

EXERCISE 28.3

Q1.

Solution:

Let us consider the equation of line,

$$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} = \lambda \quad \dots\dots (1)$$

So the general point on line (1) is

$$\{\lambda, 2\lambda + 2, 3\lambda - 3\}$$

Now, let us consider another equation of line,

$$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} = \mu \quad \dots\dots (2)$$

So the general point on line (2) is

$$\{2\mu + 2, 3\mu + 6, 4\mu + 3\}$$

If lines (1) and (2) intersect, we get a common point

So for same value of λ and μ , must have,

$$\lambda = 2\mu + 2 \Rightarrow \lambda - 2\mu = 2 \quad \dots\dots (3)$$

$$2\lambda + 2 = 3\mu + 6 \Rightarrow 2\lambda - 4\mu = 4 \quad \dots\dots (4)$$

$$3\lambda - 3 = 4\mu + 3 \Rightarrow 3\lambda - 4\mu = 6 \quad \dots\dots (5)$$

Now, let us solve (3) and (4), we get

$$\begin{array}{r} 2\lambda - 4\mu = 4 \\ 2\lambda - 3\mu = 4 \\ \hline (-) \quad (+) \quad (-) \\ -\mu = 0 \end{array}$$

$$\mu = 0$$

Now, let us substitute $\mu = 0$ in equation (3), we get

$$\begin{aligned} \lambda - 2\mu &= 2 \\ \lambda - 2(0) &= 2 \\ \lambda &= 2 \end{aligned}$$

By substituting the values λ and μ in equation (5), we get

$$\begin{aligned} 3\lambda - 4\mu &= 6 \\ 3(2) - 4(0) &= 6 \\ 6 &= 6 \end{aligned}$$

Hence,

$$\text{LHS} = \text{RHS}$$

Q2.

Solution:

Let us consider the equation of line,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \quad \dots\dots (1)$$

So the general point on line (1) is

$$(3\lambda + 1, 2\lambda - 1, 5\lambda + 1)$$

Now, let us consider another equation of line,

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \quad \dots\dots (2)$$

So the general point on line (2) is

$$(4\mu - 2, 3\mu + 1, -2\mu - 1)$$

If lines (1) and (2) intersect, we get a common point

So for same value of λ and μ , must have,

$$3\lambda + 1 = 4\mu - 2 \Rightarrow 3\lambda - 4\mu = -3 \quad \dots\dots (3)$$

$$2\lambda - 1 = 3\mu + 1 \Rightarrow 2\lambda - 3\mu = 2 \quad \dots\dots (4)$$

$$5\lambda + 1 = -2\mu - 1 \Rightarrow 5\lambda + 2\mu = -2 \quad \dots\dots (5)$$

Now, let us solve (3) and (4), we get

$$6\lambda - 8\mu = -6$$

$$\begin{array}{r} 6\lambda - 9\mu = 6 \\ (-) \quad (+) \quad (-) \end{array}$$

$$\mu = -12$$

$$\mu = -12$$

Now, let us substitute $\mu = -12$ in equation (3), we get

$$3\lambda - 4(-12) = -3$$

$$3\lambda + 48 = -3$$

$$3\lambda = -3 - 48$$

$$3\lambda = -51$$

$$\lambda = \frac{-51}{3}$$

$$\lambda = -17$$

By substituting the values λ and μ in equation (5), we get

$$5\lambda + 2\mu = -2$$

$$5(-17) + 2(-12) = -2$$

$$-85 - 24 = -2$$

$$-109 \neq -2$$

Hence,
LHS \neq RHS

Q3.

Solution:

Let us consider the equation of line,

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \quad \dots\dots (1)$$

So the general point on line (1) is

$$(3\lambda - 1, 5\lambda - 3, 7\lambda - 5)$$

Now, let us consider another equation of line,

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \quad \dots\dots (2)$$

So the general point on line (2) is

$$(\mu + 2, 3\mu + 4, 5\mu + 6)$$

If lines (1) and (2) intersect, we get a common point

So for same value of λ and μ , must have,

$$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda - \mu = 3 \quad \dots\dots (3)$$

$$5\lambda - 3 = 3\mu + 4 \Rightarrow 5\lambda - 3\mu = 7 \quad \dots\dots (4)$$

$$7\lambda - 5 = 5\mu + 6 \Rightarrow 7\lambda - 5\mu = 11 \quad \dots\dots (5)$$

Now, let us solve (3) and (4), we get

$$\begin{array}{r} 15\lambda - 5\mu = 15 \\ 15\lambda - 9\mu = 21 \\ \hline (-) \quad (+) \quad (-) \\ 4\mu = -6 \end{array}$$

$$\mu = -6/4$$

Now, let us substitute $\mu = -6/4$ in equation (3), we get

$$3\lambda - \mu = 3$$

$$3\lambda - \left(-\frac{3}{2}\right) = 3$$

$$3\lambda = 3 - \frac{3}{2}$$

$$\lambda = \frac{1}{2}$$

By substituting the values λ and μ in equation (5), we get

$$7\lambda - 5\mu = 11$$

$$7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

$$\frac{7}{2} + \frac{15}{2} = 11$$

$$\frac{22}{2} = 11$$

$$11 = 11$$

LHS = RHS

Since, the value of λ and μ obtained by solving equations (3) and (4) satisfies equation (5).

Hence, the given lines intersect each other.

So,

$$\text{The point of intersection} = (3\lambda - 1, 5\lambda - 3, 7\lambda - 5)$$

$$= \left\{ \frac{3}{2} - 1, \left(\frac{5}{2} - 3 \right), \left(\frac{7}{2} - 5 \right) \right\}$$

$$= \left(\frac{1}{2}, \frac{-1}{2}, \frac{-3}{2} \right)$$

$$\left(\frac{1}{2}, \frac{-1}{2}, \frac{-3}{2} \right)$$

\therefore The point of intersection is $\left(\frac{1}{2}, \frac{-1}{2}, \frac{-3}{2} \right)$.

Q4.

Solution:

Given points A(0, -1, -1) and B(4, 5, 1)

The equation of line passing through the points is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 0}{4 - 0} = \frac{y + 1}{5 + 1} = \frac{z + 1}{1 + 1}$$

$$\frac{x}{4} = \frac{y + 1}{6} = \frac{z + 1}{2}$$

$$\text{Let } \lambda = \frac{x}{4} = \frac{y + 1}{6} = \frac{z + 1}{2}$$

So, general point on line AB is

$$(4\lambda, 4\lambda, 2\lambda - 1)$$

Given points C(3, 9, 4) and D(-4, 4, 4)

The equation of line passing through the points is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 3}{-4 - 3} = \frac{y - 9}{4 - 9} = \frac{z - 4}{4 - 4}$$

$$\frac{x - 3}{-7} = \frac{y - 9}{-5} = \frac{z - 4}{0}$$

Let
$$\mu = \frac{x - 3}{-7} = \frac{y - 9}{-5} = \frac{z - 4}{0}$$

So, general point on line CD is

$$(-7\mu + 3, -5\mu + 9, 0\mu + 4)$$

$$(-7\mu + 3, -5\mu + 9, 4)$$

If lines AB and CD intersect, there exists a common point.

So let us find the value of λ and μ .

$$4\lambda = -7\mu + 3 \Rightarrow 4\lambda + 7\mu = 3 \dots\dots (1)$$

$$6\lambda - 1 = -5\mu + 9 \Rightarrow 6\lambda + 5\mu = 10 \dots\dots (2)$$

$$2\lambda - 1 = 4 \Rightarrow 2\lambda - 1 = 4 \dots\dots (3)$$

So from equation (3),

$$2\lambda = 4 + 1$$

$$2\lambda = 5$$

$$\lambda = 5/2$$

By substituting the value of $\lambda = 5/2$ in equation (2), we get

$$6(5/2) + 5\mu = 10$$

$$5\mu = 10 - 15$$

$$= -5$$

$$\mu = -1$$

Now, by substituting the values of λ and μ in equation (1), we get

$$4\lambda + 7\mu = 3$$

$$4(5/2) + 7(-1) = 3$$

$$10 - 7 = 3$$

$$3 = 3$$

$$\text{LHS} = \text{RHS}$$

Since, the value of λ and μ obtained by solving equations (3) and (4) satisfies equation (1).

Hence, the given lines, AB and CD intersect each other.

So,

$$\begin{aligned} \text{The point of intersection of AB and CD} &= (-7\mu+3, -5\mu+9, 4) \\ &= (-7(-)+3, -5(-1)+9, 4) \\ &= (7+3, 5+9, 4) \\ &= (10, 14, 4) \end{aligned}$$

Hence, the point of intersection of AB and CD is (10, 14, 4).

Q5.

Solution:

Given:

The equations of lines are

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$$

$$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

If these lines intersect, there exists a common point.

So for some value of λ and μ , we must have

$$(\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}) = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

$$(1 + 3\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k} = (4 + 2\mu)\hat{i} + (-1 + 3\mu)\hat{k}$$

So, the equation of coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get

$$1 + 3\lambda = 4 + 2\mu \Rightarrow 3\lambda - 2\mu = 3 \dots\dots\dots (1)$$

$$1 - \lambda = 0 \Rightarrow \lambda = 1 \dots\dots\dots (2)$$

$$-1 = -1 + 3\mu \Rightarrow \mu = 0 \dots\dots\dots (3)$$

By substituting the values of λ and μ in equation (1)

$$3\lambda - 2\mu = 3$$

$$3(1) - 2(0) = 3$$

$$3 = 3$$

LHS = RHS

Since, the value of λ and μ satisfies equation (1).

Hence, the given lines intersect each other.

So,

The point of intersection by substituting the value of λ in equation (1), we get

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + (1)(3\hat{i} - \hat{j})$$

$$= \hat{i} + \hat{j} - \hat{k} + 3\hat{i} - \hat{j}$$

$$= 4\hat{i} - \hat{k}$$

Hence, the point of intersection is (4, 0, -1).