

## 5. Factorization of Algebraic Expressions

### Exercise 5.1

#### 1. Question

Factorize:

$$x^3 + x - 3x^2 - 3$$

#### Answer

Given,

$$x^3 + x - 3x^2 - 3$$

$$= x(x^2 + 1) - 3(x^2 + 1)$$

$$= (x - 3)(x^2 + 1)$$

#### 2. Question

Factorize:

$$a(a+b)^3 - 3a^2b(a+b)$$

#### Answer

Given,

$$a(a+b)^3 - 3a^2b(a+b)$$

$$= a(a+b)\{(a+b)^2 - 3ab\} \text{ use the identity: } (a+b)^2 = a^2 + b^2 + 2ab$$

$$= a(a+b)(a^2 + b^2 + 2ab - 3ab)$$

$$= a(a+b)(a^2 + b^2 - ab)$$

#### 3. Question

Factorize:

$$x(x^3 - y^3) + 3xy - (x - y)$$

#### Answer

Given,

$$x(x^3 - y^3) + 3xy - (x - y)$$

$$\text{As } (x^3 - y^3) = (x - y)(x^2 + xy + y^2)$$

$$x(x^3 - y^3) + 3xy - (x - y) = x[(x - y)(x^2 + xy + y^2)] + 3xy - (x - y)$$

Take  $x(x - y)$  common to get,

$$x(x - y)[x^2 + xy + y^2 + 3y]$$

#### 4. Question

Factorize:

$$a^2x^2 + (ax^2 + 1)x + a$$

#### Answer

Given,

$$a^2x^2 + (ax^2 + 1)x + a = a^2x^2 + ax^3 + x + a$$

$$= ax^2(a+x) + 1(x+a)$$

$$= (ax^2 + 1)(x+a)$$

### 5. Question

Factorize:

$$x^2 + y - xy - x$$

### Answer

Given,

$$x^2 + y - xy - x = x^2 - x + y - xy$$

$$= x(x-1) + y(1-x)$$

$$= x(x-1) - y(x-1)$$

$$= (x-y)(x-1)$$

### 6. Question

Factorize:

$$x^3 - 2x^2y + 3xy^2 - 6y^3$$

### Answer

Given,

$$x^3 - 2x^2y + 3xy^2 - 6y^3$$

$$= x^2(x-2y) + 3y^2(x-2y)$$

$$= (x-2y)(x^2 + 3y^2)$$

### 7. Question

Factorize:

$$6ab - b^2 + 12ac - 2bc$$

### Answer

Given,

$$6ab - b^2 + 12ac - 2bc = b(6a-b) + 2c(6a-b)$$

$$= (b+2c)(6a-b)$$

### 8. Question

Factorize:

$$\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + 6$$

### Answer

Given,

$$\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + 6$$

$$= \left(x^2 + \frac{1}{x^2} + 2\right) - 4\left(x + \frac{1}{x}\right) + 4$$

$$= \left(x + \frac{1}{x}\right)^2 - 4\left(x + \frac{1}{x}\right) + 4$$

$$= \left\{ \left( x + \frac{1}{x} \right) - 2 \right\}^2 \quad [ \text{BY applying } (a^2 - 2ab + b^2) = (a-b)^2 ]$$

### 9. Question

Factorize:

$$x(x-2)(x-4)+4x-8$$

### Answer

Given,

$$X(x-2) (x-4)+4x-8 = x(x-2) (x-4)+4(x-2)$$

$$= (x-2) (x(x-4)+4)$$

$$= (x-2) (x^2 - 4x + 4)$$

$$= (x-2) (x-2)^2$$

$$= (x-2)^3$$

### 10. Question

Factorize:

$$(x+2)(x^2+25)-10x^2-20x$$

### Answer

Given,

$$(x+2) (x^2+25) - 10x^2 - 20x = (x+2) (x^2+25)-10x (x+2)$$

$$= (x+2) (x^2+25-10x)$$

$$= (x+2) (x-5)^2$$

### 11. Question

Factorize:

$$2a^2+2\sqrt{6} ab+3b^2$$

### Answer

Given,

$$2a^2\sqrt{6}ab+3b^2 = (\sqrt{2}a)^2 + 2(\sqrt{3}\times\sqrt{2})ab + (\sqrt{3}b)^2$$

$$= (\sqrt{2}a+\sqrt{3}b)^2$$

### 12. Question

Factorize:

$$(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$$

### Answer

Given,

$$(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$$

$$= (a-(b-c))^2 + (a+(b-c))^2 + 2(a-(b-c)) (a+(b-c))$$

[Applying identity:  $x^2 + y^2 + 2xy = (x + y)^2$ , where  $x = a-(b-c)$ ,  $y = a+(b-c)$ ]

$$= [(a-(b-c)) + (a+(b-c))]^2 = (2a)^2 = 4a^2$$

**13. Question**

Factorize:

$$a^2+b^2+2(ab+bc+ca)$$

**Answer**

Given,

$$\begin{aligned} a^2+b^2+2(ab+bc+ca) \\ &= a^2+b^2+2ab+2bc+2ca \\ &= (a+b)^2+2c(a+b) \\ &= (a+b)(a+b+2c) \end{aligned}$$

**14. Question**

Factorize:

$$4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$$

**Answer**

Given,

$$\begin{aligned} 4(x-y)^2-12(x-y)(x+y)+9(x+y)^2 &= 4(x^2-2xy+y^2)-12(x^2-y^2)+9(x^2+y^2+2xy) \\ &= 4x^2-8xy+4y^2-12x^2+12y^2+9x^2+9y^2+18xy \\ &= x^2+25y^2+10xy \\ &= (x)^2+(5y)^2+2 \times x \times 5y \\ &= (x+5y)^2 \end{aligned}$$

**15. Question**

Factorize:

$$a^2-b^2+2ab-c^2$$

**Answer**

Given,

$$\begin{aligned} a^2-b^2+2ab-c^2 &= a^2-(b^2-2bc+c^2) \\ &= a - (b-c)^2 \\ &= (a + (b-c)) (a-(b-c)) \\ &= (a+b-c) (a-b+c) \end{aligned}$$

**16. Question**

Factorize:

$$a^2+2ab+b^2-c^2$$

**Answer**

Given,

$$\begin{aligned} a^2+2ab+b^2-c^2 &= (a+b)^2 - c^2 \\ &= (a+b-c) (a+b+c) \end{aligned}$$

**17. Question**

Factorize:

$$a^2 + 4b^2 - 4ab - 4c^2$$

**Answer**

Given,

$$a^2 + 4b^2 - 4ab - 4c^2 = (a)^2 + (2b)^2 - 2 \times a \times 2b - 4c^2$$

$$= (a-2b)^2 - (2c)^2$$

$$= (a-2b-2c)(a-2b+2c)$$

### 18. Question

Factorize:

$$xy^9 - yx^9$$

**Answer**

Given,

$$xy^9 - yx^9 = xy (y^8 - x^8)$$

$$= xy ((y^4)^2 - (x^4)^2)$$

$$= xy (y^4 + x^4) (y^4 - x^4)$$

$$= xy (y^4 + x^4) (y^2 - x^2) (y^2 + x^2)$$

$$= xy (y^4 + x^4) (y^2 + x^2) (y - x) (y + x)$$

### 19. Question

Factorize:

$$x^4 + x^2y^2 + y^4$$

**Answer**

Given,

$$x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2$$

$$= (x^2y^2)^2 - (xy)^2$$

$$= (x^2 + y^2 - xy) (x^2 + y^2 + xy)$$

### 20. Question

Factorize:

$$x^2 - y^2 - 4xz + 4z^2$$

**Answer**

Given,

$$x^2 - y^2 - 4xz + 4z^2 = x^2 - 4xz + 4z^2 - y^2$$

$$= (x)^2 - 2 \times x \times 2z + (2z)^2 - y^2$$

$$= (x-2z)^2 - y^2$$

$$= (x - 2z - y) (x - 2z + y)$$

$$= (x - y - 2z) (x + y - 2z)$$

### 21. Question

Factorize:

$$x^2 + 6\sqrt{2}x + 10$$

**Answer**

Given,

$$x^2 + 6\sqrt{2}x + 10 = x^2 + \sqrt{2}x + 10$$

$$= x(x + \sqrt{2}) + 5\sqrt{2}(x + \sqrt{2})$$

$$= (x + \sqrt{2})(x + 5\sqrt{2})$$

### 22. Question

Factorize:

$$x^2 - 2\sqrt{2}x - 30$$

**Answer**

Given,

$$x^2 - 2\sqrt{2}x - 30 = x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30$$

$$= x(x - 5\sqrt{2}) + 3\sqrt{2}(x - 5\sqrt{2})$$

$$= (x - 5\sqrt{2})(x + 3\sqrt{2})$$

### 23. Question

Factorize:

$$x^2 - \sqrt{3}x - 6$$

**Answer**

Given,

$$x^2 - \sqrt{3}x - 6 = x^2 - 2\sqrt{3}x + \sqrt{3}x - 6$$

$$= x(x - 2\sqrt{3}) + \sqrt{3}(x - 2\sqrt{3})$$

$$= (x + \sqrt{3})(x - 2\sqrt{3})$$

### 24. Question

Factorize:

$$x^2 + 5\sqrt{5}x + 30$$

**Answer**

Given,

$$x^2 + 5\sqrt{5}x + 30 = x^2 + 3\sqrt{5}x + 2\sqrt{5}x + 30$$

$$= x(x + 3\sqrt{5}) + 2\sqrt{5}(x + 3\sqrt{5})$$

$$= (x + 3\sqrt{5})(x + 2\sqrt{5})$$

### 25. Question

Factorize:

$$x^2 + 2\sqrt{3}x - 24$$

**Answer**

Given,

$$\begin{aligned}
 x^2 + 2\sqrt{3}x - 24 &= x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24 \\
 &= x(x + 4\sqrt{3}) - 2\sqrt{3}(x + 4\sqrt{3}) \\
 &= (x + 4\sqrt{3})(x - 2\sqrt{3})
 \end{aligned}$$

**26. Question**

Factorize:

$$2x^2 - \frac{5}{6}x + \frac{1}{12}$$

**Answer**

Given,

$$\begin{aligned}
 2x^2 - \frac{5}{6}x + \frac{1}{12} \\
 &= 2x^2 - \frac{1}{3}x - \frac{1}{2}x + \frac{1}{12} \\
 &= x\left(2x - \frac{1}{3}\right) - \frac{1}{4}\left(2x - \frac{1}{3}\right) \\
 &= \left(2x - \frac{1}{3}\right)\left(x - \frac{1}{4}\right)
 \end{aligned}$$

**27. Question**

Factorize:

$$x^2 + \frac{12}{35}x + \frac{1}{35}$$

**Answer**

Given,

$$\begin{aligned}
 x^2 + \frac{12}{35}x + \frac{1}{35} &= x^2 - \frac{1}{7}x - \frac{1}{5}x + \frac{1}{35} \\
 x\left(x + \frac{1}{7}\right) + \frac{1}{5}\left(x + \frac{1}{7}\right) \\
 &= \left(x - \frac{1}{7}\right)\left(x + \frac{1}{5}\right)
 \end{aligned}$$

**28. Question**

Factorize:

$$21x^2 - 2x + \frac{1}{21}$$

**Answer**

Given,

$$\begin{aligned}
 21x^2 - 2x + \frac{1}{21} &= 21x^2 - x - x + \frac{1}{21} \\
 &= x(21x - 1) - \frac{1}{21}(21x - 1) \\
 &= \left(x - \frac{1}{21}\right)(21x - 1)
 \end{aligned}$$

**29. Question**

Factorize:



$$5\sqrt{5}x^2 + 20x + 3\sqrt{5}$$

**Answer**

Given,

$$\begin{aligned}5\sqrt{5}x^2 + 20x + 3\sqrt{5} &= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5} \\ &= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3) \\ &= (5x + \sqrt{5})(\sqrt{5}x + 3)\end{aligned}$$

### 30. Question

Factorize:

$$2x^2 + 3\sqrt{5}x + 531.9(2a-b)^2 - 4(2a-b) - 13$$

**Answer**

Given,

$$\begin{aligned}2x^2 + 3\sqrt{5}x + 5 &= 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5 \\ &= 2x(x + \sqrt{5}) + \sqrt{5}(x + \sqrt{5}) \\ &= (2x + \sqrt{5})(x + \sqrt{5})\end{aligned}$$

### 31. Question

Factorize:

$$9(2a - b)^2 - 4(2a - b) - 13$$

**Answer**

Given,

$$9(2a - b)^2 - 4(2a - b) - 13$$

Let us assume  $(2a - b) = x$

$$9x^2 - 4x - 13$$

$$9x^2 - 13x + 9x - 13$$

$$x(9x - 13) + 1(9x + 3)$$

$$(9x - 13)(x + 1)$$

$$[9(2a - b) - 13][2a - b + 1]$$

$$(18a - 9b - 13)(2a - b + 1)$$

### 32. Question

Factorize:

$$7(x-2y)^2 - 25(x-2y) + 12$$

**Answer**

Given,

$$7(x - 2y)^2 - 25(x - 2y) + 12$$

Let  $a = (x - 2y)$ ,

So we have,

$$= 7a^2 - 25a + 12$$

$$= 7a^2 - 21a - 4a + 12$$

$$= 7a(a - 3) - 4(a - 3)$$

$$= (7a - 4)(a - 3)$$

Put  $a = (x - 2y)$

$$= \{7(x - 2y) - 4\}(x - 2y - 3)$$

$$= (7x - 14y - 4)(x - 2y - 3)$$

### 33. Question

Factorize:

$$2(x+y)^2 - 9(x+y) - 5$$

#### Answer

Given,

$$2(x+y)^2 - 9(x+y) - 5,$$

$$= 2a^2 - 9a - 5$$

Let  $(x+y) = a$

$$= 2a^2 - 10a + a - 5$$

$$= 2a(a-5) + 1(a-5)$$

$$= (2a+1)(a-5)$$

$$= \{2(x+y)+1\}(x+y-5)$$

$$(2x+2y+1)(x+y-5)$$

### 34. Question

Give possible expressions for the length and breadth of the rectangle having  $35y^2 + 13y - 12$  as its area.

#### Answer

We know that,

$$\text{Area of rectangle} = \text{length} \times \text{breadth}$$

Given,

$$35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$$

$$= 7y(5y+4) - 3(5y+4)$$

$$= (7y - 3)(5y + 4)$$

Thus,

$$\text{Length} = (7y - 3), \text{ then breadth} = (5y + 4)$$

$$\text{Length} = (5y + 4), \text{ then breadth} = (7y - 3)$$

### 35. Question

What are the possible expressions for the dimensions of the cuboid whose volume is  $3x^2 - 12x$ .

#### Answer

Given,

We know that,

$$\text{Volume of cuboids} = \text{length} \times \text{breadth} \times \text{height}$$

Given,

$$3x^2 - 12x = 3x(x-4)$$

Thus,

Dimensions of cuboids are -

Length	breadth	height
3	x	(x-4)
x	(x-4)	3
(x-4)	3	x

## Exercise 5.2

### 1. Question

Factorize each of the following expressions:

$$p^3+27$$

**Answer**

Given,

$$p^3+27,$$

$$= p^3 + (3)^3 [\because a^3+b^3 = (a+b)(a^2 - 2ab + b^2)]$$

$$= (p + 3) (p^2 + 9 - 3p)$$

### 2. Question

Factorize each of the following expressions:

$$y^3+125$$

**Answer**

Given,

$$y^3+125,$$

$$= y^3 + (5)^3 [\because a^3+b^3 = (a+b)(a^2 - 2ab + b^2)]$$

$$= (y + 5) (y^2 - 5y + 25)$$

### 3. Question

Factorize each of the following expressions:

$$1-27a^3$$

**Answer**

Given,

$$1 - 27a^3,$$

$$= 1 - (3a)^3$$

$$= (1 - 3a) (1+9a^2+3a)$$

### 4. Question

Factorize each of the following expressions:

$$8x^3y^3+27a^3$$

**Answer**

Given,

$$\begin{aligned} &8x^3 y^3 + 27a^3, \\ &= (2xy)^3 + (3a)^3 \\ &= (2xy + 3a) (4x^2 y^2 + 9a^2 - 6axy) \end{aligned}$$

### 5. Question

Factorize each of the following expressions:

$$64a^3 - b^3$$

**Answer**

Given,

$$\begin{aligned} &64a^3 - b^3, \\ &= (4a)^3 - (b)^3 \\ &= (4a - b) (16a^2 + b^2 + 4ab) \end{aligned}$$

### 6. Question

Factorize each of the following expressions:

$$\frac{x^3}{216} - 8y^3$$

**Answer**

Given,

$$\begin{aligned} \frac{x^3}{216} - 8y^3 &= \left(\frac{x}{6}\right)^3 - (2y)^3 \\ &= \left(\frac{x}{6} - 2y\right) \left(\frac{x^2}{36} + 4y^2 + \frac{xy}{3}\right) \end{aligned}$$

### 7. Question

Factorize each of the following expressions:

$$10x^4 y - 10xy^4$$

**Answer**

Given,

$$\begin{aligned} &10x^4 y - 10xy^4, \\ &= 10xy (x^3 - y^3) \\ &= 10xy (x - y) (x^2 + xy + y^2) \end{aligned}$$

### 8. Question

Factorize each of the following expressions:

$$54x^6 y + 2x^3 y^4$$

**Answer**

Given,

$$\begin{aligned} &54x^6 y + 2x^3 y^4, \\ &= 2x^3 y (27x^3 + y^3) \end{aligned}$$

$$= 2x^3 y \{(3x)^3 + (y)^3\}$$

$$= 2x^3 y (3x + y) (9x^2 + y^2 - 3xy)$$

### 9. Question

Factorize each of the following expressions:

$$32a^2 + 108b^3$$

#### Answer

Given,

$$32a^3 + 108b^3,$$

$$= 4 (8a^3 + 27b^3)$$

$$= 4 \{ (2a)^3 + (3b)^3 \}$$

$$= 4 (2a + 3b) (4a^2 + 9b^2 - 6ab)$$

### 10. Question

Factorize each of the following expressions:

$$(a-2b)^3 - 512b^3$$

#### Answer

Given,

$$(a - 2b)^3 - 512b^3$$

$$= (a - 2b)^3 - (8b)^3$$

$$= (a - 2b - 8b) \{(a - 2b)^2 + (8b)^2 + (a - 2b) 8b\}$$

$$= (a - 10b) (a^2 + 4b^2 - 4ab + 64b^2 + 8ab - 16b^2)$$

$$= (a - 10b) (a^2 + 52b^2 + 4ab)$$

### 11. Question

Factorize each of the following expressions:

$$(a+b)^3 - 8(a-b)^3$$

#### Answer

Given,

$$(a + b)^3 - \{2(a - b)\}^3$$

$$= \{(a + b) - 2(a - b)\} \{ (a+b)^2 + 4(a-b)^2 + 2(a+b)(a - b) \} \text{ [ By using: } x^3 - y^3 = (x - y)(x^2 + y^2 + xy) \}$$

$$= (a + b - 2a + 2b)(a^2 + b^2 + 2ab + 4a^2 + 4b^2 - 8ab + 2a^2 - 2b^2)$$

$$= (3b - a) (7a^2 + 3b^2 - 6ab)$$

### 12. Question

Factorize each of the following expressions:

$$(x+2)^3 + (x-2)^3$$

#### Answer

Given,

$$(x+2)^3 + (x-2)^3 = (x + 2 + x - 2) \{ (x + 2)^2 + (x - 2)^2 - (x + 2)(x - 2) \}$$

$$= 2x(x^2 + 4 + 4x + x^2 + 4 - 4x - x^2 + 4)$$

$$= 2x(x^2 + 12)$$

### 13. Question

Factorize each of the following expressions:

$$8x^2y^3 - x^5$$

**Answer**

Given,

$$8x^2y^3 - x^5$$

$$= x^2(8y^3 - x^3)$$

$$= x^2\{(2y)^3 - (x)^3\}$$

$$= x^2(2y - x)(4y^2 + x^2 + 2xy)$$

### 14. Question

Factorize each of the following expressions:

$$1029 - 3x^3$$

**Answer**

Given,

$$1029 - 3x^3$$

$$= 3(343 - x^3)$$

$$= 3\{(7)^3 - (x)^3\}$$

$$= 3(7 - x)(49 + x^2 + 7x)$$

### 15. Question

Factorize each of the following expressions:

$$x^6 + y^6$$

**Answer**

Given,

$$x^6 + y^6 = (x^2)^3 + (y^2)^3$$

$$= (x^2 + y^2)(x^4 + y^4 - x^2y^2)$$

### 16. Question

Factorize each of the following expressions:

$$x^3y^3 + 1$$

**Answer**

Given,

$$x^3y^3 + 1 = (xy)^3 + (1)^3$$

$$= (xy + 1)(x^2y^2 + 1 - xy)$$

### 17. Question

Factorize each of the following expressions:

$$x^4y^4 - xy$$

**Answer**

Given,

$$x^4y^4 - xy = xy (x^3y^3 - 1)$$

$$= xy \{ (xy)^3 - (1)^3 \}$$

$$= xy (xy - 1) (x^2y^2 + 1 + xy)$$

**18. Question**

Factorize each of the following expressions:

$$a^{12} + b^{12}$$

**Answer**

Given,

$$a^{12} + b^{12} = (a^4)^3 + (b^4)^3$$

$$= (a^4 + b^4)(a^3 + b^3 - a^4b^4)$$

**19. Question**

Factorize each of the following expressions:

$$x^3 + 6x^2 + 12x + 16$$

**Answer**

Given,

$$x^3 + 6x^2 + 12x + 16 = (x^3 + 6x^2 + 12x + 8) + 8$$

$$= (x+2)^3 + 8 \quad [\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$$

$$= (x+2)^3 + (2)^3$$

$$= (x+2+2) \{ (x+2)^2 + 4 - 2(x+2) \}$$

$$= (x+4)(x^2 + 4 + 4x + 4 - 2x - 4)$$

$$= (x+4)(x^2 + 2x + 4)$$

**20. Question**

Factorize each of the following expressions:

$$a^3 + b^3 + a + b$$

**Answer**

Given,

$$a^3 + b^3 + a + b = (a^3 + b^3) + (a + b)$$

$$= (a + b) (a^2 - ab + b^2) + a + b$$

$$= (a + b)(a^2 - ab + b^2 + 1)$$

**21. Question**

Factorize each of the following expressions:

$$a^3 - \frac{1}{a^3} - 2a \frac{2}{a}$$

**Answer**

Given,

$$\begin{aligned}
 & a^3 - \frac{1}{a^3} - 2a + \frac{2}{a} \\
 &= a^3 - \frac{1}{a^3} - 2\left(a - \frac{1}{a}\right) \\
 &= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} + 1\right) - 2\left(a - \frac{1}{a}\right) \\
 &= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} + 1 - 2\right) \\
 &= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right)
 \end{aligned}$$

**22. Question**

Factorize each of the following expressions:

$$a^3 + 3a^2b^3 + 3ab^2 + b^3 - 8$$

**Answer**

Given,

$$\begin{aligned}
 a^3 + 3a^2b + 3ab^2 + b^3 - 8 &= (a^3 + 3a^2b + 3ab^2 + b^3) - 8 \\
 &= (a+b)^3 - 8 \\
 &= (a+b)^3 - (2)^3 \\
 &= (a+b-2) \{ (a+b)^2 + (2)^2 + 2(a+b) \} \\
 &= (a+b-2)(a^2 + b^2 + 2ab + 4 + 2a + 2b)
 \end{aligned}$$

**23. Question**

Factorize each of the following expressions:

$$8a^3 - b^3 - 4ax + 2bx$$

**Answer**

Given,

$$\begin{aligned}
 8a^3 - b^3 - 4ax + 2bx &= (2a)^3 - (b)^3 - 2x(2a - b) \\
 &= (2a - b)(4a^2 + b^2 + 2ab) - 2x(2a - b) \\
 &= (2a - b)(4a^2 + b^2 + 2ab - 2x)
 \end{aligned}$$

**24. Question**

Simplify

$$(i) \frac{173 \times 173 \times 173 - 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$$

$$(ii) \frac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 - 155 \times 55 + 55 \times 55}$$

$$(iii) \frac{1.2 \times 1.2 \times 1.2 - 0.2 \times 0.2 \times 0.2}{1.2 \times 1.2 - 1.2 \times 0.2 + 0.2 \times 0.2}$$

**Answer**

(i) Given,

$$\begin{aligned}
 &= \frac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127} \\
 &= \frac{(173)^3 + (127)^3}{(173)^2 - 173 \times 127 + (127)^2} \\
 &= \frac{(173 + 127)\{(173)^2 + (127)^2 - 173 \times 127\}}{(173)^2 - 173 \times 127 + (127)^2} = 300
 \end{aligned}$$

(ii) Given,

$$\begin{aligned}
 &\frac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55} \\
 &= \frac{(155)^3 - (55)^3}{(155)^2 + 155 \times 55 + (55)^2} \\
 &= \frac{(155 - 55)\{(155)^2 + 155 \times 55 + (55)^2\}}{(155)^2 + 155 \times 55 + (55)^2} \\
 &= (155 - 55) = 100
 \end{aligned}$$

(ii) Given,

$$\begin{aligned}
 &= \frac{1.2 \times 1.2 \times 1.2 - 0.2 \times 0.2 \times 0.2}{1.2 \times 1.2 + 1.2 \times 0.2 + 0.2 \times 0.2} \\
 &= \frac{(1.2)^3 - (0.2)^3}{(1.2)^2 + 1.2 \times 0.2 + (0.2)^2} \\
 &= \frac{(1.2 - 0.2)\{(1.2)^2 + (0.2)^2 + 1.2 \times 0.2\}}{(1.2)^2 + 1.2 \times 0.2 + (0.2)^2} \\
 &= (1.2 - 0.2) = 1
 \end{aligned}$$

### Exercise 5.3

#### 1. Question

Factorize:

$$64a^3 + 125b^3 + 240a^2b + 300ab^2$$

#### Answer

Given,

$$\begin{aligned}
 &64a^3 + 125b^3 + 240a^2b + 300ab^2, \\
 &= (4a)^3 + (5b)^3 + 60ab(4a + 5b) \\
 &= (4a)^3 + (5b)^3 + 3 \times 4a \times 5b(4a + 5b) \\
 &= (4a + 5b)^3 \\
 &= (4a + 5b)(4a + 5b)(4a + 5b)
 \end{aligned}$$

#### 2. Question

Factorize:

$$125x^3 - 27y^3 - 225x^2y + 125xy^2$$

#### Answer

Given,

$$125x^3 - 27y^3 - 225x^2y + 125xy^2,$$

$$\begin{aligned}
 &= (5x)^3 - (3y)^3 - 45xy(5x - 3y) \\
 &= (5x)^3 - (3y)^3 - 3 \times 5x \times 3y(5x - 3y) \\
 &= (5x - 3y)^3 \\
 &= (5x - 3y)(5x - 3y)(5x - 3y)
 \end{aligned}$$

### 3. Question

Factorize:

$$\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$$

### Answer

Given,

$$\begin{aligned}
 &\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x, \\
 &= \left(\frac{2}{3}x\right)^3 + (1)^3 + 2x\left(\frac{2}{3}x + 1\right) \\
 &= \left(\frac{2}{3}x\right)^3 + (1)^3 + 3 \times \frac{2}{3}x \times 1 \left(\frac{2}{3}x + 1\right) \\
 &= \left(\frac{2}{3}x + 1\right)^3 \\
 &= \left(\frac{2}{3}x + 1\right)\left(\frac{2}{3}x + 1\right)\left(\frac{2}{3}x + 1\right)
 \end{aligned}$$

### 4. Question

Factorize:

$$8x^3 + 27y^3 + 36x^2y + 54xy^2$$

### Answer

Given,

$$\begin{aligned}
 &8x^3 + 27y^3 + 36x^2y + 54xy^2, \\
 &= (2x)^3 + (3y)^3 + 18xy(2x + 3y) \\
 &= (2x)^3 + (3y)^3 + 3 \times 2x \times 3y(2x + 3y) \\
 &= (2x + 3y)^3 \\
 &= (2x + 3y)(2x + 3y)(2x + 3y)
 \end{aligned}$$

### 5. Question

Factorize:

$$a^3 - 3a^2b + 3ab^2 - b^3 + 8$$

### Answer

Given,

$$\begin{aligned}
 &a^3 - 3a^2b + 3ab^2 - b^3 + 8, \\
 &= \{(a^3 - (b^3 - 3ab(a-b))\} + 8 \\
 &= (a-b)^3 + (2)^3
 \end{aligned}$$

$$= (a - b + 2) \{(a - b)^2 + (2)^2 + 2(a - b)\}$$

$$= (a - b + 2)(a^2 - 2ab + b^2 + 4 + 2a - 2b)$$

### 6. Question

Factorize:

$$x^3 + 8y^3 + 6x^2y + 12xy^2$$

### Answer

Given,

$$x^3 + 8y^3 + 6x^2y + 12xy^2,$$

$$= (x)^3 + (2y)^3 + 6xy(x + 2y)$$

$$= (x)^3 + (2y)^3 + 3 \times x \times 2y(x + 2y)$$

$$= (x + 2y)^3$$

$$= (x + 2y)(x + 2y)(x + 2y)$$

### 7. Question

Factorize:

$$8x^3 + y^3 + 12x^2y + 6xy^2$$

### Answer

Given,

$$8x^3 + y^3 + 12x^2y + 6xy^2,$$

$$= (2x)^3 + (y)^3 + 6xy(2x + y)$$

$$= (2x)^3 + (y)^3 + 3 \times 2x \times y(2x + y)$$

$$= (2x + y)^3$$

$$= (2x + y)(2x + y)(2x + y)$$

### 8. Question

Factorize:

$$8a^3 + 27b^3 + 36a^2b + 54ab^2$$

### Answer

Given,

$$8a^3 + 27b^3 + 36a^2b + 54ab^2,$$

$$= (2a)^3 + (3b)^3 + 18ab(2a + 3b)$$

$$= (2a)^3 + (3b)^3 + 3 \times 2a \times 3b(2a + 3b)$$

$$= (2a + 3b)^3$$

$$= (2a + 3b)(2a + 3b)(2a + 3b)$$

### 9. Question

Factorize:

$$8a^3 - 27b^3 - 36a^2b + 54ab^2$$

### Answer



Given,

$$\begin{aligned} & 8a^3 - 27b^3 - 36a^2b + 54ab^2, \\ & = (2a)^3 - (3b)^3 - 18ab(2a - 3b) \\ & = (2a)^3 - (3b)^3 - 3 \times 2a \times 3b(2a - 3b) \\ & = (2a - 3b)^3 \\ & = (2a - 3b)(2a - 3b)(2a - 3b) \end{aligned}$$

### 10. Question

Factorize:

$$x^3 - 12x(x-4) - 64$$

**Answer**

Given,

$$\begin{aligned} & x^3 - 12x(x-4) - 64, \\ & = (x)^3 - 12x(x-4) - (4)^3 \\ & = (x)^3 - (4)^3 - 3 \times x \times 4(x-4) \\ & = (x-4)^3 \\ & = (x-4)(x-4)(x-4) \end{aligned}$$

### 11. Question

Factorize:

$$a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$$

**Answer**

Given,

$$\begin{aligned} & a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3, \\ & = (ax)^3 - 3abx(ax-b) - (b)^3 \\ & = (ax)^3 - (b)^3 - 3abx(ax-b) \\ & = (ax-b)^3 \\ & = (ax-b)(ax-b)(ax-b) \end{aligned}$$

## Exercise 5.4

### 1. Question

Factorize each of the following expressions:

$$a^3 + 8b^3 + 64c^3 - 24abc$$

**Answer**

Given,

$$= a^3 + 8b^3 + 64c^3 - 24abc$$

This can be written in form

$$= a^3 + 8b^3 + 64c^3 - 24abc = (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

Hence,

$$= a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c)\{(a)^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a\}$$

$$= (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$$

Thus the required factors of  $a^3 + 8b^3 + 64c^3 - 24abc$  is  $(a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$

## 2. Question

Factorize each of the following expressions:

$$x^3 - 8y^3 + 27z^3 + 18xyz$$

**Answer**

Given,

$$= x^3 - 8y^3 + 27z^3 + 18xyz$$

This can be written in form,

$$= x^3 - 8y^3 + 27z^3 + 18xyz = (x)^3 + (-2y)^3 + (3z)^3 - 3 \times x \times (-2y) \times 3z$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$= x^3 - 8y^3 + 27z^3 + 18xyz = \{x + (-2y) + 3z\}\{(x)^2 + (-2y)^2 + (3z)^2 - (x)(-2y) - (-2y)(3z) - (3z)(x)\}$$

$$= (x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$$

Thus the required factors of  $x^3 - 8y^3 + 27z^3 + 18xyz$  is  $(x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

## 3. Question

Factorize each of the following expressions:

$$27x^3 - y^3 - z^3 - 9xyz$$

**Answer**

Given,

$$= 27x^3 - y^3 - z^3 - 9xyz$$

This can be written in form ,

$$= 27x^3 - y^3 - z^3 - 9xyz = (3x)^3 + (-y)^3 + (-z)^3 - 3(3x)(-y)(-z)$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

So ,

$$= 27x^3 - y^3 - z^3 - 9xyz = \{3x + (-y) + (-z)\}\{(3x)^2 + (-y)^2 + (-z)^2 - (3x)(-y) - y(-z) - z(3x)\}$$

$$= (3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3zx)$$

Thus the factors of  $27x^3 - y^3 - z^3 - 9xyz$  is  $(3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3zx)$

## 4. Question

Factorize:

$$\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$$

**Answer**

Given,

$$= \frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz$$

This can be written in form ,

$$= \frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz = \left(\frac{1}{3}x\right)^3 + (-y)^3 + (5z)^3 - 3\left(\frac{1}{3}x\right)(-y)(5z)$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$= \frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz = \left\{\left(\frac{1}{3}x\right) + (-y) + (5z)\right\} \left\{\left(\frac{1}{3}x\right)^2 + (-y)^2 + (5z)^2 - \left(\frac{1}{3}x\right)(-y) - (-y)(5z) - (5z)\left(\frac{1}{3}x\right)\right\}$$

$$= \left(\frac{1}{3}x - y + 5z\right) \left(\frac{1}{9}x^2 + y^2 + 25z^2 + \frac{1}{3}xy + 5yz - \frac{5}{3}zx\right)$$

Thus the factors of  $\frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz$  is  $\left(\frac{1}{3}x - y + 5z\right) \left(\frac{1}{9}x^2 + y^2 + 25z^2 + \frac{1}{3}xy + 5yz - \frac{5}{3}zx\right)$

### 5. Question

Factorize each of the following expressions:

$$8x^3 + 27y^3 - 216z^3 + 108xyz$$

#### Answer

Given,

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz$$

This can be written in form ,

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz = (2x)^3 + (3y)^3 + (-6z)^3 - 3(2x)(3y)(-6z)$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz = \{(2x) + (3y) + (-6z)\} \{(2x)^2 + (3y)^2 + (-6z)^2 - (2x)(3y) - (3y)(-6z) - (-6z)(2x)\}$$

$$= (2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$$

Thus the factors of

$$8x^3 + 27y^3 - 216z^3 + 108xyz \text{ is } (2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$$

### 6. Question

Factorize each of the following expressions:

$$125 + 8x^3 - 27y^3 + 90xy$$

#### Answer

Given,

$$= 125 + 8x^3 - 27y^3 + 90xy$$

This can be written in form ,

$$= 125 + 8x^3 - 27y^3 + 90xy = (5)^3 + (2x)^3 + (-3y)^3 - 3(5)(2x)(-3y)$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$= 125 + 8x^3 - 27y^3 + 90xy = \{5 + 2x - 3y\} \{(5)^2 + (2x)^2 + (-3y)^2 - (5)(2x) - (2x)(-3y) - (-3y)(5)\}$$

$$= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$$

Thus the factors of  $125 + 8x^3 - 27y^3 + 90xy$  is  $(5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

### 7. Question

Factorize:

$$(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$$

### Answer

Given,

$$= (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3$$

$$\text{Let } a = (3x - 2y), b = (2y - 4z), c = (4z - 3x)$$

$$= (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = a^3 + b^3 + c^3$$

Here ,

$$= a + b + c = (3x - 2y + 2y - 4z + 4z - 3x) = 0$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

Hence ,

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = 3(3x - 2y)(2y - 4z)(4z - 3x)$$

### 8. Question

Factorize each of the following expressions:

$$(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$$

### Answer

Given,

$$= (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$$

$$\text{Let } a = (2x - 3y), b = (4z - 2x), c = (3y - 4z)$$

Then ,

$$= (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = a^3 + b^3 + c^3$$

Here ,

$$= a + b + c = 2x - 3y + 4z - 2x + 3y - 4z = 0$$

Hence ,

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = 3(2x - 3y)(4z - 2x)(3y - 4z)$$

### 9. Question

Factorize each of the following expressions:

$$\left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} - y\right)^3 - \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3$$

### Answer

Given,

$$= \left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3$$

$$\text{Let } a = \left(\frac{x}{2} + y + \frac{z}{3}\right), b = \left(\frac{x}{3} - \frac{2y}{3} + z\right), c = \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)$$

Then ,

$$= \left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3 = a^3 + b^3 + c^3$$

Here ,

$$= a + b + c = \left(\frac{x}{2} + y + \frac{z}{3}\right) + \left(\frac{x}{3} - \frac{2y}{3} + z\right) + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right) = 0$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

Hence ,

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= \left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3 = 3 \left(\frac{x}{2} + y + \frac{z}{3}\right) \left(\frac{x}{3} - \frac{2y}{3} + z\right) \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right).$$

### 10. Question

Factorize each of the following expressions:

$$(a-3b)^3 + (3b-c)^3 + (c-a)^3$$

**Answer**

Given,

$$= (a-3b)^3 + (3b-c)^3 + (c-a)^3$$

$$\text{Let } x = (a-3b), y = (3b-c), z = (c-a)$$

Then,

$$= (a-3b)^3 + (3b-c)^3 + (c-a)^3 = x^3 + y^3 + z^3$$

Here ,

$$= x + y + z = a - 3b + 3b - c + c - a = 0$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (a-3b)^3 + (3b-c)^3 + (c-a)^3 = 3(a-3b)(3b-c)(c-a)$$

### 11. Question

Factorize each of the following expressions:

$$2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$$

**Answer**

Given,

$$= 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$$

This can be written in form ,

$$= 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc = (\sqrt{2}a)^3 + (\sqrt{3}b)^3 + (c)^3 - 3(\sqrt{2}a)(\sqrt{3}b)(c)$$

And,  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$  By formula

Hence,

$$= 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc = \{(\sqrt{2}a) + (\sqrt{3}b) + c\} \{(\sqrt{2}a)^2 + (\sqrt{3}b)^2 + (c)^2 - (\sqrt{2}a)(\sqrt{3}b) - (\sqrt{3}b)(c) - (c)(\sqrt{2}a)\}$$

$$= \sqrt{2}a + \sqrt{3}b + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ca$$

## 12. Question

Factorize each of the following expressions:

$$3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$$

**Answer**

Given,

$$= 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$$

This can be written in form .

$$= 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = (\sqrt{3}a)^3 + (-b)^3 + (-\sqrt{5}c)^3 - 3(\sqrt{3}a)(-b)(-\sqrt{5}c)$$

And,  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$  By formula

$$= 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = \{(\sqrt{3}a) + (-b) + (-\sqrt{5}c)\} \{(\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - (\sqrt{3}a)(-b) - (-\sqrt{5}c)(\sqrt{3}a) - (-b)(-\sqrt{5}c)\}$$

$$= (\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ca$$

## 13. Question

Factorize each of the following expressions:

$$8x^3 - 125y^3 + 180xy + 216$$

**Answer**

Given,

$$= 8x^3 - 125y^3 + 180xy + 216$$

This can be written in form ,

$$= 8x^3 - 125y^3 + 180xy + 216 = (2x)^3 + (-5y)^3 + 6^3 - 3(2x)(-5y)(6)$$

And,  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$  By formula

Hence ,

$$= 8x^3 - 125y^3 + 180xy + 216 = \{2x + (-5y) + 6\} \{(2x)^2 + (-5y)^2 + 6^2 - (2x)(-5y) - (-5y)(6) - (6)(2x)\}$$

$$= (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

Thus the factors of  $8x^3 - 125y^3 + 180xy + 216$  is

$$(2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

## 14. Question

Factorize each of the following expressions:

$$2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$$

**Answer**

Given,

$$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$$

This can be written in form ,

$$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + c^3 - 3(\sqrt{2}a)(2\sqrt{2}b)(c)$$

And ,  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$  By formula

Hence ,

$$= \frac{2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc}{(\sqrt{2}a)(2\sqrt{2}b - (2\sqrt{2}b)(c) - (c)(\sqrt{2}a))} = \{\sqrt{2}a + 2\sqrt{2}b + c\} \{(\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 -$$

$$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ca)$$

Thus the factors of  $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$  is  $(\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ca)$

### 15. Question

Find the value of  $x^3 + y^3 - 12xy + 64$ , when  $x + y = -4$ .

**Answer**

Given,

$$= x^3 + y^3 - 12xy + 64$$

$$= x + y = -4 \text{ Given}$$

$$= x + y + 4 = 0$$

This can be written in form ,

$$= x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 4^3 - 3(x)(y)(4)$$

And ,  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$  By formula

$$= x^3 + y^3 - 12xy + 64 = (x + y + 4)\{(x)^2 + (y)^2 + 4^2 - xy - 4y - 4x\}$$

$$= 0 \times (x^2 + y^2 + 16 - xy - 4y - 4x)$$

$$= 0$$

### 16. Question

Multiply:

(i)  $x^2 + y^2 + z^2 - xy + xz + yz$  by  $x + y - z$

(ii)  $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$  by  $x - 2y - z$

(iii)  $x^2 + 4y^2 + 2xy - 3x + 6y + 9$  by  $x - 2y + 3$

(iv)  $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$  by  $3x - 5y + 4$

**Answer**

(i) Given,

$$= x^2 + y^2 + z^2 - xy + xz + yz$$

Multiply the above expression by  $(x + y - z)$

$$\begin{aligned}
&= (x + y - z)(x^2 + y^2 + z^2 - xy + xz + yz) \\
&= \{x + y + (-z)\}\{(x)^2 + (y)^2 + (-z)^2 - xy + yz + zx\} \\
&= x^3 + y^3 + (-z)^3 - 3 \cdot x \cdot y \cdot (-z) \\
&= x^3 + y^3 - z^3 + 3xyz
\end{aligned}$$

(ii) Given,

$$= x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$$

Multiply above expression by  $(x-2y-z)$

Then ,

$$\begin{aligned}
&= (x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz) \\
&= \{x + (-2y) + (-z)\}\{x^2 + 4y^2 + z^2 + 2xy - 2yz + zx\}
\end{aligned}$$

By formula...

$$= x^3 - 8y^3 - z^3 - 6xyz$$

iii) we have

$$= x^2 + 4y^2 + 2xy - 3x + 6y + 9$$

(iii) Given,

$$= x^2 + 4y^2 + 2xy - 3x + 6y + 9$$

Multiply above equation by  $(x - 2y + 3)$

$$\begin{aligned}
&= (x - 2y + 3)(x^2 + 4y^2 + 2xy - 3x + 6y + 9) \\
&= \{x - 2y + 3\}\{x^2 + 4y^2 + 9 + 2xy + 6y - 3x\} \\
&= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula} \\
&= x^3 - 8y^3 + 27 + 18xy
\end{aligned}$$

(iv) Given,

$$= 9x^2 + 25y^2 + 15xy + 12x - 20y + 16$$

Multiply above equation by  $(3x - 5y + 4)$

We got,

$$\begin{aligned}
&= (3x - 5y + 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16) = \{3x + (-5y) + 4\}\{(3x)^2 + (-5y)^2 + 4^2 - 3x(-5y) - (-5y)(4) - (4)(3x)\} \\
&= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula} \\
&= (3x)^3 + (-5y)^3 + (4)^3 - 3 \cdot 3x \cdot (-5y) \cdot (4) \\
&= 27x^3 - 125y^3 + 64 + 180xy
\end{aligned}$$

## CCE - Formative Assessment

### 1. Question

Factorize:  $x^4 + x^2 + 25$ .

### Answer

We have,

First we rewrite the question,

$$\begin{aligned}
 x^4 + x^2 + 25 &= (x^2)^2 + 2 \cdot x^2 \cdot 5 + 5^2 - 9x^2 \\
 &= \{(x^2)^2 + 2 \cdot x^2 \cdot 5 + 5^2\} - (3x)^2 \quad [\text{By using } a^2 + 2ab + b^2 = (a + b)^2] \\
 &= \{x^2 + 5\}^2 - (3x)^2 \quad [\text{By using } a^2 - b^2 = (a + b)(a - b)] \\
 &= (x^2 + 5 + 3x)(x^2 + 5 - 3x)
 \end{aligned}$$

Thus, the factors of  $x^4 + x^2 + 25$  are  $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$ .

## 2. Question

Factorize:  $x^2 - 1 - 2a - a^2$ .

### Answer

We have,

$$= x^2 - 1 - 2a - a^2$$

Taking -1 as common from last three terms

$$= x^2 - (1 + 2a + a^2)$$

$$= x^2 - (1^2 + 2 \times 1 \times a + a^2)$$

$$= x^2 - (1 + a)^2 [a^2 - b^2 = (a + b)(a - b)]$$

$$= (x + 1 + a)(x - 1 - a)$$

Thus the factors of  $x^2 - 1 - 2a - a^2$  are  $(x + 1 + a)(x - 1 - a)$ .

## 3. Question

If  $a + b + c = 0$ , then write the value of  $a^3 + b^3 + c^3$ .

### Answer

We have,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

When  $(a + b + c) = 0$  Given

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 - 3abc = 0$$

$$= a^3 + b^3 + c^3 = 3abc.$$

## 4. Question

If  $a^2 + b^2 + c^2 = 20$ , and  $a + b + c = 0$ , find  $ab + bc + ca$ .

### Answer

We have,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \text{ By formula}$$

$$= a^2 + b^2 + c^2 = 20 \text{ Given}$$

$$= (a + b + c) = 0 \text{ Given}$$

Then,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$= (0)^2 = 20 + 2(ab + bc + ca)$$

$$= 2(ab + bc + ca) = -20$$

$$= (ab + bc + ca) = -10$$

### 5. Question

If  $a + b + c = 9$  and  $ab + bc + ca = 40$ , find  $a^2 + b^2 + c^2$ .

### Answer

We have ,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \text{ By formula}$$

$$= a + b + c = 9 \text{ Given}$$

$$= ab + bc + ca = 40 \text{ Given}$$

Then,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$= (9)^2 = a^2 + b^2 + c^2 + 2 \times 40$$

$$= a^2 + b^2 + c^2 = 81 - 80$$

$$= a^2 + b^2 + c^2 = 1$$

### 6. Question

If  $a^2 + b^2 + c^2 = 250$  and  $ab + bc + ca = 3$ , find  $a + b + c$ .

### Answer

We have,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \text{ By formula}$$

$$= a^2 + b^2 + c^2 = 250 \text{ Given}$$

$$= ab + bc + ca = 3 \text{ Given}$$

Then,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$= (a + b + c)^2 = 250 + 2 \times 3$$

$$= (a + b + c)^2 = 256$$

$$= (a + b + c) = \sqrt{256} = \pm 16.$$

### 7. Question

Write the value of:  $25^3 - 75^3 + 50^3$ .

### Answer

We have,

$$= 25^3 - 75^3 + 50^3$$

$$\text{Let } a = 25, b = -75, c = 50,$$

Then the expression becomes as ,

$$= 25^3 - 75^3 + 50^3 = a^3 + b^3 + c^3$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$\text{Here, } (a + b + c) = 25 + (-75) + 50 = 0$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= 25^3 - 75^3 + 50^3 = 3 \times 25 \times -75 \times 50$$

$$= 25^3 - 75^3 + 50^3 = -281250.$$

### 8. Question

Write the value of:  $48^3 - 30^3 - 18^3$ .

### Answer

We have,

$$= 48^3 - 30^3 - 18^3 \text{ Given}$$

$$\text{Let } a = 48, b = -30, c = -18$$

Then the expression becomes,

$$= 48^3 - 30^3 - 18^3 = a^3 + b^3 + c^3$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

Here,

$$= (a + b + c) = 48 + (-30) + (-18) = 0$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= 48^3 - 30^3 - 18^3 = 3 \times 48 \times -30 \times -18$$

$$= 48^3 - 30^3 - 18^3 = 77760.$$

### 9. Question

Write the value of:  $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$

### Answer

We have,

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 \text{ Given}$$

Let  $a = \frac{1}{2}, b = \frac{1}{3}, c = -\frac{5}{6}$ , then the expression becomes

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = a^3 + b^3 + c^3$$

Here,

$$= a + b + c = \frac{1}{2} + \frac{1}{3} - \frac{5}{6} = 0$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = 3 \times \frac{1}{2} \times \frac{1}{3} \times -\frac{5}{6}$$

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = -\frac{5}{12}$$

### 10. Question

Write the value of:  $30^3 + 20^3 - 50^3$ .

#### Answer

We have ,

$$= 30^3 + 20^3 - 50^3 \text{ given}$$

Let  $a = 30, b = 20, c = -50$ , then the expression becomes,

$$= 30^3 + 20^3 - 50^3 = a^3 + b^3 + c^3$$

Here ,

$$= a + b + c = 30 + 20 - 50 = 0$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$$

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= 30^3 + 20^3 - 50^3 = 3 \times 30 \times 20 \times -50$$

$$= 30^3 + 20^3 - 50^3 = -90000$$

### 1. Question

The factors of  $a^2 - 1 - 2x - x^2$  are

- A.  $(a-x+1)(a-x-1)$
- B.  $(a+x-1)(a-x+1)$
- C.  $(a+x+1)(a-x-1)$
- D. none of these

#### Answer

We have,

$$= a^2 - 1 - 2x - x^2$$

$$= a^2 - (1 + 2x + x^2) = a^2 - (1 + x)^2$$

$$= (a - 1 - x)(a + 1 + x) [a^2 - b^2 = (a + b)(a - b)]$$

Thus, the factors of  $a^2 - 1 - 2x - x^2$  are  $(a - 1 - x)(a + 1 + x)$ .

### 2. Question

The factors of  $x^4 + x^2 = 25$  are

- A.  $(x^2 + 3x + 5)(x^2 - 3x + 5)$

B.  $(x^2+3x+5)(x^2+3x-5)$

C.  $(x^2+x+5)(x^2-x+5)$

D. none of these

**Answer**

We have,

$$= x^4 + x^2 + 25$$

Adding and subtracting  $9x^2$  in the equation

$$= x^4 + 25 + x^2 + 9x^2 - 9x^2$$

$$= x^4 + 25 + 10x^2 - 9x^2 = (x^2 + 5)^2 - (3x)^2$$

$$= (x^2 + 5 - 3x)(x^2 + 5 + 3x)$$

Thus the factors of  $(x^4+x^2+25)$  are  $(x^2 + 5 - 3x)(x^2 + 5 + 3x)$ .

**3. Question**

The factors of  $x^2+4y^2+4y-4xy-2x-8$  are

A.  $(x-2y-4)(x-2y+2)$

B.  $(x-y+2)(x-4y-4)$

C.  $(x+2y-4)(x+2y+2)$

D. none of these

**Answer**

We have ,

$$= x^2 + 4y^2 + 4y - 4xy - 2x - 8$$

$$= x^2 + 4y^2 - 4xy + 4y - 2x - 8 = (x - 2y)^2 - 2(x - 2y) - 8$$

Let  $a = (x - 2y)$  , then the expression becomes ,

$$= x^2 + 4y^2 + 4y - 4xy - 2x - 8 = a^2 - 2a - 8$$

$$= a^2 - 4a + 2a - 8$$

$$= a(a-4) + 2(a - 4)$$

$$=(a - 4)(a + 2)$$

Put  $a = (x - 2y)$

$$= x^2 + 4y^2 + 4y - 4xy - 2x - 8 = (x - 2y - 4)(x - 2y + 2)$$

Thus the factors of  $x^2 + 4y^2 + 4y - 4xy - 2x - 8$  are  $(x - 2y - 4)(x - 2y + 2)$

**4. Question**

The factors of  $x^3-x^2y-xy^2+y^3$  are

A.  $(x+y)(x^2-xy+y^2)$

B.  $(x+y)(x^2+xy+y^2)$

C.  $(x+y)^2(x-y)$

D.  $(x-y)^2(x+y)$

**Answer**

We have,

$$\begin{aligned}
 &= x^3 - x^2y - xy^2 + y^3 \\
 &= x^3 + y^3 - x^2y - xy^2 = (x + y)^3 - 3x^2y - 3xy^2 - x^2y - xy^2 \\
 \text{As } x^3 + y^3 &= (x + y)^3 - 3x^2y - 3xy^2 \\
 &= (x + y)^3 - 4xy(x + y) \\
 &= (x + y) \{(x + y)^2 - 4xy\} \\
 &= (x + y)(x^2 + y^2 + 2xy - 2xy) \\
 &= (x + y)(x^2 + y^2 - 2xy) = (x + y)(x - y)^2
 \end{aligned}$$

**5. Question**

The factors of  $x^3 - 1 + y^3 + 3xy$  are

- A.  $(x - 1 + y)(x^2 + 1 + y^2 + x + y - xy)$
- B.  $(x + y + 1)(x^2 + y^2 + 1 - xy - x - y)$
- C.  $(x - 1 + y)(x^2 - 1 - y^2 + x + y + xy)$
- D.  $3(x + y - 1)(x^2 + y^2 - 1)$

**Answer**

We have ,

$$\begin{aligned}
 &= x^3 - 1 + y^3 + 3xy \\
 &= (x)^3 + (-1)^3 + (y)^3 - 3(x)(-1)(y) \\
 &= (x - 1 + y)(x^2 + 1 + y^2 + x + y - xy) \{(a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)\}
 \end{aligned}$$

Thus the factors of  $x^3 - 1 + y^3 + 3xy$  are  $(x - 1 + y)(x^2 + 1 + y^2 + x + y - xy)$

**6. Question**

The factors of  $8a^2 + b^3 - 6ab + 1$  are

- A.  $(2a + b - 1)(4a^2 + b^2 + 1 - 3ab - 2a)$
- B.  $(2a - b + 1)(4a^2 + b^2 - 4ab + 1 - 2a + b)$
- C.  $(2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$
- D.  $(2a - 1 + b)(4a^2 + 1 - 4a - b - 2ab)$

**Answer**

We have ,

$$\begin{aligned}
 &= 8a^3 + b^3 - 6ab + 1 \\
 &= (2a)^3 + (b)^3 - 3(2a)(b)(1) + (1)^3 \text{ try to make } (a^3 + b^3 + c^3 - 3abc) \\
 &= (2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)
 \end{aligned}$$

Thus the factors of  $8a^3 + b^3 - 6ab + 1$  are  $(2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$  .

**7. Question**

$(x + y)^3 - (x - y)^3$  can be factorized as:

- A.  $2y(3x^2+y^2)$
- B.  $2x(3x^2+y^2)$
- C.  $2y(3y^2+x^2)$
- D.  $2x(x^2+3y^2)$

**Answer**

We have ,

$$=(x+y)^3 - (x-y)^3$$

Applying formulas,

$$=x^3 + y^3 + 3x^2y + 3xy^2 - (x^3 - y^3 - 3x^2y + 3xy^2)$$

$$=x^3 + y^3 + 3x^2y + 3xy^2 - x^3 + y^3 + 3x^2y - 3xy^2$$

$$=2y^3 + 6x^2y$$

$$=2y(y^2 + 3x^2)$$

Thus the factors of  $(x+y)^3 - (x-y)^3$  are  $2y(y^2 + 3x^2)$

**8. Question**

The factors of  $x^2-7x+6$  are

- A.  $x(x-6)(x-1)$
- B.  $(x^2-6)(x-1)$
- C.  $(x+1)(x+2)(x-3)$
- D.  $(x-1)(x+3)(x-2)$

**Answer**

We have,

$$=x^3 - 7x - 6$$

Adding and subtracting 1 in the equation

$$=x^3 - 7x - 6 - 1 + 1 = x^3 + 1^3 - 7x - 7$$

$$= (x+1)(x^2 + 1 + x) - 7(x+1)$$

$$=(x+1)(x^2 - 6 + x)$$

$$=(x+1)(x^3 - 3x + 2x + x) = (x+1)(x+2)(x-3)$$

Thus the factors of  $x^3 - 7x - 6$  are  $(x+1)(x+2)(x-3)$ .

**9. Question**

The expression  $(a-b)^3+(b-c)^3+(c-a)^3$  can be factorized as:

- A.  $(a-b)(b-c)(c-a)$
- B.  $3(a-b)(b-c)(c-a)$
- C.  $-3(a-b)(b-c)(c-a)$
- D.  $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

**Answer**

We have,



$$\text{Let } x = (a - b), y = (b - c), z(c - a)$$

$$\text{So, } x + y + z = a - b + b - c + c - a$$

If  $a+b+c = 0$ , then,

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a).$$

### 10. Question

The expression  $x^4+4$  can be factorized as

A.  $(x^2+2x+2)(x^2-2x+2)$

B.  $(x^2+2x+2)(x^2+2x-2)$

C.  $(x^2-2x-2)(x^2-2x+2)$

D.  $(x^2+2)(x^2-2)$

### Answer

We have ,

$$= x^4 + 4$$

$$= x^4 + 4x^2 + 4 - 4x^2$$

$$= (x^2 + 2)^2 - 4x^2$$

$$= (x^2 + 2)^2 - (2x)^2 [x^2 - y^2 = (x - y)(x + y)]$$

$$= (x^2 + 2 + 2x)(x^2 + 2 - 2x)$$

### 11. Question

If  $3x=a+b+c$ , then the value of  $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$  is

A.  $a+b+c$

B.  $(a-b)(b-c)(c-a)$

D. 0

D. none of these

### Answer

We have,

$$= 3x = a+b+c$$

$$\text{Let } a_1 = x - a, b_1 = x - b, c_1 = x - c$$

$$\text{So, } a_1 + b_1 + c_1 = x - a + x - b + x - c = 3x - (a + b + c)$$

$$= 3x - 3x = 0 [ a + b + c = 3x ] \text{ given}$$

$$= (x - a)^3 + (x - b)^3 + (x - c)^3 = 3(x - a)(x - b)(x - c)$$

$$\text{Now, } (x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c) = 0.$$

### 12. Question

If  $(x+y)^3-(x-y)^3-6y(x^2-y^2)=ky^2$  then  $k=$

A. 1

B. 2

C. 4

D. 8

**Answer**

We have,

$$=(x+y)^3 - (x-y)^3 - 6y(x^2 - y^2) = ky^3$$

$$= (x+y-x+y)^3 + 3(x+y)(x-y)(x+y-x+y) - 6y(x^2 - y^2) = ky^3$$

$$= 2y^3 + 6y(x^2 - y^2) - 6y(x^2 - y^2) = ky^3$$

$$= 8y^3 = ky^3$$

$$= k = 8 .$$

**13. Question**

If  $x^3 - 3x^2 + 3x - 7 = (x+1)(ax^2 + bx + c)$ , then  $a+b+c=$

A. 4

B. 12

C. -10

D. 3

**Answer**

We have,

$$= x^3 - 3x^2 + 3x - 7 = (x+1)(ax^2 + bx + c)$$

$$= x^3 - 3x^2 + 3x - 7 = ax^3 + bx^2 + cx + ax^2 + bx + c$$

$$= x^3 - 3x^2 + 3x - 7 = ax^3 + (a+b)x^2 + (b+c)x + c$$

By comparing both sides ,

$$= a = 1$$

$$= a + b = -3$$

$$= b + c = 3$$

$$= c = -7$$

Thus ,  $a + (b + c) = 1 + 3 = 4$ .

**14. Question**

The value of  $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$  is

A. 2

B. 3

C. 2.327

D. 2.273

**Answer**

We have,

$$= \frac{2.3^3 - 0.027}{2.3^2 + 0.69 + 0.09}$$

$$= \frac{2.3^2 - 0.027}{2.3^2 + 0.69 + 0.09} = \frac{2.3^2 - 0.3^2}{2.3^2 + 0.3^2 + 2.3 \times 0.3} [a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$

Hence,

$$= \frac{(a-b)(a^2 + b^2 + ab)}{a^2 + b^2 + ab} = a - b$$

$$= (2.3 - 0.3) = 2.$$

### 15. Question

The value of  $\frac{(0.013)^3 + (0.007)^3}{(0.013)^2 - 0.013 \times 0.013 \times 0.007 + (0.007)^2}$  is

- A. 0.006
- B. 0.02
- C. 0.0091
- D. 0.00185

### Answer

We have,

$$= \frac{0.013^3 + 0.007^3}{0.013^2 - 0.013 \times 0.007 + 0.007^2} [a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$$

Hence,

$$= \frac{(a+b)(a^2 + b^2 - ab)}{a^2 + b^2 - ab} = a + b$$

$$= 0.013 + 0.007 = 0.020$$

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