Solution 1: **Exercise 17(C)**

In the given figure, ΔABC is an equilateral triangle.
Hence all the three angles of the triangle will be equal to 60°.
i.e. $\angle A = \angle B = \angle C = 60°$
As the triangle is an equilateral triangle,
BO and CO will be the angle bisectors of $\angle B$ and $\angle C$ respectively.

$$\text{Hence } \angle OBC = \frac{\angle ABC}{2} = 30°$$

and as given in the figure we can see that OB and OC are the radii of the given circle
Hence they are of equal length.
The ΔOBC is an isosceles triangle with OB = OC
In ΔOBC, $\angle OBC = \angle OCB$ as they are angles opposite to the two equal sides of an isosceles triangle.
Hence, $\angle OBC = 30°$ and $\angle OCB = 30°$
Since the sum of all the angles of a triangle is 180°
Hence in triangle OBC, $\angle OCB + \angle OBC + \angle BOC = 180°$

$30° + 30° + \angle BOC = 180°$

$60° + \angle BOC = 180°$

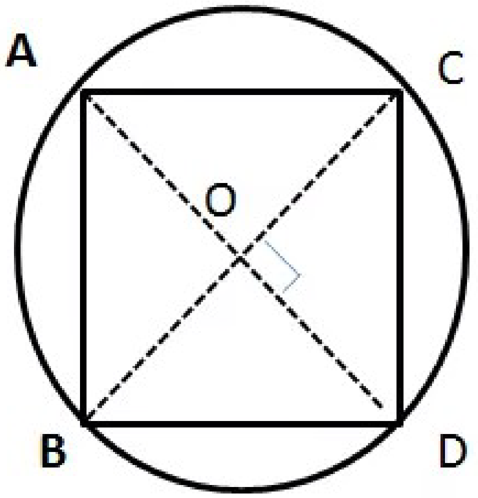
$\angle BOC = 180° - 60°$

$\angle BOC = 120°$

Hence $\angle BOC = 120°$ and $\angle OBC = 30°$

Solution 2:

In the given figure we can extend the staight line OB to BD and CO to CA
Then we get the diagonals of the square which intersect each other at 90° by the property of Square.



From the above statement we can see that

$\angle COD = 90°$.

The sum of the angle $\angle BOC$ and $\angle OCD$ is 180° as BD is a straight line.

Hence $\angle BOC + \angle OCD = \angle BOD = 180°$

$\angle BOC + 90° = 180°$

$\angle BOC = 180° - 90°$

$\angle BOC = 90°$

We can see that the ΔOCB is an isosceles triangle with sides OB and OC of equal length as they are the radii of the same circle.

In ΔOCB , $\angle OBC = \angle OCB$ as they are oppsite angles to the two equal sides of an isosceles triangle.

Sum of all the angles of a triangle is 180°

so, $\angle OBC + \angle OCB + \angle BOC = 180°$

$\angle OBC + \angle OBC + 90° = 180°$ as, $\angle OBC = \angle OCB$

$2\angle OBC = 180° - 90°$

$2\angle OBC = 90°$

$\angle OBC = 45°$

as $\angle OBC = \angle OCB$ So,

$\angle OBC = \angle OCB = 45°$

Yes BD is the diameter of the circle.

Solution 3:

As given that AB is the side of a pentagon the angle subtended by each arm of the pentagon at

the centre of the circle is $= \frac{360^\circ}{5} = 72^\circ$

Thus angle $\angle AOB = 72^\circ$

Similarly as BC is the side of a hexagon hence the angle subtended

by BC at the centre is $= \frac{360^\circ}{6}$

i.e. 60°

$\angle BOC = 60^\circ$

Now $\angle AOC = \angle AOB + \angle BOC = 72^\circ + 60^\circ = 132^\circ$

The triangle thus formed , ΔAOB is an isosceles triangle with $OA = OB$ as they are radii of the same circle.

Thus $\angle OBA = \angle BAO$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle AOB + \angle OBA + \angle BAO = 180^\circ$

$2\angle OBA + 72^\circ = 180^\circ$ as $, \angle OBA = \angle BAO$

$2\angle OBA = 180^\circ - 72^\circ$

$2\angle OBA = 108^\circ$

$\angle OBA = 54^\circ$

as $\angle OBA = \angle BAO$ So,

$\angle OBA = \angle BAO = 54^\circ$

The triangle thus formed , ΔBOC is an isosceles triangle with $OB = OC$ as they are radii of the same circle.

Thus $\angle OBC = \angle OCB$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle BOC + \angle OBC + \angle OCB = 180^\circ$

$2\angle OBC + 60^\circ = 180^\circ$ as $, \angle OBC = \angle OCB$

$2\angle OBC = 180^\circ - 60^\circ$

$2\angle OBC = 120^\circ$

$\angle OBC = 60^\circ$

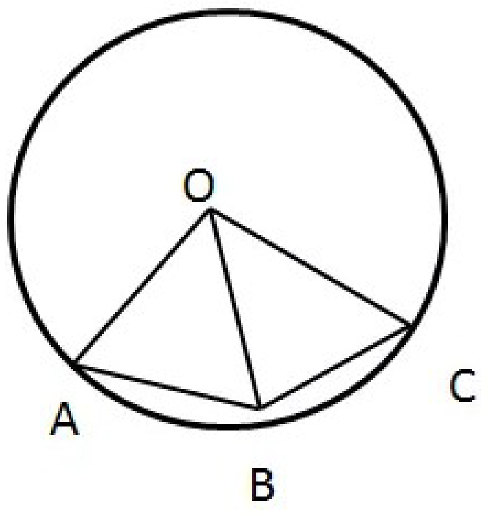
as $\angle OBC = \angle OCB$

So, $\angle OBC = \angle OCB = 60^\circ$

$\angle ABC = \angle OBA + \angle OBC = 54^\circ + 60^\circ = 114^\circ$

Solution 4:

We know that the arc of equal lengths subtend equal angles at the centre.



hence $\angle AOB = \angle BOC = 48^\circ$

Then $\angle AOC = \angle AOB + \angle BOC = 48^\circ + 48^\circ = 96^\circ$

The triangle thus formed , ΔBOC is an isosceles triangle with OB = OC as they are radii of the same circle.

Thus $\angle OBC = \angle OCB$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle BOC + \angle OBC + \angle OCB = 180^\circ$

$2\angle OBC + 48^\circ = 180^\circ$ as $\angle OBC = \angle OCB$

$2\angle OBC = 180^\circ - 48^\circ$

$2\angle OBC = 132^\circ$

$\angle OBC = 66^\circ$

as $\angle OBC = \angle OCB$

So, $\angle OBC = \angle OCB = 66^\circ$

The triangle thus formed , ΔAOC is an isosceles triangle with OA = OC as they are radii of the same circle.

Thus $\angle OAC = \angle OCA$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle COA + \angle OAC + \angle OCA = 180^\circ$

$2\angle OAC + 96^\circ = 180^\circ$ as ,$\angle OAC = \angle OCA$

$2\angle OAC = 180^\circ - 96^\circ$

$2\angle OAC = 84^\circ$

$\angle OAC = 42^\circ$

as $\angle OCA = \angle OAC$

So,$\angle OCA = \angle OAC = 42^\circ$

Solution 5:

We know that for two arcs are in ratio 3:2 then

$\angle AOB : \angle BOC = 3:2$

As give $\angle AOC = 96°$

So, $3x=96$

$x = 32$

Therefore $\angle BOC = 2 \times 32 = 64°$

The triangle thus formed , ΔAOB is an isosceles triangle with OA = OB as they are radii of the same circle.

Thus $\angle OBA = \angle BAO$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle AOB+\angle OBA+\angle BAO=180°$

$2\angle OBA + 96° = 180°$ as ,$\angle OBA = \angle BAO$

$2\angle OBA = 180° - 96°$

$2\angle OBA = 84°$

$\angle OBA = 42°$

as $\angle OBA = \angle BAO$ So,

$\angle OBA = \angle BAO = 42°$

The triangle thus formed , ΔBOC is an isosceles triangle with OB = OC as they are radii of the same circle.

Thus $\angle OBC = \angle OCB$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle BOC+\angle OBC+\angle OCB =180°$

$2\angle OBC+64° = 180°$ as ,$\angle OBC = \angle OCB$

$2\angle OBC = 180° - 64°$

$2\angle OBC= 116°$

$\angle OBC= 58°$

as $\angle OBC = \angle OCB$ So,

$\angle OBC = \angle OCB = 58°$

$\angle ABC = \angle BOA + \angle OBC = 42° + 58° = 100°$

Solution 6:

Since arc AB and BC are equal

so, $\angle AOB = \angle BOC = 50^\circ$

Now

$\angle AOC = \angle AOB + \angle BOC = 50^\circ + 50^\circ = 100^\circ$

As arc AB, arc BC and arc CD so,

$\angle AOB = \angle BOC = \angle COD = 50^\circ$

$\angle AOD = \angle AOB + \angle BOC + \angle COD = 50^\circ + 50^\circ + 50^\circ = 150^\circ$

Now, $\angle BOD = \angle BOC + \angle BOD$

$\angle BOD = 50^\circ + 50^\circ$

$\angle BOD = 100^\circ$

The triangle thus formed , ΔAOC is an isosceles triangle with OA = OC as they are radii of the same circle.

Thus $\angle OAC = \angle OCA$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle AOC + \angle OAC + \angle OCA = 180^\circ$

$2\angle OAC + 100^\circ = 180^\circ$ as ,$\angle OAC = \angle OCA$

$2\angle OAC = 180^\circ - 100^\circ$

$2\angle OAC = 80^\circ$

$\angle OAC = 40^\circ$

as $\angle OCA = \angle OAC$ So,

$\angle OCA = \angle OAC = 40^\circ$

The triangle thus formed , ΔAOD is an isosceles triangle with OA = OD as they are radii of the same circle.

Thus $\angle OAD = \angle ODA$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle AOD + \angle OAD + \angle ODA = 180^\circ$

$2\angle ODA + 150^\circ = 180^\circ$ as ,$\angle OAD = \angle ODA$

$2\angle ODA = 180^\circ - 150^\circ$

$2\angle ODA = 30^\circ$

$\angle ODA = 15^0$

as $\angle OAD = \angle ODA$ So,

$\angle OAD = \angle ODA = 15^\circ$

Solution 7:

As AB is the side of a hexagon so the

$\angle AOB = \frac{360°}{6} = 60°$

AC is the side of an eight sided polygon so,

$\angle AOC = \frac{360°}{8} = 45°$

From the given figure we can see that:

$\angle BOC = \angle AOB + \angle AOC = 60° + 45° = 105°$

Again, from the figure we can see that ΔBOC is an isosceles triangle with sides BO = OC as they are the radii of the same circle.

Angles $\angle OBC = \angle OCB$ as they are opposite angles to the equal sides of an isoseles triangle.

Sum of all the angles of a triangle is 180°

$\angle OBC + \angle OCB + \angle BOC = 180°$

$2\angle OBC + 105° = 180°$ as, $\angle OBC = \angle BOC$

$2\angle OBC = 180° - 105°$

$2\angle OBC = 75°$

$\angle OBC = 37.5° = 37°30'$

As, $\angle OBC = \angle BOC$

$\angle OBC = \angle BOC = 37.5° = 37°30'$

Solution 8:

We know that when two arcs are in ratio 2:1 then the subtended by them is also in ratio 2:1

As given arc AB is twice the length of arc BC

Therefore, arc AB: arc BC = 2:1

Hence, $\angle AOB : \angle BOC = 2:1$

Now given that $\angle AOB = 100°$

so, $\angle BOC = \frac{1}{2}\angle AOB = \frac{1}{2} \times 100° = 50°$

Now, $\angle AOC = \angle AOB + \angle BOC = 100° + 50° = 150°$

The triangle thus formed , ΔAOC is an isosceles triangle with OA = OC as they are radii of the same circle.

Thus $\angle OAC = \angle OCA$ as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, $\angle COA + \angle OAC + \angle OCA = 180°$

$2\angle OAC + 150° = 180°$ as $\angle OAC = \angle OCA$

$2\angle OAC = 180° - 150°$

$2\angle OAC = 30°$

$\angle OAC = 15^0$

as $\angle OCA = \angle OAC$ So,

$\angle OCA = \angle OAC = 15°$