value of x and y are 3 and 5.

6. The difference between the squares of two numbers is 45. The square of the smaller number is 4 times the larger number. Determine the numbers.
Solution:
Let us consider the larger number be ‘x’
Smaller number be ‘y’
So according to the question,
$x^2 - y^2 = 45$ … (i)
$y^2 = 4x$ … (ii)
Now substitute the value of y in equation (i), we get
$x^2 - 4x = 45$
$x^2 - 4x - 45 = 0$

let us factorize,
$x^2 - 9x + 5x - 45 = 0$
$x(x - 9) + 5 (x - 9) = 0$
$(x - 9) (x + 5) = 0$
So,
$(x - 9) = 0$ or $(x + 5) = 0$
$x = 9$ or $x = -5$
When x = 9, then
The larger number = x = 9
Smaller number = $y \Rightarrow y^2 = 4x$
$y = \sqrt{4x} = \sqrt{4(9)} = \sqrt{36} = 6$

When x = -5, then
The larger number = x = -5
Smaller number = $y \Rightarrow y^2 = 4x$
$y = \sqrt{4x} = \sqrt{4(-5)} = \sqrt{-20}$ (which is not possible)
∴ The value of x and y are 9, 6.

7. There are three consecutive positive integers such that the sum of the square of the first and the product of the other two is 154. What are the integers?
Solution:
Let us consider the first integer be 'x'
Second integer be 'x + 1'
Third integer be 'x + 2'
So, according to the question,
$x^2 + (x + 1) (x + 2) = 154$
let us simplify,
$x^2 + x^2 + 3x + 2 - 154 = 0$
$2x^2 + 3x - 152 = 0$
Let us factorize,
$2x^2 + 19x - 16x - 152 = 0$
$x(2x + 19) - 8 (2x + 19) = 0$
$(2x + 19) (x - 8) = 0$
So,
$(2x + 19) = 0$ or $(x - 8) = 0$
$2x = -19$ or $x = 8$
$x = -19/2$ or $x = 8$
∴ The value of x = 8 [since -19/2 is a negative value]
So,

First integer = x = 8
Second integer = x + 1 = 8 + 1 = 9
Third integer = x + 2 = 8 + 2 = 10
∴ The numbers are 8, 9, 10.

8. (i) Find three successive even natural numbers, the sum of whose squares is 308.
(ii) Find three consecutive odd integers, the sum of whose squares is 83.
Solution:
(i) Find three successive even natural numbers, the sum of whose squares is 308.
Let us consider first even natural number be '2x'
Second even number be '2x + 2'
Third even number be '2x + 4'
So according to the question,
$(2x)^2 + (2x + 2)^2 + (2x + 4)^2 = 308$
$4x^2 + 4x^2 + 8x + 4 + 4x^2 + 16x + 16 – 308 = 0$
$12x^2 + 24x – 288 = 0$
Divide by 12, we get
$x^2 + 2x – 24 = 0$
Let us factorize,
$x^2 + 6x – 4x – 24 = 0$
$x(x + 6) – 4(x + 6) = 0$
$(x + 6) (x - 4) = 0$
So,
$(x + 6) = 0$ or $(x - 4) = 0$
$x = -6$ or $x = 4$
∴ Value of x = 4 [since, -6 is not positive]
First even natural number = 2x = 2 (4) = 8
Second even natural number = 2x + 2 = 2(4) + 2 = 10
Third even natural number = 2x + 4 = 2(4) + 4 = 12
∴ The numbers are 8, 10, 12.

(ii) Find three consecutive odd integers, the sum of whose squares is 83.
Let the three numbers be 'x', 'x + 2', 'x + 4'
So according to the question,
$(x)^2 + (x + 2)^2 + (x + 4)^2 = 83$
$x^2 + x^2 + 4x + 4 + x^2 + 8x + 16 – 83 = 0$
$3x^2 + 12x – 63 = 0$
Divide by 3, we get
$x^2 + 4x – 21 = 0$

let us factorize,
$x^2 + 7x - 3x - 21 = 0$
$x(x + 7) - 3(x + 7) = 0$
$(x + 7)(x - 3) = 0$
So,
$(x + 7) = 0$ or $(x - 3) = 0$
$x = -7$ or $x = 3$
∴ The numbers will be x, x+2, x+4 => -7, -7+2, -7+4 => -7, -5, -3
Or the numbers will be x, x+2, x+4 => 3, 3+2, 3+4, => 3, 5, 7

9. In a certain positive fraction, the denominator is greater than the numerator by 3. If 1 is subtracted from both the numerator and denominator, the fraction is decreased by 1/14. Find the fraction.

Solution:
Let the numerator be 'x'
Denominator be 'x+3'
So the fraction is x/(x+3)
According to the question,

$$\frac{x-1}{x+3-1} = \frac{x}{x+3} - \frac{1}{14}$$

Firstly let us simplify RHS

$$\frac{x-1}{x+2} = \frac{14x - x - 3}{14(x+3)}$$

$$\frac{x-1}{x+2} = \frac{13x-3}{14x+42}$$

By cross multiplying, we get
$(x - 1)(14x + 42) = (x + 2)(13x - 3)$
$14x^2 + 42x - 14x - 42 = 13x^2 - 3x + 26x - 6$
$14x^2 + 42x - 14x - 42 - 13x^2 + 3x - 26x + 6 = 0$
$x^2 + 5x - 36 = 0$
let us factorize,
$x^2 + 9x - 4x - 36 = 0$
$x(x + 9) - 4(x + 9) = 0$
$(x + 9)(x - 4) = 0$
So,
$(x + 9) = 0$ or $(x - 4) = 0$
$x = -9$ or $x = 4$

So the value of x = 4 [since, -9 is a negative number]
When substitute the value of x = 4 in the fraction x/(x+3), we get
4/(4+3) = 4/7
∴ The required fraction is = 4/7

10. The sum of the numerator and denominator of a certain positive fraction is 8. If 2 is added to both the numerator and denominator, the fraction is increased by 4/35. Find the fraction.

Solution:

Let the denominator be 'x'
So the numerator will be '8-x'
The obtained fraction is (8-x)/x
So according to the question,

$$\frac{8-x+2}{x+2}=\frac{8-x}{x}+\frac{4}{35}$$

$$\frac{10-x}{x+2}=\frac{8-x}{x}+\frac{4}{35}$$

$$\frac{10-x}{x+2}-\frac{8-x}{x}=\frac{4}{35}$$

By taking LCM

$$\frac{10x-x^2-8x+x^2-16+2x}{x(x+2)}=\frac{4}{35}$$

$$\frac{4x-16}{x^2+2x}=\frac{4}{35}$$

By cross multiplying,
$35(4x - 16) = 4(x^2 + 2x)$
$140x – 560 = 4x^2 + 8x$
$4x^2 + 8x – 140x + 560 = 0$
$4x^2 – 132x + 560 = 0$
Divide by 4, we get
$x^2 – 33x + 140 = 0$
let us factorize,
$x^2 – 28x – 5x + 140 = 0$
$x(x - 28) – 5 (x – 28) = 0$
$(x - 28) (x - 5) = 0$
So,

(x - 28) = 0 or (x - 5) = 0
x = 28 or x = 5
So the value of x = 5 [since, x = 28 is not possible as sum of numerator and denominator is 8]
When substitute the value of x = 5 in the fraction (8-x)/x, we get
(8 - 5)/5 = 3/5
∴ The required fraction is = 3/5

11. A two digit number contains the bigger at ten's place. The product of the digits is 27 and the difference between two digits is 6. Find the number.
Solution:
Let us consider unit's digit be 'x'
Ten's digit = x+6
Number = x + 10(x+6)
= x + 10x + 60
= 11x + 60
So according the question,
x(x + 6) = 27
$x^2 + 6x - 27 = 0$
let us factorize,
$x^2 + 9x - 3x - 27 = 0$
x(x + 9) – 3 (x + 9) = 0
(x + 9) (x - 3) = 0
So,
(x + 9) = 0 or (x - 3) = 0
x = -9 or x = 3
so, value of x = 3 [since, -9 is a negative number]
∴ The number = 11x + 60
= 11(3) + 60
= 33 + 60
= 93

12. A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. (2014)
Solution:
Let us consider 2-digit number be 'xy' = 10x + y
Reversed digits = yx = 10y + x
So according to the question,
10x + y + 9 = 10y + x

It is given that,
$xy = 6$
$y = 6/x$
so, by substituting the value in above equation, we get
$10x + 6/x + 9 = 10(6/x) + x$
By taking LCM,
$10x^2 + 6 + 9x = 60 + x^2$
$10x^2 + 6 + 9x – 60 – x^2 = 0$
$9x^2 + 9x – 54 = 0$
Divide by 9, we get
$x^2 + x – 6 = 0$
let us factorize,
$x^2 + 3x – 2x – 6 = 0$
$x(x + 3) – 2 (x + 3) = 0$
$(x + 3) (x - 2) = 0$
So,
$(x + 3) = 0$ or $(x - 2) = 0$
$x = -3$ or $x = 2$
Value of $x = 2$ [since, -3 is a negative value]
Now, substitute the value of x in $y = 6/x$, we get
$y = 6/2 = 3$
∴ 2-digit number = $10x + y = 10(2) + 3 = 23$

13. A rectangle of area 105 cm² has its length equal to x cm. Write down its breadth in terms of x. Given that the perimeter is 44 cm, write down an equation in x and solve it to determine the dimensions of the rectangle.
Solution:
Given:
Perimeter of rectangle = 44cm
Length + breadth = 44/2 = 22cm
Let us consider length be 'x'
Breadth be '22 – x'
So according to the question,
$x(22 - x) = 105$
$22x – x^2 – 105 = 0$
$x^2 – 22x + 105 = 0$
let us factorize,
$x^2 – 15x – 7x + 105 = 0$
$x(x - 15) – 7(x - 15) = 0$

(x - 15) (x – 7) = 0
So,
(x - 15) = 0 or (x – 7) = 0
x = 15 or x = 7
Since length > breadth, x = 7 is not admissible.
∴ Length = 15cm
Breadth = 22 – x = 22 – 15 = 7cm

14. A rectangular garden 10 m by 16 m is to be surrounded by a concrete walk of uniform width. Given that the area of the walk is 120 square meters, assuming the width of the walk to be x, form an equation in x and solve it to find the value of x. (1992)
Solution:
Given:
Length of garden = 16cm
Width = 10cm
Let the width of walk be 'x' meter
Outer length = 16 + 2x
Outer width = 10 + 2x
So according to the question,
(16 + 2x) (10 + 2x) – 16(10) = 120
160 + 32x + 20x + $4x^2$ – 160 – 120 = 0
$4x^2 + 52x – 120 = 0$
Divide by 4, we get
$x^2 + 13x – 30 = 0$
$x^2 + 15x – 2x – 30 = 0$
x(x + 15) – 2 (x + 15) = 0
(x + 15) (x - 2) = 0
So,
(x + 15) = 0 or (x - 2) = 0
x = -15 or x = 2
∴ Value of x is 2 [Since, -15 is a negative value]

15. The length of a rectangle exceeds its breadth by 5 m. If the breadth was doubled and the length reduced by 9 m, the area of the rectangle would have increased by 140 m². Find its dimensions.
Solution:
(ii) In first case:
Let us consider length of the rectangle be 'x' meter

Width = (x - 5) meter
Area = lb
= x(x - 5) sq.m

In second case:
Length = (x – 9) meter
Width = 2 (x - 5) meter
Area = (x - 9) 2(x - 5) = 2(x - 9) (x - 5) sq.m

So according to the question,
$2(x - 9)(x - 5) = x(x - 5) + 140$
$2(x^2 - 14x + 45) = x^2 - 5x + 140$
$2x^2 - 28x + 90 - x^2 + 5x - 140 = 0$
$x^2 - 23x - 50 = 0$
let us factorize,
$x^2 - 25x + 2x - 50 = 0$
$x(x - 25) + 2(x - 25) = 0$
$(x - 25)(x + 2) = 0$
So,
$(x - 25) = 0$ or $(x + 2) = 0$
$x = 25$ or $x = -2$
∴ Length of the first rectangle = 25meters. [Since, -2 is a negative value]
Width = x – 5 = 25 – 5 = 20meters
Area = lb
= $25 \times 20 = 500\ m^2$

16. The perimeter of a rectangular plot is 180 m and its area is 1800 m². Take the length of the plot as x m. Use the perimeter 180 m to write the value of the breadth in terms of x. Use the values of length, breadth and the area to write an equation in x. Solve the equation to calculate the length and breadth of the plot. (1993)
Solution:

Given,
The perimeter of a rectangular field = 180 m
And area = 1800 m²
Let's assume the length of the rectangular field as 'x' m
We know that,
Perimeter of rectangular field = 2 (length + breadth)
So, (length + breadth) = perimeter/ 2
x + breadth = 180/2

⇒ breadth = 90 - x
Now, the area of the area of the rectangular field is given as
Length × breadth = 1800
$x \times (90 - x) = 1800$
$90x - x^2 = 1800$
$x^2 - 90x + 1800 = 0$
Upon factorization, we have
$x^2 - 60x - 30x + 1800 = 0$
$x(x - 60) - 30(x - 60) = 0$
$(x - 30)(x - 60) = 0$
So,
$x - 30 = 0$ or $x - 60 = 0$
$x = 30$ or $x = 60$
As length is greater than its breadth,
Therefore, for the rectangular field
Length = 60 m and breadth = (90 - 60) = 30 m

17. The lengths of the parallel sides of a trapezium are (x + 9) cm and (2x – 3) cm and the distance between them is (x + 4) cm. If its area is 540 cm², find x.
Solution:

We know that,
Area of a trapezium = ½ × (sum of parallel sides) × (height)
Given, the length of parallel sides are (x + 9) and (2x - 3)
And height = (x + 4)
Now, according the conditions in the problem
$\frac{1}{2} \times (x + 9 + 2x - 3) \times (x + 4) = 540$
$(3x + 6)(x + 4) = 540 \times 2$
$3x^2 + 12x + 6x + 24 = 1080$
$3x^2 + 18x - 1056 = 0$
$x^2 + 6x - 352 = 0$ [Dividing by 3]
By factorization method, we have
$x^2 + 22x - 16x - 352 = 0$
$x(x + 22) - 16(x + 22) = 0$
$(x - 16)(x + 22) = 0$
So,
$x - 16 = 0$ or $x + 22 = 0$
$x = 16$ or $x = -22$
As measurements cannot be negative x = -22 is not possible

Therefore, $x = 16$

18. If the perimeter of a rectangular plot is 68 m and the length of its diagonal is 26 m, find its area.
Solution:

Given,
Perimeter = 68 m and diagonal = 26 m
So, Length + breadth = Perimeter/2
= 68/2
= 34 m
Let's consider the length of the rectangular plot to be 'x' m
Then, breadth = (34 - x) m
Now, the diagonal of the rectangular plot is given by
$length^2 + breadth^2 = diagonal^2$ [By Pythagoras Theorem]
$x^2 + (34 - x)^2 = 26^2$
$x^2 + 1156 + x^2 – 68x = 676$
$2x^2 – 68x + 1156 – 676 = 0$
$2x^2 – 68x + 480 = 0$
$x^2 – 34x + 240 = 0$ [Dividing by 2]
By factorization method, we have
$x^2 – 24x – 10x + 240 = 0$
$x(x - 24) – 10(x - 24) = 0$
$(x - 10) (x - 24) = 0$
So,
$x – 10 = 0$ or $x – 24 = 0$
$x = 10$ or $x = 24$
As length is greater than breadth,
Thus, length = 24 m and breadth = (34 – 24) m = 10 m
And, area of the rectangular plot = 24 × 10 = 240 m^2.

19. If the sum of two smaller sides of a right-angled triangle is 17cm and the perimeter is 30cm, then find the area of the triangle.
Solution:

Given,
The perimeter of the triangle = 30 cm
Let's assume the length of one of the two small sides as x cm
Then, the other side will be (17 – x) cm

Now, length of hypotenuse = perimeter – sum of other two sides
= (30 – 17) cm
= 13 cm

According to the problem, by Pythagoras theorem we have

$x^2 + (17 - x)^2 = 13^2$

$x^2 + 289 + x^2 – 34x = 169$

$2x^2 – 34x + 289 – 169 = 0$

$2x^2 – 34x + 120 = 0$

$x^2 – 17x + 60 = 0$ [Dividing by 2]

By factorization method, we have

$x^2 – 12x – 5x + 60 = 0$

$x(x - 12) – 5(x - 12) = 0$

$(x - 5) (x - 12) = 0$

So,

$(x - 5) = 0$ or $(x - 12) = 0$

$x = 5$ or $x = 12$

When, x = 5

First side = 5 cm and second side = (17 - 5) = 12 cm

And when x = 12

First side = 12 cm and second side = (17 - 12) = 5 cm

Thus,

Area of the triangle = ½ (5 × 12)
= 60/2
= 30 cm^2

20. The hypotenuse of grassy land in the shape of a right triangle is 1 metre more than twice the shortest side. If the third side is 7 metres more than the shortest side, find the sides of the grassy land.

Solution:

Let's consider the shortest side to be 'x' cm

Hypotenuse = 2x + 1

And third side = x + 7

Now, by Pythagoras theorem we have

$(2x + 1)^2 = x^2 + (x + 7)^2$

$4x^2 + 1 + 4x = x^2 + x^2 + 49 + 14x$

$4x^2 – 2x^2 + 4x – 14x + 1 – 49 = 0$

$2x^2 – 10x – 48 = 0$

$x^2 – 5x – 24 = 0$ [Dividing by 2]

By factorization method, we have

$x^2 - 8x + 3x - 24 = 0$

$x(x - 8) + 3(x - 8) = 0$

$(x - 8)(x + 3) = 0$

So,

$x - 8 = 0$ or $x + 3 = 0$

$x = 8$ or $x = -3$

As measurement of a side cannot be negative, $x = 8$

Therefore,

The shortest side = 8 m

Third side = $x + 7 = 8 + 7 = 13$ m

And hypotenuse = $2x + 1 = 8 \times 2 + 1 = 16 + 1 = 17$ m

CHAPTER TEST

Solve the following equations (1 to 4) by factorisation:

1.(i) $x^2 + 6x - 16 = 0$ (ii) $3x^2 + 11x + 10 = 0$

Solution:

(i) $x^2 + 6x - 16 = 0$

Let us factorize the given expression,

$x^2 + 8x - 2x - 16 = 0$ [As $8 \times (-2) = -16$ and $8 - 2 = 6$]

$x(x + 8) - 2(x + 8) = 0$

$(x - 2)(x + 8) = 0$

So now,

$(x - 2) = 0$ or $(x + 8) = 0$

$x = 2$ or $x = -8$

∴ Value of $x = 2, -8$

(ii) $3x^2 + 11x + 10 = 0$

Let us factorize the given expression,

$3x^2 + 6x + 5x + 10 = 0$ [As $3 \times 10 = 30$ and $6 + 5 = 11$]

$3x(x + 2) + 5(x + 2) = 0$

$(3x + 5)(x + 2) = 0$

So now,

$(3x + 5) = 0$ or $(x + 2) = 0$

$3x = -5$ or $x = -2$

$x = -5/3$ or $x = -2$

∴ Value of $x = -5/3, -2$

2. (i) $2x^2 + ax - a^2 = 0$ (ii) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Solution:

(i) $2x^2 + ax - a^2 = 0$

Let us factorize the given expression,

$2x^2 + 2ax - ax - a^2 = 0$ [As $2 \times (-a^2) = -2a^2$ and $2a - a = a$]

$2x(x + a) - a(x + a) = 0$

$(2x - a)(x + a) = 0$

So now,

$(2x - a) = 0$ or $(x + a) = 0$

$2x = a$ or $x = -a$

$x = a/2$ or $x = -a$

∴ Value of $x = a/2, -a$

(ii) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$
Let us factorize the given expression,
$\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$ [As $\sqrt{3} \times (7\sqrt{3}) = 7 \times (\sqrt{3})^2 = 21$ and $7 + 3 = 10$]
$\sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$
$(\sqrt{3}x + 7)(x + \sqrt{3}) = 0$
So now,
$(\sqrt{3}x + 7) = 0$ or $(x + \sqrt{3}) = 0$
$\sqrt{3}x = -7$ or $x = -\sqrt{3}$
$x = -7/\sqrt{3}$ or $x = -\sqrt{3}$
∴ Value of $x = -7/\sqrt{3}, -\sqrt{3}$

3. (i) $x(x + 1) + (x + 2)(x + 3) = 42$ (ii) $6/x – 2/(x - 1) = 1/(x - 2)$
Solution:

(i) $x(x + 1) + (x + 2)(x + 3) = 42$
Let us simplify the given expression,
$x^2 + x + x^2 + 2x + 3x + 6 = 42$
$2x^2 + 6x + 6 – 42 = 0$
$2x^2 + 6x – 36 = 0$
$x^2 + 3x – 18 = 0$ [Dividing by 2]
Now, let us factorize
$x^2 + 6x – 3x – 18 = 0$ [As $6 \times (-3) = -18$ and $6 – 3 = 3$]
$x(x + 6) -3(x + 6) = 0$
$(x + 6)(x – 3) = 0$
So now,
$(x + 6) = 0$ or $(x - 3) = 0$
$x = -6$ or $x = 3$
∴ Value of $x = -6, 3$

(ii) $6/x – 2/(x - 1) = 1/(x – 2)$
Let us simplify the given expression,
$[6(x - 1) – 2x]/ [x(x - 1)] = 1/(x - 2)$ [Taking L.C.M]
$(6x – 6 – 2x)/(x^2- x) = 1/(x - 2)$
$(4x – 6)/(x^2- x) = 1/(x - 2)$
$(4x – 6)(x - 2) = (x^2- x)$
$4x^2 – 6x – 8x + 12 = x^2- x$
$4x^2 – x^2 – 14x + x + 12 = 0$
$3x^2 – 13x + 12 = 0$

Now, let us factorize
$3x^2 - 9x - 4x + 12 = 0$
$3x(x - 3) - 4(x - 3) = 0$
$(3x - 4)(x - 3) = 0$
So now,
$(3x - 4) = 0$ or $(x - 3) = 0$
$3x = 4$ or $x = 3$
$x = 4/3$ or $x = 3$
∴ Value of $x = 4/3, 3$

4. (i) $\sqrt{(x + 15)} = x + 3$ (ii) $\sqrt{(3x^2 - 2x - 1)} = 2x - 2$
Solution:

(i) $\sqrt{(x + 15)} = x + 3$
Let us simplify the given expression,
$x + 15 = (x + 3)^2$ [Squaring on both sides]
$x + 15 = x^2 + 9 + 6x$
$x^2 + 6x - x + 9 - 15 = 0$
$x^2 + 5x - 6 = 0$
Now, let us factorize
$x^2 + 6x - x - 6 = 0$
$x(x + 6) -1(x + 6) = 0$
$(x - 1)(x + 6) = 0$
So now,
$(x - 1) = 0$ or $(x + 6) = 0$
$x = 1$ or $x = -6$
∴ Value of $x = 1, -6$

<u>Let's check:</u>
When $x = 6$, then
L.H.S $= \sqrt{(x + 15)}$
$= \sqrt{(-6 + 15)}$
$= \sqrt{9}$
$= 3$
R.H.S $= x + 3$
$= -6 + 3$
$= -3$
Thus, L.H.S ≠ R.H.S
So, $x = -6$ is not a root

And, when $x = 1$, then

L.H.S $= \sqrt{(x + 15)}$

$= \sqrt{(1 + 15)}$

$= \sqrt{16}$

$= 4$

R.H.S $= x + 3$

$= 1 + 3$

$= 4$

Thus, L.H.S = R.H.S

So, $x = 1$ is a root of this equation

Therefore, $x = 1$.

(ii) $\sqrt{(3x^2 - 2x - 1)} = 2x - 2$

Let us simplify the given expression,

$3x^2 - 2x - 1 = (2x - 2)^2$ [Squaring on both sides]

$3x^2 - 2x - 1 = 4x^2 + 4 - 8x$

$4x^2 - 3x^2 - 8x + 2x + 4 + 1 = 0$

$x^2 - 6x + 5 = 0$

Now, let us factorize

$x^2 - 5x - x + 5 = 0$

$x(x - 5) -1(x - 5) = 0$

$(x - 1)(x - 5) = 0$

So now,

$(x - 1) = 0$ or $(x - 5) = 0$

$x = 1$ or $x = 5$

$\therefore$ Value of $x = 1, 5$

<u>Let's check:</u>

When $x = 5$, then

L.H.S $= \sqrt{(3x^2 - 2x - 1)}$

$= \sqrt{(3(5)^2 - 2(5) - 1)}$

$= \sqrt{(3 \times 25 - 2 \times 5 - 1)}$

$= \sqrt{64} = 8$

R.H.S $= 2x - 2$

$= 2(5) - 2$

$= 10 - 2 = 8$

Thus, L.H.S = R.H.S

So, $x = 5$ is a root

And, when x = 1, then
L.H.S = $\sqrt{(3x^2 - 2x - 1)}$
$= \sqrt{(3(1)^2 - 2(1) - 1)}$
$= \sqrt{(3 \times 1 - 2 \times 1 - 1)}$
$= \sqrt{0} = 0$
R.H.S = 2x - 2
= 2(1) - 2
= 0
Thus, L.H.S = R.H.S
So, x = 1 is a root
Therefore, x = 1, 5.

Solve the following equations (5 to 8) by using formula:
5. (i) $2x^2 - 3x - 1 = 0$ (ii) x(3x + ½) = 6
Solution:

(i) $2x^2 - 3x - 1 = 0$
Let us consider,
a = 2, b = -3, c = -1
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-3)^2 - 4(2)(-1)$
= 9 + 8
= 17
So,
$x = [-(-3) \pm \sqrt{17}] / 2(2)$
$= [3 \pm \sqrt{17}] / 4$
$= [3 + \sqrt{17}]/ 4$ or $[3 - \sqrt{17}]/4$
∴ Value of x = $(3 + \sqrt{17})/4, (3 - \sqrt{17})/4$

(ii) x(3x + ½) = 6
Let us simplify the given expression,
$3x^2 + x/2 = 6$

$(6x^2 + x)/2 = 6$ [Taking L.C.M]
$6x^2 + x = 12$
$6x^2 + x – 12 = 0$
Let us consider,
a = 6, b = 1, c = -12
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 – 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 – 4ac$
$= (1)^2 – 4(6)(-12)$
$= 1 + 288$
$= 289$
So,
$x = [-(1) \pm \sqrt{289}] / 2(6)$
$= [-1 \pm 17] / 12$
$= [-1 + 17]/ 12$ or $[-1 - 17]/12$
$= 16/12$ or $-18/12$
$= 4/3$ or $-3/2$
∴ Value of x = 4/3, -3/2

6. (i) (2x + 5)/(3x + 4) = (x + 1)/(x + 3)
(ii) 2/(x + 2) – 1/(x + 1) = 4/(x + 4) – 3/(x + 3)
Solution:

(i) $(2x + 5)/(3x + 4) = (x + 1)/(x + 3)$
Let's simply the given expression,
$(2x + 5)(x + 3) = (x + 1)(3x + 4)$
$2x^2 + 6x + 5x + 15 = 3x^2 + 3x + 4x + 4$
$2x^2 + 11x + 15 = 3x^2 + 7x + 4$
$3x^2 – 2x^2 + 7x – 11x + 4 – 15 = 0$
$x^2 – 4x – 11 = 0$
Let us consider,
a = 1, b = -4, c = -11
So, by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (-4)^2 - 4(1)(-11)$
$= 16 + 44$
$= 60$

So,
$x = [-(-4) \pm \sqrt{60}] / 2(1)$
$= [4 \pm 2\sqrt{15}] / 2$
$= [4 + 2\sqrt{15}]/ 2$ or $[4 - 2\sqrt{15}]/ 2$
$= 2(2 + \sqrt{15})/2$ or $2(2 - \sqrt{15})/2$
$= (2 + \sqrt{15})$ or $(2 - \sqrt{15})$

∴ Value of $x = (2 + \sqrt{15}), (2 - \sqrt{15})$

(ii) $2/(x + 2) - 1/(x + 1) = 4/(x + 4) - 3/(x + 3)$

$$\frac{2}{x+2} - \frac{1}{x+1} = \frac{4}{x+4} - \frac{3}{x+3}$$

$$\frac{2}{x+2} - \frac{1}{x+1} = \frac{4}{x+4} - \frac{3}{x+3}$$

$$\frac{2x+2-x-2}{(x+2)(x+1)} = \frac{4x+12-3x-12}{(x+4)(x+3)}$$

$$\frac{x}{(x+2)(x+1)} = \frac{x}{(x+4)(x+3)}$$

$$\frac{1}{(x+2)(x+1)} = \frac{1}{(x+4)(x+3)} \quad \text{[Dividing by } x \text{ if } x \neq 0]$$

$$\frac{1}{x^2+3x+2} = \frac{1}{x^2+7x+12}$$

So, we have
$x^2 + 7x + 12 - x^2 - 3x - 2 = 0$
$4x + 10 = 0$
$2x + 5 = 0$
$x = -5/2$
But if $x = 0$, then

$$\frac{0}{(x+2)(x+1)} = \frac{0}{(x+4)(x+3)}$$

Which is actually true
Therefore, x = 0, -5/2.

7. (i) (3x - 4)/7 + 7/(3x - 4) = 5/2, x ≠ 4/3
(ii) 4/x – 3 = 5/(2x + 3), x ≠ 0, -3/2
Solution:

(i) (3x - 4)/7 + 7/(3x - 4) = 5/2
Taking L.C.M, we get
$[(3x - 4)^2 + 7^2]/ [7(3x - 4)] = 5/2$
$2[(3x - 4)^2 + 7^2] = 5 \times [7(3x - 4)]$
$2(9x^2 + 16 – 24x + 49) = 35 (3x - 4)$
$2(9x^2 – 24x + 65) = 35 (3x - 4)$
$18x^2 – 48x + 130 = 105x – 140$
$18x^2 – 48x – 105x + 130 + 140 = 0$
$18x^2 – 153x + 270 = 0$
$2x^2 – 17x + 30 = 0$ [Dividing by 9]
Let us consider,
a = 2, b = -17, c = 30
So, by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 – 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 – 4ac$
$= (-17)^2 – 4(2)(30)$
$= 289 – 240$
$= 49$
So,
x = [-(-17) ± √49] / 2(2)
= [17 ± 7] / 4
= [17 + 7]/ 4 or [17 - 7]/4
= 24/4 or 10/4
= 6 or 5/2
∴ Value of x = 6, 5/2

(ii) $4/x - 3 = 5/(2x + 3)$, $x \neq 0, -3/2$
Let's simplify the given equation,
$(4 - 3x)/x = 5/(2x + 3)$ [Taking L.C.M]
$(4 - 3x)(2x + 3) = 5x$
$8x + 12 - 6x^2 - 9x = 5x$
$6x^2 + 5x + x - 12 = 0$
$6x^2 + 6x - 12 = 0$
$x^2 + x - 2 = 0$ [Dividing by 6]
Let us consider,
$a = 1, b = 1, c = -2$
So, by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (1)^2 - 4(1)(-2)$
$= 1 + 8$
$= 9$
So,
$x = [-(1) \pm \sqrt{9}] / 2(1)$
$= [-1 \pm 3] / 2$
$= [-1 + 3]/ 2$ or $[-1 - 3]/ 2$
$= 2/2$ or $-4/2$
$= 1$ or -2
∴ Value of $x = 1, -2$

8. (i) $x^2 + (4 - 3a)x - 12a = 0$
(ii) $10ax^2 - 6x + 15ax - 9 = 0$, $a \neq 0$
Solution:

(i) $x^2 + (4 - 3a)x - 12a = 0$
Let us consider,
$a = 1, b = (4 - 3a), c = -12a$
So, by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (4 - 3a)^2 - 4(1)(-12a)$
$= 16 + 9a^2 - 24a + 48a$
$= 16 + 9a^2 + 24a$
$= (4 + 3a)^2$

So,

$x = [-(4 - 3a) \pm \sqrt{(4 + 3a)^2}] / 2(1)$
$= [-4 + 3a \pm (4 + 3a)] / 2$
$= [-4 + 3a + (4 + 3a)] / 2$ or $[-4 + 3a - (4 + 3a)] / 2$
$= 6a / 2$ or $-8 / 2$
$= 3a$ or -4

∴ Value of $x = 3a, -4$

(ii) $10ax^2 - 6x + 15ax - 9 = 0, a \neq 0$

$10ax^2 - (6 - 15a)x - 9 = 0$

Let us consider,

$a = 10, b = -(6 - 15a), c = -9$

So, by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac$
$= (6 - 15a)^2 - 4(10a)(-9)$
$= 36 + 225a^2 - 180a + 360a$
$= 36 + 225a^2 + 180a$
$= (6 + 15a)^2$

So,

$x = [-(-(6 - 15a)) \pm \sqrt{(6 + 15a)^2}] / 2(10a)$
$= [6 - 15a \pm (6 + 15a)] / 20a$
$= [6 - 15a + (6 + 15a)] / 20a$ or $[6 - 15a - (6 + 15a)] / 20a$
$= 12/20a$ or $-30/20a$
$= 3/5a$ or $-3/2$

∴ Value of x = 3/5a, -3/2

9. Solve for x using the quadratic formula. Write your answer correct to two significant figures: $(x - 1)^2 – 3x + 4 = 0$.
Solution:

Given quadratic equation,
$(x - 1)^2 – 3x + 4 = 0$
$x^2 - 2x – 3x + 1 + 4 = 0$
$x^2 – 5x + 5 = 0$
Let us consider,
a = 1, b = -5, c = 5
So, by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 – 4ac = D$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 – 4ac$
$= (-5)^2 – 4(1)(5)$
$= 25 - 20$
$= 5$
So,
$x = [-(-5) \pm \sqrt{5}] / 2(1)$
$= [5 \pm \sqrt{5}] / 2$
$= [5 + \sqrt{5}]/ 2$ or $[5 - \sqrt{5}]/ 2$
$= (5 + 2.236) /2$ or $(5 - 2.236)/2$
$= 7.236 /2$ or $2.764 /2$
$= 3.618$ or 1.382
∴ Value of x = 3.618, 1.382

10. Discuss the nature of roots of the following equations:
(i) $3x^2 – 7x + 8 = 0$ (ii) $x^2 – ½ x – 4 = 0$
(iii) $5x^2 – 6\sqrt{5}x + 9 = 0$ (iv) $\sqrt{3}x^2 – 2x – \sqrt{3} = 0$
In case the real roots exist, then find them.
Solution:

(i) $3x^2 – 7x + 8 = 0$
Let us consider,

$a = 3$, $b = -7$, $c = 8$
By using the formula,
$D = b^2 – 4ac$
$= (-7)^2 – 4(3)(8)$
$= 49 - 96$
$= -47$
So,
Discriminate, $D = -47$
$D < 0$
∴ Roots are not real.

(ii) $x^2 – ½ x – 4 = 0$
Let us consider,
$a = 1$, $b = -1/2$, $c = -4$
By using the formula,
$D = b^2 – 4ac$
$= (-1/2)^2 – 4(1)(-4)$
$= 1/4 + 16$
$= 65/16$
So,
Discriminate, $D = 65/16$
$D > 0$
∴ Roots are real and distinct.
So,
$x = [-(-1/2) \pm \sqrt{(65/16)}] / 2(1)$
$= [1/2 \pm \sqrt{65}/4] / 2$
$= [1/2 + \sqrt{65}/4]/ 2$ or $[1/2 - \sqrt{65}/4]/ 2$
$= (2 + \sqrt{65})/4 /2$ or $(2 - \sqrt{65})/4 /2$
$= (2 + \sqrt{65})/ 8$ or $(2 - \sqrt{65})/ 8$
∴ Value of $x = (2 + \sqrt{65})/ 8, (2 - \sqrt{65})/ 8$

(iii) $5x^2 – 6\sqrt{5}x + 9 = 0$
Let us consider,
$a = 5$, $b = -6\sqrt{5}$, $c = 9$
By using the formula,
$D = b^2 – 4ac$
$= (-6\sqrt{5})^2 – 4(5)(9)$
$= 180 - 180$
$= 0$

So,
Discriminate, D = 0
D = 0
∴ Roots are equal and real.
So,
$x = [-(-6\sqrt{5}) \pm \sqrt{0}] / 2(5)$
$= 6\sqrt{5}/ 10$
$= 3\sqrt{5}/ 5$
∴ Value of $x = 3\sqrt{5}/ 5$

(iv) $\sqrt{3}x^2 – 2x – \sqrt{3}$
Let us consider,
$a = \sqrt{3}, b = -2, c = -\sqrt{3}$
By using the formula,
$D = b^2 – 4ac$
$= (-2)^2 – 4(\sqrt{3}) (-\sqrt{3})$
$= 4 + 4(3)$
$= 4 + 12$
$= 16$
So,
Discriminate, D = 16
D > 0
∴ Roots are real and distinct.
So,
$x = [-(-2) \pm \sqrt{16}] / 2(\sqrt{3})$
$= [2 \pm 4] / 2\sqrt{3}$
$= [2 + 4]/ 2\sqrt{3}$ or $[2 - 4]/ 2\sqrt{3}$
$= 6/(2\sqrt{3})$ or $-2/(2\sqrt{3})$
$= 3/\sqrt{3}$ or $-1/\sqrt{3}$
$= \sqrt{3}$ or $-1/\sqrt{3}$
∴ Value of $x = \sqrt{3}, -1/\sqrt{3}$

11. Find the values of k so that the quadratic equation $(4 - k)x^2 + 2(k + 2)x + (8k + 1) = 0$ has equal roots.
Solution:

Given quadratic equation,
$(4 - k)x^2 + 2(k + 2)x + (8k + 1) = 0$
Let us consider,

$a = (4 - k)$, $b = 2(k + 2)$, $c = (8k + 1)$
By using the formula,
$D = b^2 – 4ac$
$= [2(k + 2)]^2 – 4(4 - k)(8k + 1)$
$= 4(k^2 + 4k + 4) – 4(32k – 8k^2 + 4 - k)$
$= 4k^2 + 16k + 16 – 128k + 32k^2 – 16 + 4k$
$= 36k^2 - 108k$
$= 36k(k - 3)$
So,
Discriminate, $D = 36k(k - 3)$
As the roots are equal
Hence, $D = 0$
$36k(k - 3) = 0$
So,
$36k = 0$ or $k – 3 = 0$
$k = 0$ or $k = 3$
Therefore, the value of $x = 0, 3$.

12. Find the values of m so that the quadratic equation $3x^2 – 5x – 2m = 0$ has two distinct real roots.
Solution:

Given quadratic equation,
$3x^2 - 5x – 2m = 0$
Let us consider,
$a = 3$, $b = -5$, $c = -2m$
By using the formula,
$D = b^2 – 4ac$
$= (-5)^2 – 4(3)(-2m)$
$= 25 + 24m$
So,
Discriminate, $D = 25 + 24m$
As the roots are real and distinct
Hence, $D > 0$
$25 + 24m > 0$
$24m > -25$
So,
$m > -25/24$
Therefore, the value of m must be greater than $-25/24$.

13. Find the value(s) of k for which each of the following quadratic equation has equal roots:
(i) $3kx^2 = 4(kx - 1)$ (ii) $(k + 4)x^2 + (k + 1)x + 1 = 0$
Also, find the roots for that value(s) of k in each case.
Solution:

For a quadratic equation to have equal roots, discriminant (D) = 0

(i) $3kx^2 = 4(kx - 1)$
Let's rearrange the given equation,
$3kx^2 = 4kx – 4$
$3kx^2 - 4kx + 4 = 0$
Let us consider,
a = 3k, b = -4k, c = 4
By using the formula,
$D = b^2 – 4ac$
$= (-4k)^2 – 4(3k) (4)$
$= 16k^2 – 48k$
Now,
$16k^2 – 48 = 0$
$16k(k - 3) = 0$
So,
k = 0 or k – 3 = 0
Thus, k = 0 or 3
As D = 0, by the quadratic formula we have
x = -b/2a
= 4k/(2 × 3k)
= 4k/6k
= 2/3
Hence, the roots are 2/3, 2/3.

(ii) $(k + 4)x^2 + (k + 1)x + 1 = 0$
Let us consider,
a = k + 4, b = k + 1, c = 1
By using the formula,
$D = b^2 – 4ac$
$= (k + 1)^2 – 4(k + 4) (1)$
$= k^2 + 1 + 2k – 4k – 16$

$= k^2 - 2k - 15$
Now,
$k^2 - 2k - 15 = 0$
$k^2 - 5k + 3k - 15 = 0$
$k(k - 5) + 3(k - 5) = 0$
$(k + 3)(k - 5) = 0$
So,
$k + 3 = 0$ or $k - 5 = 0$
$k = -3$ or $k = 5$
Thus, $k = -3, 5$
As $D = 0$, by the quadratic formula we have
$x = -b/2a$
$= -(k + 1)/[2 \times (k + 4)]$
$= (-k - 1)/(2k + 8)$
Now, when $k = 5$, we get
$x = (-5 - 1)/(2 \times 5 + 8)$
$= -6/18$
$= -1/3$
Hence, the roots are $-1/3, -1/3$
And, when $k = -3$, we get
$x = (-(-3) - 1)/(2 \times (-3) + 8)$
$= (3 - 1)/(-6 + 8)$
$= 2/2 = 1$
Hence, the roots are 1, 1.

14. Find two natural numbers which differ by 3 and whose squares have the sum 117.
Solution:

Let the first natural number be x
Then, second the natural number will be x + 3
According to the condition given in the problem,
$x^2 + (x + 3)^2 = 117$
$x^2 + x^2 + 9 + 6x = 117$
$2x^2 + 9 + 6x - 117 = 0$
$2x^2 + 6x - 108 = 0$
$x^2 + 3x - 54 = 0$ [Dividing by 2]
$x(x + 9) - 6(x + 9) = 0$
$(x - 6)(x + 9) = 0$

So,
$x - 6 = 0$ or $x + 9 = 0$
$x = 6$ or $x = -9$
As x should be a natural number,
$x = 6$
Hence, first number = 6 and second number = 6 + 3 = 9.

15. Divide 16 into two parts such that the twice the square of the larger part exceeds the square of the smaller part by 164.
Solution:

Let the larger part be considered as x
Then, the smaller part will be (16 - x)
According to the conditions given in the problem, we have
$2x^2 - (16 - x)^2 = 164$
$2x^2 - (256 - 32x + x^2) = 164$
$2x^2 - 256 + 32x - x^2 - 164 = 0$
$x^2 + 32x - 420 = 0$
Now, by factorization method
$x^2 + 42x - 10x - 420 = 0$
$x(x + 42) - 10(x + 42) = 0$
$(x - 10)(x + 42) = 0$
So,
$x - 10 = 0$ or $x + 42 = 0$
$x = 10$ or $x = -42$, which is not possible as its negative
Thus, $x = 10$
Therefore, the larger part = 10 and the smaller part = 16 – 10 = 6.

16. Two natural numbers are in the ratio 3 : 4. Find the numbers if the difference between their squares is 175.
Solution:

Given, ratio of two natural numbers is 3 : 4
Let the numbers be taken as 3x and 4x
Then, according to the conditions in the problem, we have
$(4x)^2 - (3x)^2 = 175$
$16x^2 - 9x^2 = 175$
$7x^2 = 175$
$x^2 = 175/7 = 25$

So,
$x = \sqrt{25} = \pm 5$
But, the value of x cannot be -5 as its not a natural number
Thus, $x = 5$
Therefore,
The natural numbers are 3(5) and 4(5) i.e. 15 and 20.

17. Two squares have sides x cm and (x + 4) cm. The sum of their areas is 656 sq. cm. Express this as an algebraic equation and solve it to find the sides of the squares.
Solution:

We have,
Side of first square = x cm
And the side of second square = (x + 4) cm
Now according to the given conditions in the problem, we have
$x^2 + (x + 4)^2 = 656$
$x^2 + x^2 + 16 + 8x = 656$
$2x^2 + 8x + 16 – 656 = 0$
$2x^2 + 8x – 640 = 0$
$x^2 + 4x – 320 = 0$ [Dividing by 2]
By factorization method, we have
$x^2 + 20x – 16x – 320 = 0$
$x(x + 20) – 16(x + 20) = 0$
$(x + 20) (x - 16) = 0$
So,
$x + 20 = 0$ or $x – 16 = 0$
$x = -20$ or $x = 16$
Since, side of a square cannot be negative.
Thus, $x = 16$
Therefore,
Side of first square = 16 cm
And, the side of the second square = (16 + 4) = 20 cm

18. The length of a rectangular garden is 12 m more than its breadth. The numerical value of its area is equal to 4 times the numerical value of its perimeter. Find the dimensions of the garden.
Solution:

Let's assume the breadth of the rectangular garden as x m
Then, length = (x + 12) m
So,
Area = l × b m^2
= x × (x + 12) m^2
And, perimeter = 2(l + b)
= 2 (x + x + 12)
= 2 (2x + 12) m
Now according to the given condition in the problem, we have
$x(x + 12) = 4 \times 2(2x + 12)$
$x^2 + 12x = 16x + 96$
$x^2 - 4x - 96 = 0$
$x^2 - 12x + 8x - 96 = 0$
$x(x - 12) + 8(x - 12) = 0$
$(x + 8)(x - 12) = 0$
So,
x + 18 = 0 or x – 12 = 0
x = -18 or x = 12
But x = -18 is not possible as it negative
Thus, x = 12
Therefore,
Breadth = 12 m and length = 12 + 12 = 24 m.

19. A farmer wishes to grow a 100 m² rectangular vegetable garden. Since he has with him only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side fence. Find the dimensions of his garden.
Solution:

Given,
Area of rectangular garden = 100 cm^2
Length of barbed wire = 30 m
Let's assume the length of the side opposite to wall to be x
And the length of other side = (30 - x)/ 2
So, the area = (30 - x)/ 2 × x
= $(30x - x^2)/ 2$
⇒ $(30x - x^2)/ 2 = 100$
$30x - x^2 = 200$
$x^2 - 30x + 200 = 0$

By factorization method, we have
$x^2 - 20x - 10x + 200 = 0$
$x(x - 20) - 10(x - 20) = 0$
$(x - 20)(x - 10) = 0$
So,
$x - 20$ or $x - 10 = 0$
$x = 20$ or $x = 10$
Hence,
(i) If $x = 20$, then side opposite to the wall = 20 m
And other side will be = $(30 - 20)/2 = 10/2 = 5$ m
(ii) If $x = 10$, then side opposite to the wall = 10 m
And other side will be = $(30 - 10)/2 = 20/2 = 10$ m
Therefore,
Sides of the rectangular can be 20 m, 5 m or 10 m, 10 m.

20. The hypotenuse of a right-angled triangle is 1 m less than twice the shortest side. If the third side is 1 m more than the shortest side, find the sides of the triangle.
Solution:

Let's consider the length of the shortest side = x m
Length of hypotenuse = $2x - 1$
And third side = $x + 1$
Now according to the given condition in the problem, we have
$x^2 + (x + 1)^2 = (2x - 1)^2$ [By Pythagoras theorem]
$x^2 + x^2 + 2x + 1 = 4x^2 + 1 - 4x$
$4x^2 - 2x^2 - 4x - 2x - 1 + 1 = 0$
$2x^2 - 6x = 0$
$x^2 - 3x = 0$ [Dividing by 2]
$x(x - 3) = 0$
So,
$x = 0$ or $x - 3 = 0$
$x = 0$ or $x = 3$
But, $x = 0$ is not possible
Hence, $x = 3$
So,
Shortest side = 3 m
Hypotenuse = $2 \times 3 - 1 = 6 - 1 = 5$ m
And, third side = $x + 1 = 3 + 1 = 4$ m
Therefore, the sides of the triangle are 3m, 5m and 4m.