

Chapter 8: Remainder and Factor Theorems

Exercise 8(C)

Solution:

$$\text{Let } f(x) = x^3 - 7x^2 + 14x - 8$$

Then, for $x = 1$

$$f(1) = (1)^3 - 7(1)^2 + 14(1) - 8 = 1 - 7 + 14 - 8 = 0$$

Thus, $(x - 1)$ is a factor of $f(x)$.

Now, performing long division we have

$$\begin{array}{r} x^2 - 6x + 8 \\ x - 1 \overline{) x^3 - 7x^2 + 14x - 8} \\ \underline{x^3 - x^2} \\ -6x^2 + 14x - 8 \\ \underline{-6x^2 + 6x} \\ 8x - 8 \\ \underline{8x - 8} \\ 0 \end{array}$$

$$\begin{aligned} \text{Hence, } f(x) &= (x - 1)(x^2 - 6x + 8) \\ &= (x - 1)(x^2 - 4x - 2x + 8) \\ &= (x - 1)[x(x - 4) - 2(x - 4)] \\ &= (x - 1)(x - 4)(x - 2) \end{aligned}$$

Solution:

$$\text{Let } f(x) = x^3 + 10x^2 - 37x + 26$$

From remainder theorem, we know that

For $x = 1$, the value of $f(x)$ is the remainder

$$f(1) = (1)^3 + 10(1)^2 - 37(1) + 26 = 1 + 10 - 37 + 26 = 0$$

As $f(1) = 0$, $x - 1$ is a factor of $f(x)$.

Now, performing long division we have

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$$\begin{array}{r} x^2 + 11x - 26 \\ x - 1 \overline{) x^3 + 10x^2 - 37x + 26} \\ \underline{x^3 - x^2} \\ 11x^2 - 37x + 26 \\ \underline{11x^2 - 11x} \\ -26x + 26 \\ \underline{-26x + 26} \\ 0 \end{array}$$

$$\begin{aligned} \text{Thus, } f(x) &= (x - 1)(x^2 + 11x - 26) \\ &= (x - 1)(x^2 + 13x - 2x - 26) \\ &= (x - 1)[x(x + 13) - 2(x + 13)] \\ &= (x - 1)(x + 13)(x - 2) \end{aligned}$$

1. When $x^3 + 3x^2 - mx + 4$ is divided by $x - 2$, the remainder is $m + 3$. Find the value of m .

Solution:

$$\text{Let } f(x) = x^3 + 3x^2 - mx + 4$$

From the question, we have

$$f(2) = m + 3$$

$$(2)^3 + 3(2)^2 - m(2) + 4 = m + 3$$

$$8 + 12 - 2m + 4 = m + 3$$

$$24 - 3 = m + 2m$$

$$3m = 21$$

$$\text{Thus, } m = 7$$

2. What should be subtracted from $3x^3 - 8x^2 + 4x - 3$, so that the resulting expression has $x + 2$ as a factor?

Solution:

Let's assume the required number to be k .

$$\text{And let } f(x) = 3x^3 - 8x^2 + 4x - 3 - k$$

From the question, we have

$$f(-2) = 0$$

$$3(-2)^3 - 8(-2)^2 + 4(-2) - 3 - k = 0$$

$$-24 - 32 - 8 - 3 - k = 0$$

$$-67 - k = 0$$

$$k = -67$$

Therefore, the required number is -67 .

3. If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, find the values of a and b . And then, factorise the given expression completely.

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Solution:

Let's take $f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$

As, $(x + 1)$ is a factor of $f(x)$.

Then, remainder = $f(-1) = 0$

$$(-1)^3 + (a + 1)(-1)^2 - (b - 2)(-1) - 6 = 0$$

$$-1 + (a + 1) + (b - 2) - 6 = 0$$

$$a + b - 8 = 0 \dots (1)$$

And as, $(x - 2)$ is a factor of $f(x)$.

Then, remainder = $f(2) = 0$

$$(2)^3 + (a + 1)(2)^2 - (b - 2)(2) - 6 = 0$$

$$8 + 4a + 4 - 2b + 4 - 6 = 0$$

$$4a - 2b + 10 = 0$$

$$2a - b + 5 = 0 \dots (2)$$

Adding (1) and (2), we get

$$3a - 3 = 0$$

Thus, $a = 1$

Substituting the value of a in (i), we get,

$$1 + b - 8 = 0$$

Thus, $b = 7$

Hence, $f(x) = x^3 + 2x^2 - 5x - 6$

Now as $(x + 1)$ and $(x - 2)$ are factors of $f(x)$.

So, $(x + 1)(x - 2) = x^2 - x - 2$ is also a factor of $f(x)$.

$$\begin{array}{r} x + 3 \\ \hline x^2 - x - 2 \overline{) x^3 + 2x^2 - 5x - 6} \\ \underline{x^3 - x^2 - 2x} \\ 3x^2 - 3x - 6 \\ \underline{3x^2 - 3x - 6} \\ 0 \end{array}$$

Therefore, $f(x) = x^3 + 2x^2 - 5x - 6 = (x + 1)(x - 2)(x + 3)$

4. If $x - 2$ is a factor of $x^2 + ax + b$ and $a + b = 1$, find the values of a and b .

Solution:

Let $f(x) = x^2 + ax + b$

Given, $(x - 2)$ is a factor of $f(x)$.

Then, remainder = $f(2) = 0$

$$(2)^2 + a(2) + b = 0$$

$$4 + 2a + b = 0$$

$$2a + b = -4 \dots (1)$$

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And also given that,
 $a + b = 1 \dots (2)$

Subtracting (2) from (1), we have
 $a = -5$

On substituting the value of a in (2), we have
 $b = 1 - (-5) = 6$

5. Factorise $x^3 + 6x^2 + 11x + 6$ completely using factor theorem.

Solution:

Let $f(x) = x^3 + 6x^2 + 11x + 6$

For $x = -1$, the value of $f(x)$ is

$$\begin{aligned} f(-1) &= (-1)^3 + 6(-1)^2 + 11(-1) + 6 \\ &= -1 + 6 - 11 + 6 = 12 - 12 = 0 \end{aligned}$$

Thus, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r} x^2 + 5x + 6 \\ x + 1 \overline{) x^3 + 6x^2 + 11x + 6} \\ \underline{x^3 + x^2} \\ 5x^2 + 11x + 6 \\ \underline{5x^2 + 5x} \\ 6x + 6 \\ \underline{6x + 6} \\ 0 \end{array}$$

$$\begin{aligned} \text{Therefore, } f(x) &= (x + 1)(x^2 + 5x + 6) \\ &= (x + 1)(x^2 + 3x + 2x + 6) \\ &= (x + 1)[x(x + 3) + 2(x + 3)] \\ &= (x + 1)(x + 3)(x + 2) \end{aligned}$$

6. Find the value of 'm', if $mx^3 + 2x^2 - 3$ and $x^2 - mx + 4$ leave the same remainder when each is divided by $x - 2$.

Solution:

Let $f(x) = mx^3 + 2x^2 - 3$ and $g(x) = x^2 - mx + 4$

From the question, it's given that $f(x)$ and $g(x)$ leave the same remainder when divided by $(x - 2)$. So, we have:

$$f(2) = g(2)$$

$$m(2)^3 + 2(2)^2 - 3 = (2)^2 - m(2) + 4$$

$$8m + 8 - 3 = 4 - 2m + 4$$

$$10m = 3$$

$$\text{Thus, } m = 3/10$$