EXERCISE 23C

Solve the following equations for A, if:
(i) 2 sin A = 1
(ii) 2 cos 2 A = 1
(iii) sin 3 A = √3/2
(iv) sec 2 A = 2
(v) √3 tan A = 1
(vi) tan 3 A = 1
(vii) 2 sin 3 A = 1
(viii) √3 cot 2 A = 1

Solution:
(i) Given 2 sin A = 1
Sin A = ½
Sin A = sin 30°
Therefore, A = 30°

(ii) Given 2 cos 2A = 1
Cos 2A = ½
Cos 2A = cos 60°
2A = 60°
A = 30°

(iii) Given sin 3A = √3/2
Sin 3A = sin 60°
3A = 60°
A = 20°

(iv) Given sec 2A = 2
Sec 2A = sec 60°
2A = 60°
A = 30°

(v) Given √3 tan A = 1
Tan A = 1/√3
Tan A = tan 30°
A = 30°

(vi) Given tan 3A = 1
Tan 3A = tan 45°

3A = 45°
A = 15°

(vii) Given 2 sin 3A = 1
Sin 3A = ½
Sin 3A = sin 30°
3A = 30°
A = 10°

(viii) Given √3 cot 3A = 1
Cot 2A = 1/√3
Cot 2A = cot 60°
2A = 60°
A = 30°

2. Calculate the value of A, if:
(i) (sin A - 1) (2 cos A - 1) = 0
(ii) (tan A - 1) (cosec 3A - 1) = 0
(iii) (sec 2A - 1) (cosec 3A - 1) = 0
(iv) cos 3A. (2 sin 2A - 1) = 0
(v) (cosec 2A - 2) (cot 3A - 1) = 0

Solution:
(i) Given (sin A - 1) (2 cos A - 1) = 0
It can be written as
(sin A - 1) = 0 and (2 cos A - 1) = 0
Sin A = 1 and cos A = ½
Sin A = sin 90° and cos A = cos 60°
A = 90° and A = 60°

(ii) Given (tan A - 1) (cosec 3A - 1) = 0
It can be written as
(tan A - 1) = 0 and (cosec 3A - 1) = 0
tan A = 1 and cosec 3A = 1
tan A = tan 45° and cosec 3A = cosec 90°
A = 45° and A = 30°

(iii) Given (sec 2A - 1) (cosec 3A - 1) = 0
It can be written as
(sec 2A - 1) = 0 and (cosec 3A - 1) = 0
sec 2A = 1 and cosec 3A = 1
sec 2A = sec 60° and cosec 3A = cosec 90°
A = 0° and A = 30°

(iv) Given cos 3A (2 sin 2A – 1) = 0
It can be written as
Cos 3A = 0 and 2 sin 2A – 1 = 0
Cos 3A = cos 90^o and sin 2A = ½
3A = 90^o and sin 2A = sin 30^o
A = 30^o and 2A = 30^o which implies A = 15^o

(v) Given (cosec 2A - 2) (cot 3A - 1) = 0
It can be written as
(cosec 2A - 2) = 0 and (cot 3A - 1) = 0
cosec 2A = 2 and cot 3A = 1
cosec 2A = cosec 30^o and cot 3A = cot 45^o
2A = 30^o and 3A = 45^o
A = 15^o and A = 15^o

3. If 2 sin x^o - 1 = 0 and x^o is an acute angle; find:
(i) sin x^o
(ii) x^o
(iii) cos x^o and tan x^o.

Solution:
(i) Given 2 sin x^o - 1 = 0
2 sin x^o = 1
sin x^o = ½

(ii) we have sin x^o = ½
sin x^o = sin 30^o
x^o = 30^o

(iii) we have x^o = 30^o
Cos x^o = cos 30^o = √3/2
Tan x^o = tan 30^o = 1/√3

4. If 4 $\cos^2 x^o$ - 1 = 0 and 0 ≤ x^o ≤ 90^o, find:
(i) x^o
(ii) $\sin^2 x^o + \cos^2 x^o$
(iii) $\frac{1}{\cos^2 x^o} - \tan^2 x^o$

Solution:
(i) Given 4 $\cos^2 x^o$ - 1 = 0
4 $\cos^2 x^o$ = 0

$\cos^2 x^o = (½)^2$
$\cos x^o = ½$
$\cos x^o = \cos 60^o$
$x^o = 60^o$

(ii) $\sin^2 x^o + \cos^2 x^0 = \sin^2 60^o + \cos^2 60^o$
$= (\sqrt{3}/2)^2 + (½)^2$
$= ¾ + ¼$
$= 1$

(iii) Given
$$\frac{1}{\cos^2 x^o} - \tan^2 x^o = \frac{1}{\cos^2 60^o} - \tan^2 60^o$$
$= 1/ (½)^2 – (\sqrt{3})^2$
$= 4 – 3$
$= 1$

5. If $4 \sin^2 θ - 1= 0$ and angle θ is less than 90^o, find the value of θ and hence the value of $\cos^2 θ + \tan^2 θ$.

Solution:
Given $4 \sin^2 θ - 1= 0$
$\sin^2 θ = ¼$
$\sin θ = ½$
$\sin θ = \sin 30^o$
$θ = 30^o$
$\cos^2 θ + \tan^2 θ = \cos^2 30^o + \tan^2 30^o$
$= (\sqrt{3}/2)^2 + (1/\sqrt{3})^2$
$= ¾ + 1/3$
$= (9 + 4)/12$
$= 13/12$

6. If $\sin 3A = 1$ and $0 ≤ A ≤ 90^o$, find:
(i) sin A
(ii) cos 2A
(iii) $\tan^2 A – 1/\cos^2 A$

Solution:
Given $\sin 3A = 1$
$\text{Sin } 3A = \sin 90^o$
$3A = 90^o$
$A = 30^o$

(i) according to the question we have,
Sin A = sin 30°
Sin A = ½

(ii) cos 2A = cos 2(30°)
= cos 60°
= ½

(iii) According to the question we have,

$$\tan^2 A - \frac{1}{\cos^2 A} = \tan^2 30^0 - \frac{1}{\cos^2 30^0}$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 - \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$$

= 1/3 – 4/3
= -3/3
= -1