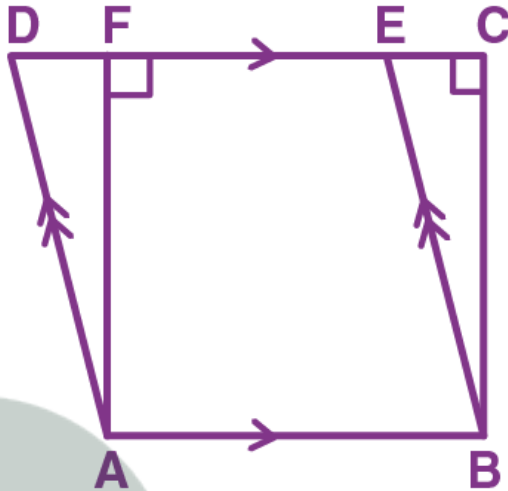


Exercise 16(A)

1. In the given figure, if area of triangle ADE is 60 cm^2 , state, given reason, the area of:
- Parallelogram ABED
 - Rectangle ABCF
 - Triangle ABE



Solution:

(i) As $\triangle ADE$ and parallelogram ABED are on the same base AB and between the same parallels $DE \parallel AB$, the area of the $\triangle ADE$ will be half the area of parallelogram ABED.

So,

$$\begin{aligned} \text{Area of parallelogram ABED} &= 2 \times \text{Area of } \triangle ADE \\ &= 2 \times 60 \text{ cm}^2 \\ &= 120 \text{ cm}^2 \end{aligned}$$

(ii) Area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e., between the same parallels

So,

$$\text{Area of rectangle ABCF} = \text{Area of parallelogram ABED} = 120 \text{ cm}^2$$

(iii) We know that, area of triangles on the same base and between same parallel lines are equal

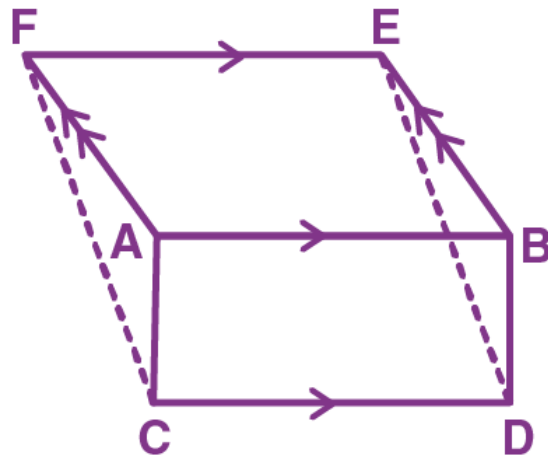
So,

$$\text{Area of } \triangle ABE = \text{Area of } \triangle ADE = 60 \text{ cm}^2$$

2. The given figure shows a rectangle ABDC and a parallelogram ABEF; drawn on opposite sides of AB. Prove that:

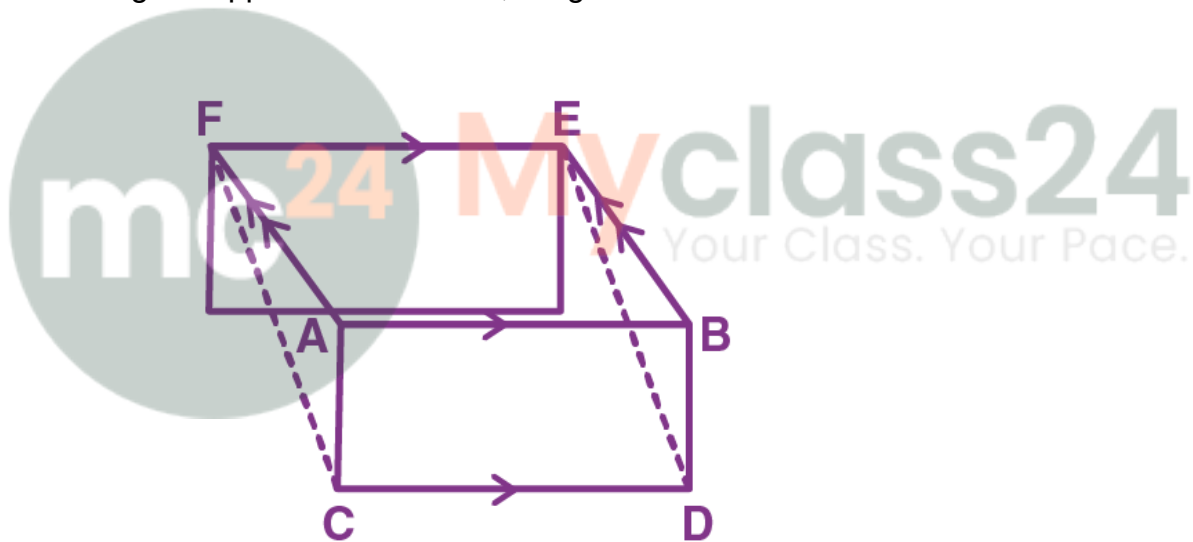
(i) Quadrilateral CDEF is a parallelogram.

(ii) Area of quad. CDEF = Area of rect. ABDC + Area of || gm. ABEF.



Solution:

After drawing the opposite sides of AB, we get



It's seen from the figure that $CD \parallel FE$. Therefore, FC must be parallel to DE .

Thus, it is proved that the quadrilateral $CDEF$ is a parallelogram.

We know that,

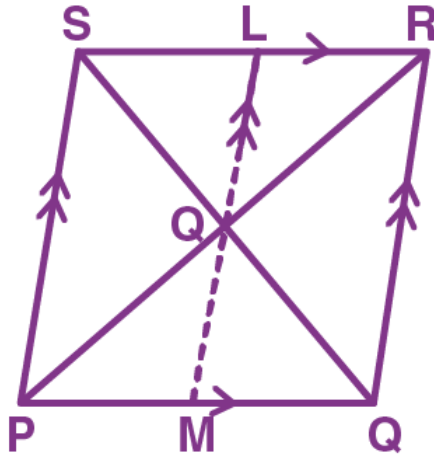
Area of parallelograms on same base and between same parallel lines is always equal. Also, area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e., between same parallel lines.

So,

Area of $CDEF$ = Area of $ABDC$ + Area of $ABEF$

- Hence Proved.

3. In the given figure, diagonals PR and QS of the parallelogram $PQRS$ intersect at point O and LM is parallel to PS . Show that:



(i) $2 \text{ Area } (\triangle POS) = \text{Area } (\parallel\text{gm PMLS})$

(ii) $\text{Area } (\triangle POS) + \text{Area } (\triangle QOR) = \frac{1}{2} \text{Area } (\parallel\text{gm PQRS})$

(iii) $\text{Area } (\triangle POS) + \text{Area } (\triangle QOR) = \text{Area } (\triangle POQ) + \text{Area } (\triangle SOR)$.

Solution:

(i) As $\triangle POS$ and parallelogram $PMLS$ are on the same base PS and between the same parallels i.e., $SP \parallel LM$

Since O is the center of LM and ratio of areas of triangles with same vertex and bases along the same line is equal to ratio of their respective bases

Also, the area of the parallelogram is twice the area of the triangle if they lie on the same base and in between the same parallels

Thus, $2 \text{ Area } (\triangle PSO) = \text{Area}(PMLS)$

(ii) Taking LHS, we have

LM is parallel to PS and PS is parallel to RQ , therefore, LM is parallel to RQ

And, since $\triangle POS$ lie on the base PS and in between the parallels PS and LM , we have

$\text{Area } (\triangle POS) = \frac{1}{2} \text{Area } (PSLM)$

Also, since $\triangle QOR$ lie on the base QR and in between the parallels LM and RQ , we have

$\text{Area } (\triangle QOR) = \frac{1}{2} \text{Area } (LMQR)$

Now,

$$\begin{aligned} \text{Area}(\triangle POS) + \text{Area}(\triangle QOR) &= \frac{1}{2} \text{Area}(PSLM) + \frac{1}{2} \text{Area}(LMQR) \\ &= \frac{1}{2} [\text{Area}(PSLM) + \text{Area}(LMQR)] \\ &= \frac{1}{2} \text{Area}(PQRS) \end{aligned}$$

(iii) We know that, the diagonals in a parallelogram bisect each other.

So, $OS = OQ$

In $\triangle PQS$, as $OS = OQ$

OP is the median of the $\triangle PQS$.

We know that median of a triangle divides it into two triangles of equal area.

Therefore,

$\text{Area } (\triangle POS) = \text{Area } (\triangle POQ) \dots (1)$

Similarly, as OR is the median of the triangle QRS , we have

$$\text{Area } (\triangle QOR) = \text{Area } (\triangle SOR) \dots (2)$$

Now, adding (1) and (2) we get

$$\text{Area } (\triangle POS) + \text{area } (\triangle QOR) = \text{Area } (\triangle POQ) + \text{Area } (\triangle SOR)$$

- Hence Proved.

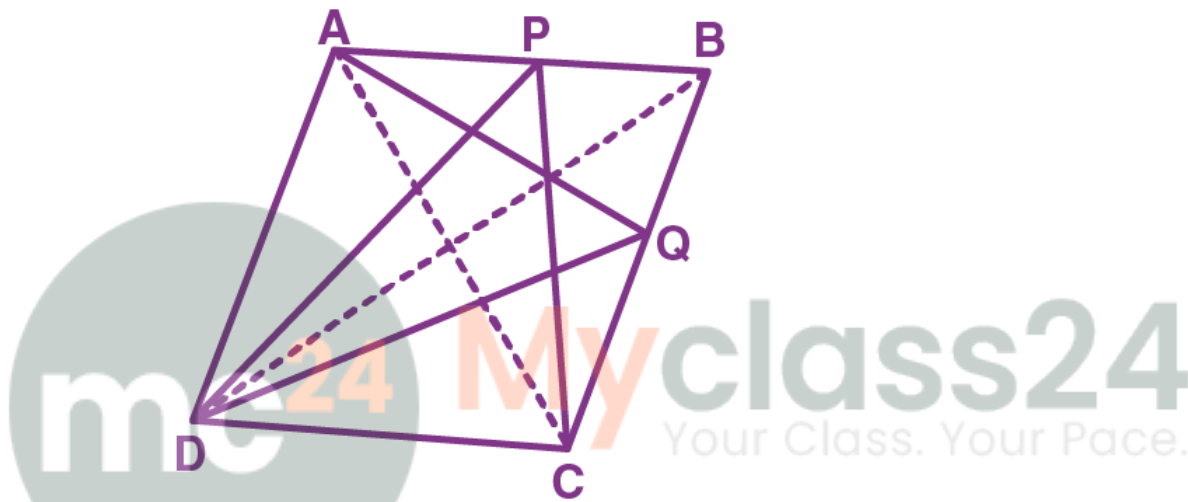
4. In parallelogram ABCD, P is a point on side AB and Q is a point on side BC.

Prove that:

(i) $\triangle CPD$ and $\triangle AQD$ are equal in area.

(ii) $\text{Area } (\triangle AQD) = \text{Area } (\triangle APD) + \text{Area } (\triangle CPB)$

Solution:



Given, ABCD is a parallelogram.

P and Q are any points on the sides AB and BC respectively.

Join diagonals AC and BD.

Proof:

(i) As triangles with same base and between same set of parallel lines have equal areas

$$\text{Area } (\triangle CPD) = \text{Area } (\triangle BCD) \dots\dots (1)$$

Again, diagonals of the parallelogram bisect area in two equal parts

$$\text{Area } (\triangle BCD) = \frac{1}{2} \text{ area of parallelogram ABCD} \dots\dots (2)$$

From (1) and (2)

$$\text{Area } (\triangle CPD) = \frac{1}{2} \text{ Area } (ABCD) \dots\dots (3)$$

Similarly,

$$\text{Area } (\triangle AQD) = \text{Area } (\triangle ABD) = \frac{1}{2} \text{ Area } (ABCD) \dots\dots (4)$$

From (3) and (4), we get

$$\text{Area } (\triangle CPD) = \text{Area } (\triangle AQD)$$

- Hence proved.

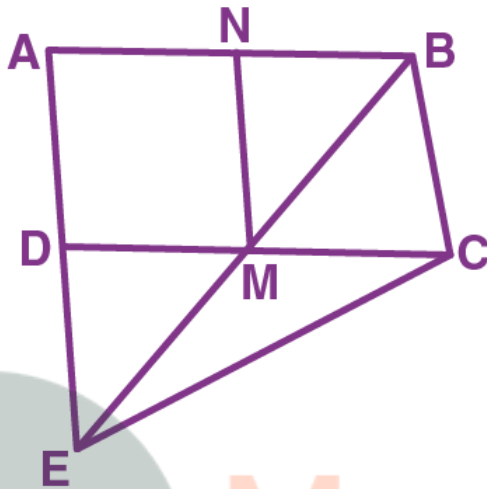
(ii) We know that, area of triangles on the same base and between same parallel lines are equal

So,

Area of $\triangle AQD = \text{Area of } \triangle ACD = \text{Area of } \triangle PDC = \text{Area of } \triangle BDC = \text{Area of } \triangle ABC = \text{Area of } \triangle APD + \text{Area of } \triangle BPC$

- Hence Proved

5. In the given figure, M and N are the mid-points of the sides DC and AB respectively of the parallelogram ABCD.



If the area of parallelogram ABCD is 48 cm^2

(i) State the area of the triangle BEC.

(ii) Name the parallelogram which is equal in area to the triangle BEC.

Solution:

(i) As $\triangle BEC$ and parallelogram ABCD are on the same base BC and between the same parallels i.e., $BC \parallel AD$.

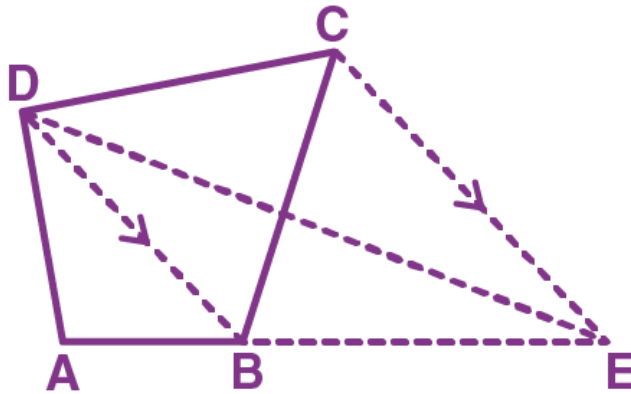
$$\text{Area } (\triangle BEC) = \frac{1}{2} \times \text{Area } (ABCD) = \frac{1}{2} \times 48 = 24 \text{ cm}^2$$

$$\begin{aligned} \text{(ii) Area } (ANMD) &= \text{Area } (BNMC) \\ &= \frac{1}{2} \times \text{Area } (ABCD) \\ &= \frac{1}{2} \times 2 \times \text{Area } (\triangle BEC) \\ &= \text{Area } (\triangle BEC) \end{aligned}$$

Therefore, the parallelograms ANMD and NBCM have areas equal to $\triangle BEC$.

6. In the following figure, CE is drawn parallel to diagonals DB of the quadrilateral ABCD which meets AB produced at point E.

Prove that $\triangle ADE$ and quadrilateral ABCD are equal in area.



Solution:

As, $\triangle DCB$ and $\triangle DEB$ are on the same base DB and between the same parallels i.e., $DB \parallel CE$,
 We have,
 $\text{Area}(\triangle DCB) = \text{Area}(\triangle DEB)$
 $\text{Area}(\triangle DCB + \triangle ADB) = \text{Area}(\triangle DEB + \triangle ADB)$
 $\text{Area}(ABCD) = \text{Area}(\triangle ADE)$
 - Hence proved

7. $ABCD$ is a parallelogram a line through A cuts DC at point P and BC produced at Q .
 Prove that triangle BCP is equal in area to triangle DPQ .



Solution:

Its seen that, $\triangle APB$ and parallelogram $ABCD$ are on the same base AB and between the same parallel lines AB and CD .

So,

$$\text{Ar}(\triangle APB) = \frac{1}{2} \text{Ar}(\text{||gm } ABCD) \dots (i)$$

Now,

$\triangle ADQ$ and parallelogram $ABCD$ are on the same base AD and between the same parallel lines

AD and BQ.

So,

$$\text{Ar}(\triangle ADQ) = \frac{1}{2} \text{Ar}(\text{||gm ABCD}) \dots \text{(ii)}$$

On adding equation (i) and (ii), we get

$$\text{Ar}(\triangle APB) + \text{Ar}(\triangle ADQ) = \frac{1}{2} \text{Ar}(\text{||gm ABCD}) + \frac{1}{2} \text{Ar}(\text{||gm ABCD}) = \text{Ar}(\text{||gm ABCD})$$

$$\text{Ar}(\text{quad. ADQB}) - \text{Ar}(\triangle BPQ) = \text{Ar}(\text{||gm ABCD})$$

$$\text{Ar}(\text{quad. ADQB}) - \text{Ar}(\triangle BPQ) = \text{Ar}(\text{quad. ADQB}) - \text{Ar}(\triangle DCQ)$$

$$\text{Ar}(\triangle BPQ) = \text{Ar}(\triangle DCQ)$$

Subtracting $\text{Ar}(\triangle PCQ)$ from both the sides, we get

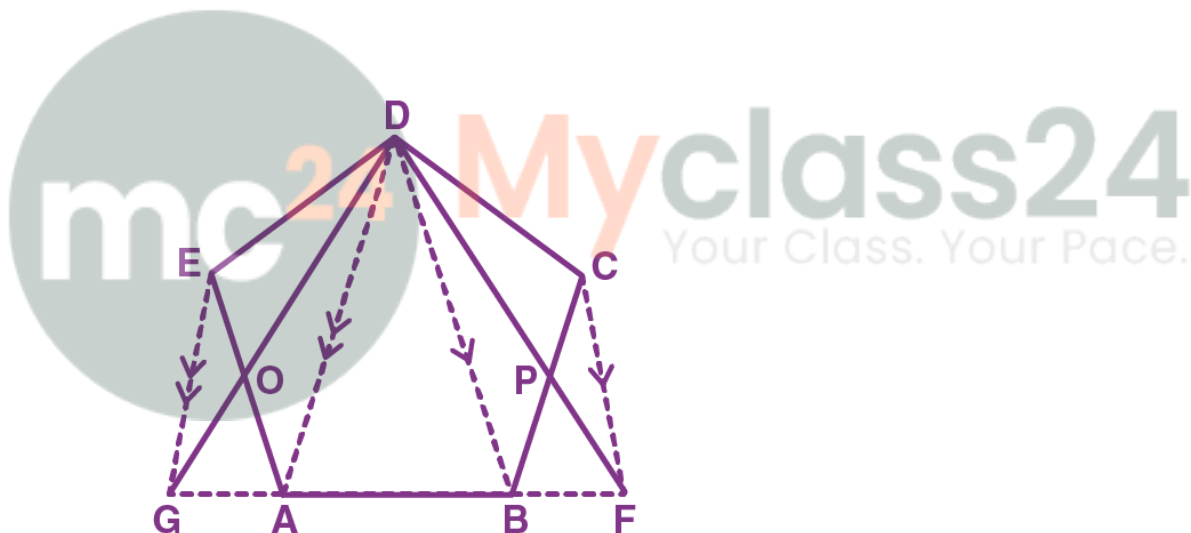
$$\text{Ar}(\triangle BPQ) - \text{Ar}(\triangle PCQ) = \text{Ar}(\triangle DCQ) - \text{Ar}(\triangle PCQ)$$

$$\text{Ar}(\triangle BCP) = \text{Ar}(\triangle DPQ)$$

- Hence proved.

8. The given figure shows a pentagon ABCDE. EG drawn parallel to DA meets BA produced at G and CF draw parallel to DB meets AB produced at F. Prove that the area of pentagon ABCDE is equal to the area of triangle GDF.

Solution:



It's seen that triangles EDG and EGA are on the same base EG and between the same parallel lines EG and DA, so

$$\text{Ar}(\triangle EDG) = \text{Ar}(\triangle EGA)$$

On subtracting $\triangle EOG$ from both sides, we have

$$\text{Ar}(\triangle EDG) - \text{Ar}(\triangle EOG) = \text{Ar}(\triangle EGA) - \text{Ar}(\triangle EOG)$$

$$\text{Ar}(\triangle EOD) = \text{Ar}(\triangle GOA) \dots \text{(i)}$$

Similarly,

$$\text{Ar}(\triangle DPC) = \text{Ar}(\triangle BPF) \dots \text{(ii)}$$

Now,

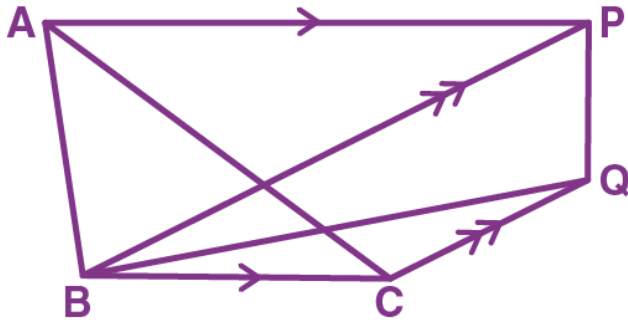
$$\text{Ar}(\text{GDF}) = \text{Ar}(\text{GOA}) + \text{Ar}(\text{BPF}) + \text{Ar}(\text{pen. ABPDO})$$

$$= \text{Ar}(\text{EOD}) + \text{Ar}(\text{DPC}) + \text{Ar}(\text{pen. ABPDO})$$

$$= \text{Ar}(\text{pen. ABCDE})$$

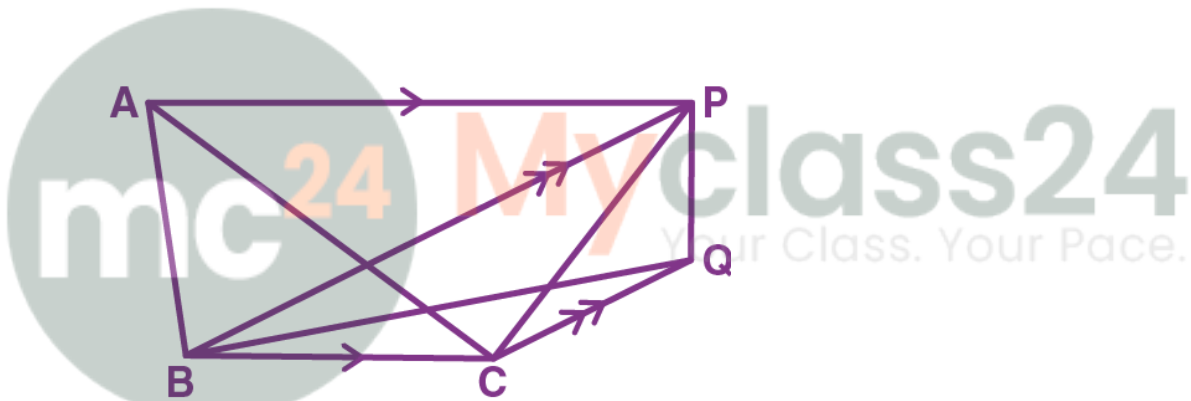
- Hence proved

9. In the given figure, AP is parallel to BC, BP is parallel to CQ. Prove that the area of triangles ABC and BQP are equal.



Solution:

On joining PC, we see that



$\triangle ABC$ and $\triangle BPC$ are on the same base BC and between the same parallel lines AP and BC.

$$\text{Ar}(\triangle ABC) = \text{Ar}(\triangle BPC) \dots (i)$$

And, $\triangle BPC$ and $\triangle BQP$ are on the same base BP and between the same parallel lines BP and CQ.

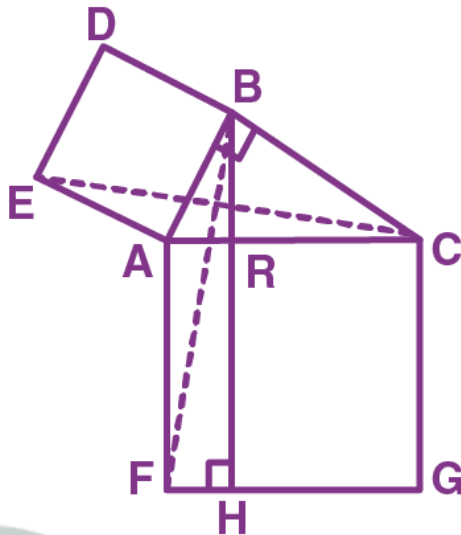
$$\text{Ar}(\triangle BPC) = \text{Ar}(\triangle BQP) \dots (ii)$$

From (i) and (ii), we get

$$\text{Ar}(\triangle ABC) = \text{Ar}(\triangle BQP)$$

- Hence proved.

10. In the figure given alongside, squares ABDE and AFGC are drawn on the side AB and the hypotenuse AC of the right triangle ABC.



If BH is perpendicular to FG prove that:

(i) $\triangle EAC \cong \triangle BAF$.

(ii) Area of the square ABDE = Area of the rectangle ARHF.

Solution:

(i) From figure,

$$\begin{aligned} \angle EAC &= \angle EAB + \angle BAC \\ &= 90^\circ + \angle BAC \dots (i) \end{aligned}$$

$$\begin{aligned} \angle BAF &= \angle FAC + \angle BAC \\ &= 90^\circ + \angle BAC \dots (ii) \end{aligned}$$

From (i) and (ii), we get

$$\angle EAC = \angle BAF$$

In $\triangle EAC$ and $\triangle BAF$, we have,

$$EA = AB$$

$$\angle EAC = \angle BAF \text{ (proved above)}$$

$$AC = AF$$

Therefore, $\triangle EAC \cong \triangle BAF$ (SAS axiom of congruency)

(ii) As $\triangle ABC$ is a right triangle, we have

$$AC^2 = AB^2 + BC^2 \quad \text{[Using Pythagoras theorem in } \triangle ABC]$$

$$AB^2 = AC^2 - BC^2$$

$$AB^2 = (AB^2 + BC^2) - [BR^2 + RC^2] \quad \text{[Since } AC^2 = AR^2 + RC^2 \text{ and using Pythagoras Theorem in } \triangle BRC]$$

$$AB^2 = AR^2 + 2AR \times RC + RC^2 - (BR^2 + RC^2) \quad \text{[Using the identity]}$$

$$AB^2 = AR^2 + 2AR \times RC + RC^2 - (AB^2 - AR^2 + RC^2) \quad \text{[Using Pythagoras Theorem in } \triangle ABR]$$

$$2AB^2 = 2AR^2 + 2AR \times RC$$

$$AB^2 = AR (AR + RC)$$

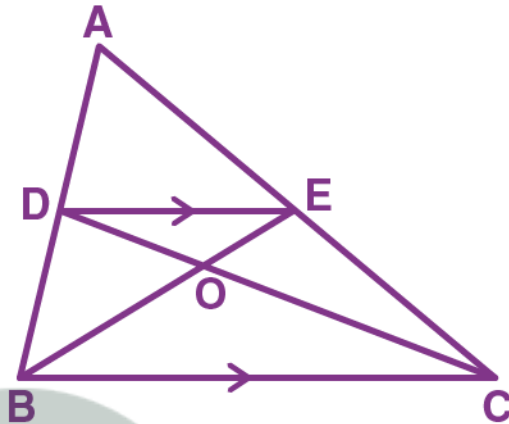
$$AB^2 = AR \times AF$$

$$\therefore \text{Area}(\triangle ABDE) = \text{Area}(\text{rectangle } ARHF)$$

11. In the following figure, DE is parallel to BC. Show that:

(i) $\text{Area}(\triangle ADC) = \text{Area}(\triangle AEB)$.

(ii) $\text{Area}(\triangle BOD) = \text{Area}(\triangle COE)$.



Solution:

(i) In $\triangle ABC$, D is midpoint of AB and E is the midpoint of AC

And, DE is parallel to BC

So, by mid-point theorem, we have

$$AD/AB = AE/AC$$

$$\text{Ar}(\triangle ADC) = \text{Ar}(\triangle BDC) = \frac{1}{2} \text{Ar}(\triangle ABC) \dots \text{(i)}$$

Again,

$$\text{Ar}(\triangle AEB) = \text{Ar}(\triangle BEC) = \frac{1}{2} \text{Ar}(\triangle ABC) \dots \text{(ii)}$$

From equations (i) and (ii), we have

$$\text{Area}(\triangle ADC) = \text{Area}(\triangle AEB).$$

- Hence Proved

(ii) We know that, area of triangles on the same base and between same parallel lines are equal

$$\text{So, Area}(\triangle DBC) = \text{Area}(\triangle BCE)$$

$$\text{Area}(\triangle DOB) + \text{Area}(\triangle BOC) = \text{Area}(\triangle BOC) + \text{Area}(\triangle COE)$$

$$\text{Thus, Area}(\triangle DOB) = \text{Area}(\triangle COE) \text{ [On subtracting Area}(\triangle BOC) \text{ on both sides]}$$

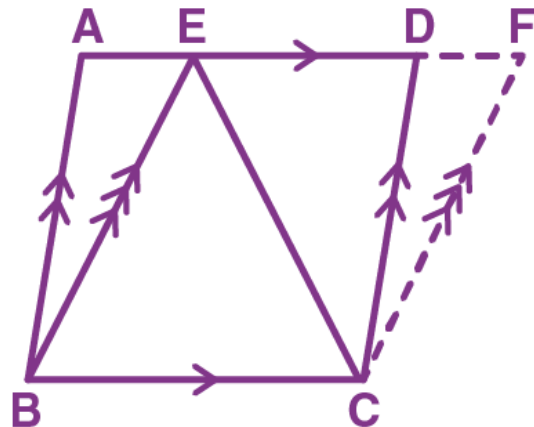
12. ABCD and BCFE are parallelograms. If area of triangle EBC = 480 cm²; AB = 30 cm and BC = 40 cm; Calculate:

(i) Area of parallelogram ABCD

(ii) Area of the parallelogram BCFE

(iii) Length of altitude from A on CD

(iv) Area of triangle ECF



Solution:

(i) As $\triangle EBC$ and parallelogram $ABCD$ are on the same base BC and between the same parallels i.e., $BC \parallel AD$.

$$\text{So, Area}(\triangle EBC) = \frac{1}{2} \text{Area}(\text{||gm } ABCD)$$

$$\begin{aligned} \text{Area}(\text{||gm } ABCD) &= 2 \times \text{Ar}(\triangle EBC) \\ &= 2 \times 480 \text{ cm}^2 \\ &= 960 \text{ cm}^2 \end{aligned}$$

(ii) We know that, parallelograms on same base and between same parallels are equal in area
So, Area of $BCFE$ = Area of $ABCD$ = 960 cm^2

$$\begin{aligned} \text{(iii) Area of } \triangle ACD &= 960/2 = 480 \\ &= (1/2) \times 30 \times \text{Altitude} \end{aligned}$$

$$\begin{aligned} \text{Thus,} \\ \text{Altitude} &= 480/15 \\ &= 32 \text{ cm} \end{aligned}$$

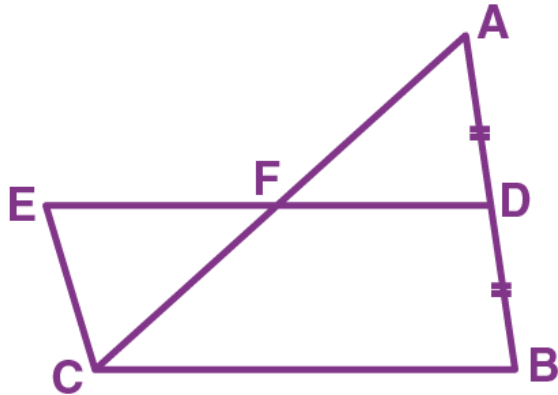
(iv) We know that, the area of a triangle is half that of a parallelogram on the same base and between the same parallels

$$\begin{aligned} \text{Therefore,} \\ \text{Area}(\triangle ECF) &= \frac{1}{2} \text{Area}(\text{||gm } CBEF) \end{aligned}$$

$$\begin{aligned} \text{Similarly,} \\ \text{Area}(\triangle BCE) &= \frac{1}{2} \text{Area}(\text{||gm } CBEF) \end{aligned}$$

$$\begin{aligned} \text{Thus,} \\ \text{Area}(\triangle ECF) = \text{Area}(\triangle BCE) &= 960/2 = 480 \text{ cm}^2 \end{aligned}$$

13. In the given figure, D is mid-point of side AB of $\triangle ABC$ and BDEC is a parallelogram.



Prove that: Area of $\triangle ABC$ = Area of $\parallel\text{gm BDEC}$.

Solution:

Here, $AD = DB$ and $EC = CB$ (Given)

So, $EC = AD$

It's seen that $\angle EFC = \angle AFD$ (Vertically opposite angles)

And, as ED and CB are parallel lines with AC cutting these lines, we have

$\angle ECF = \angle FAD$ (Alternate interior angles)

From the above conditions, we have

$\triangle EFC \cong \triangle AFD$ by AAS Congruency criterion

So, Area ($\triangle EFC$) = Area ($\triangle AFD$)

Now, adding quadrilateral $CBDF$ in both sides, we get

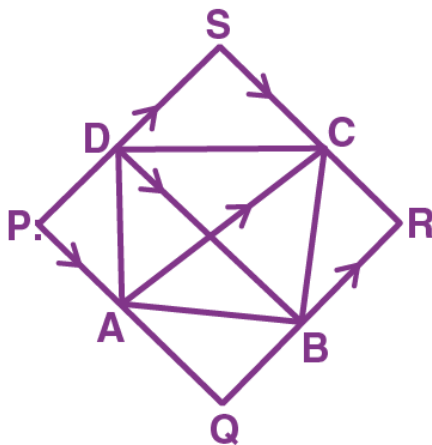
Area of $\parallel\text{gm BDEC}$ = Area of $\triangle ABC$

- Hence proved

14. In the following, $AC \parallel PS \parallel QR$ and $PQ \parallel DB \parallel SR$.

Prove that:

Area of quadrilateral PQRS = $2 \times$ Area of quad. ABCD.



Solution:

In parallelogram PQRS, $AC \parallel PS \parallel QR$ and $PQ \parallel DB \parallel SR$

Similarly, AQRC and APSC are also parallelograms.

As, $\triangle ABC$ and parallelogram AQRC are on the same base AC and between the same parallels, we have

$$\text{Ar}(\triangle ABC) = \frac{1}{2} \text{Ar}(\text{AQRC}) \dots (i)$$

Similarly,

$$\text{Ar}(\triangle ADC) = \frac{1}{2} \text{Ar}(\text{APSC}) \dots (ii)$$

On adding (i) and (ii), we get

$$\text{Ar}(\triangle ABC) + \text{Ar}(\triangle ADC) = \frac{1}{2} \text{Ar}(\text{AQRC}) + \frac{1}{2} \text{Ar}(\text{APSC})$$

$$\text{Area (quad. ABCD)} = \frac{1}{2} \text{Area (quad. PQRS)}$$

Therefore,

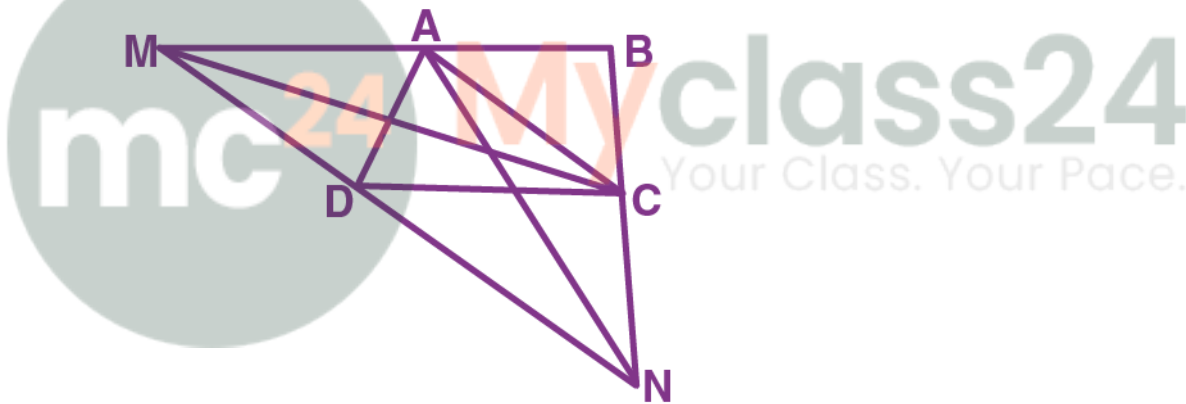
$$\text{Area of quad. PQRS} = 2 \times \text{Area of quad. ABCD}$$

15. ABCD is trapezium with $AB \parallel DC$. A line parallel to AC intersects AB at point M and BC at point N. Prove that: area of $\triangle ADM$ = area of $\triangle ACN$.

Solution:

Given: ABCD is a trapezium

$AB \parallel CD$, $MN \parallel AC$



Let's join C and M

We know that, area of triangles on the same base and between same parallel lines are equal.

So, Area of $\triangle AMD$ = Area of $\triangle AMC$... (i)

Similarly, considering quad. AMNC where $MN \parallel AC$, we have

$\triangle ACM$ and $\triangle ACN$ are on the same base and between the same parallel lines

Thus, their areas should be equal.

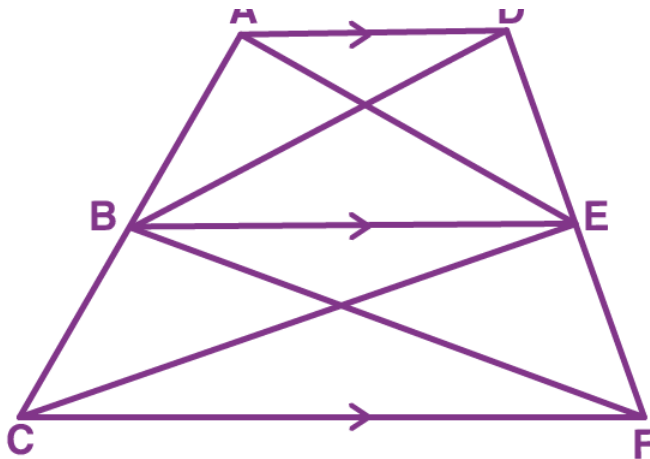
i.e., Area of $\triangle ACM$ = Area of $\triangle ACN$... (ii)

From equations (i) and (ii), we get

$$\text{Area of } \triangle ADM = \text{Area of } \triangle ACN$$

- Hence Proved.

16. In the given figure, $AD \parallel BE \parallel CF$. Prove that area ($\triangle AEC$) = area ($\triangle DBF$)



Solution:

We know that,

Area of triangles on the same base and between same parallel lines are equal.

So, in ABED quadrilateral and $AD \parallel BE$

With common base, BE and between AD and BE parallel lines, we have

Area of $\triangle ABE = \text{Area of } \triangle BDE \dots (i)$

Similarly, in BEFC quadrilateral and $BE \parallel CF$

With common base BC and between BE and CF parallel lines, we have

Area of $\triangle BEC = \text{Area of } \triangle BEF \dots (ii)$

On adding equations (i) and (ii), we get

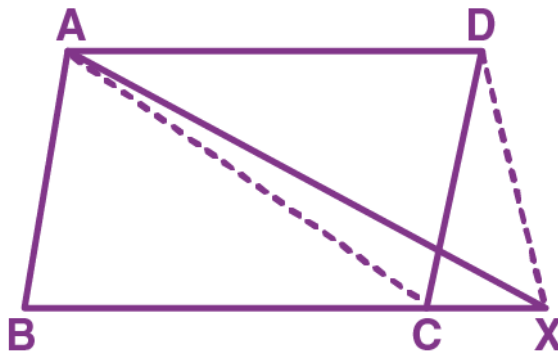
Area of $\triangle ABE + \text{Area of } \triangle BEC = \text{Area of } \triangle BEF + \text{Area of } \triangle BDE$

Thus,

Area of $\triangle AEC = \text{Area of } \triangle DBF$

- Hence Proved

17. In the given figure, ABCD is a parallelogram; BC is produced to point X. Prove that: area ($\triangle ABX$) = area (quad. ACXD).



Solution:

Given: ABCD is a parallelogram.

We know that,

Area of $\triangle ABC$ = Area of $\triangle ACD$ (Diagonal divides a ||gm into 2 triangles of equal area)

Now, consider $\triangle ABX$

Area of $\triangle ABX$ = Area of $\triangle ABC$ + Area of $\triangle ACX$

We also know that, area of triangles on the same base and between same parallel lines are equal.

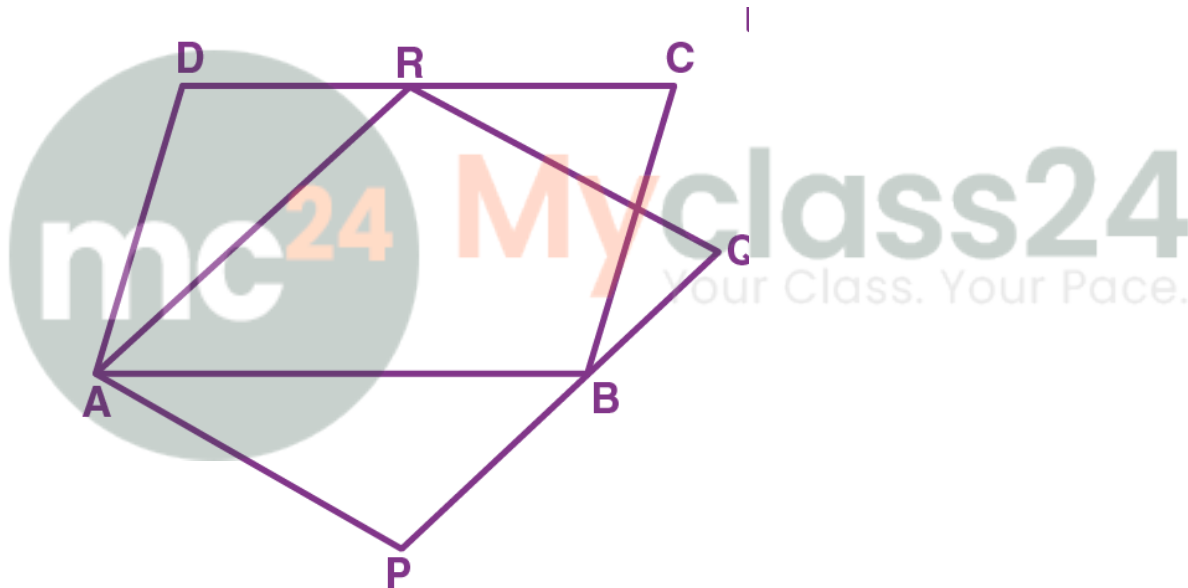
So, Area of $\triangle ACX$ = Area of $\triangle CXD$

From above equations, we have

$$\begin{aligned} \text{Area of } \triangle ABX &= \text{Area of } \triangle ABC + \text{Area of } \triangle ACX \\ &= \text{Area of } \triangle ACD + \text{Area of } \triangle CXD \\ &= \text{Area of quadrilateral ACXD} \end{aligned}$$

- Hence Proved

18. The given figure shows parallelograms ABCD and APQR. Show that these parallelograms are equal in area.



Solution:

Let's join B and R and also P and R.

We know that, the area of the parallelogram is equal to twice the area of the triangle, if the triangle and the parallelogram are on the same base and between the parallels

Taking ABCD parallelogram:

As ||gm ABCD and $\triangle ABR$ lie on AB and between the parallels AB and DC, we have

$$\text{Area (||gm ABCD)} = 2 \times \text{Area } (\triangle ABR) \dots (i)$$

Also, the area of triangles with same base and between the same parallel lines are equal.

As the triangles ABR and APR lie on the same base AR and between the parallels AR and QP, we have,

$$\text{Area } (\triangle ABR) = \text{Area } (\triangle APR) \dots (ii)$$

From equations (i) and (ii), we have

Area (||gm ABCD) = 2 x Area (Δ APR) ... (iii)

Also, its seen that APR and ||gm ARQP lie on the same base AR and between the same parallels AR and QP

So, Area (Δ APR) = $\frac{1}{2}$ Area (||gm ARQP) ... (iv)

Using (iv) in (iii), we get

Area (||gm ABCD) = 2 x $\frac{1}{2}$ x Area (||gm ARQP)

Area (||gm ABCD) = Area (||gm ARQP)

- Hence proved



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