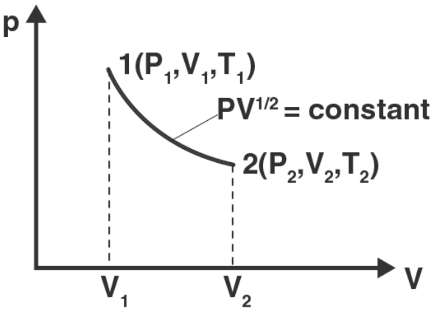
Exemplar Solutions for Class 11 Physics Chapter 11 - Thermodynamics

Long Answers

23. Consider a P-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown in the figure.



The process follows: $PV^{(1/2)} = K$ = constant

a) Find the work done when the gas is taken from state 1 to state 2 b) What is the ratio of temperature T_1/T_2 if $V_2 = 2V_1$

c) Given the internal energy for one mole of gas at temperature T is (3/2)RT, find the heat supplied to the gas when it is taken from state 1 to 2 with $V_2 = 2V_1$.

Solution:

a) Work done: For process $PV^{(1/2)} = K$: $W_{1-2} = \int[V_1 \text{ to } V_2]\, P\, dV = \int[V_1 \text{ to } V_2]\, (K/V^{(1/2)})\, dV$
$W_{1-2} = K \int[V_1 \text{ to } V_2]\, V^{(-1/2)}\, dV = K[2V^{(1/2)}]_1{}^2$ $W_{1-2} = 2K(V_2{}^{(1/2)} - V_1{}^{(1/2)})$
Since $K = P_1V_1{}^{(1/2)} = P_2V_2{}^{(1/2)}$: $W_{1-2} = 2P_2V_2{}^{(1/2)}(V_2{}^{(1/2)} - V_1{}^{(1/2)}) = 2P_2V_2{}^{(1/2)}(\sqrt{V_2} - \sqrt{V_1})$

b) Temperature ratio: Using ideal gas law: $T = PV/nR$ $(n = 1)$ $T \propto PV = (K/V^{(1/2)}) \times V = KV^{(1/2)}$

Therefore: $T_2/T_1 = (V_2/V_1)^{(1/2)} = (2V_1/V_1)^{(1/2)} = \sqrt{2}$

So: $T_1/T_2 = 1/\sqrt{2}$

c) Heat supplied: $\Delta U = (3/2)R(T_2 - T_1) = (3/2)RT_1(\sqrt{2} - 1)$ $\Delta W = 2P_1V_1{}^{(1/2)}(V_2{}^{(1/2)} - V_1{}^{(1/2)}) = 2RT_1(\sqrt{2} - 1)$

From first law: $\Delta Q = \Delta U + \Delta W$ $\Delta Q = (3/2)RT_1(\sqrt{2} - 1) + 2RT_1(\sqrt{2} - 1) = (7/2)RT_1(\sqrt{2} - 1)$

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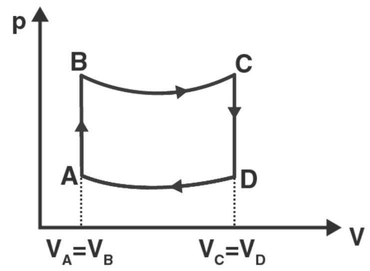
24. A cycle followed by an engine is shown in the figure.

Process details:

- A to B: volume constant (isochoric)
- B to C: adiabatic
- C to D: volume constant (isochoric)
- D to A: adiabatic
- $V^c = V_D = 2V_a = 2V_\beta$



a) In which part of the cycle heat is supplied to the engine from outside? b) In which part of the cycle heat is being given to the surrounding by the engine?
c) What is the work done by the engine in one cycle in terms of P_a, P_β, V_a? d) What is the efficiency of the engine?

Solution:

a) Heat is supplied during process A→B (isochoric heating at constant volume V_a)

b) Heat is rejected during process C→D (isochoric cooling at constant volume $V^c = 2V_a$)

c) Work done in one cycle:

- A→B: $W_1 = 0$ (constant volume)
- B→C: W_2 = adiabatic expansion work
- C→D: $W_3 = 0$ (constant volume)
- D→A: W_4 = adiabatic compression work

For adiabatic process: $W = (P_1V_1 - P_2V_2)/(\gamma-1)$

Total work = $W_2 + W_4 = (3/2)[1 - (1/2)^{\wedge}(2/3)][P_\beta - P_a]V_a$

d) Efficiency: Heat input: $Q_{in} = C_v\Delta T = (3/2)(P_\beta - P_a)V_a$

$\eta = W/Q_{in} = [1 - (1/2)^{\wedge}(2/3)]$

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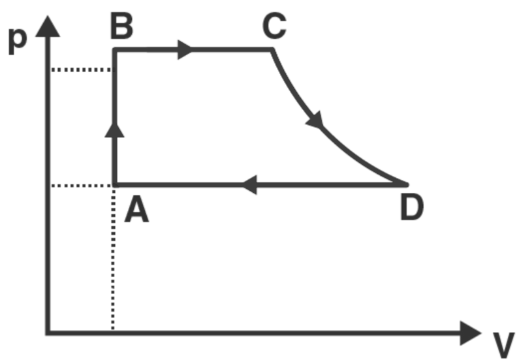
25. A cycle followed by an engine is shown in the figure. Find heat exchanged by the engine, with the surroundings for each section of the cycle considering $C_v = (3/2)R$.



Process details:

- AB: constant volume
- BC: constant pressure
- CD: adiabatic
- DA: constant pressure

Solution:

a) AB: Constant volume (isochoric) $\Delta Q = \Delta U + \Delta W = \Delta U + 0 = C_v\Delta T = (3/2)(P_\beta - P_a)V_a$

b) BC: Constant pressure (isobaric)

$\Delta Q = \Delta U + \Delta W = C_v\Delta T + P\Delta V = (5/2)P_\beta(V^c - V_a)$

c) CD: Adiabatic $\Delta Q = 0$

d) DA: Constant pressure (compression) $\Delta Q = \Delta U + \Delta W = C_v\Delta T + P\Delta V = (5/2)P_a(V_a - V_D)$

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26. Consider that an ideal gas is expanding in a process given by P = f(V), which passes through a point (V_0, P_0). Show that the gas is absorbing heat at (P_0, V_0) if the slope of the curve P = f(V) is larger than the slope of the adiabat passing through (P_0, V_0).

Solution:

For any process: $dQ = dU + dW = C_v dT + PdV$

Using ideal gas law $PV = nRT$: $dT = (1/nR)(PdV + VdP)$

Substituting: $dQ = (C_v/nR)(PdV + VdP) + PdV$ $dQ = dV[P(C_v/nR + 1) + V(C_v/nR)(dP/dV)]$ $dQ = dV[P(\gamma/(\gamma-1)) + V(C_v/nR)(dP/dV)]$

For adiabatic process: $dQ = 0$, which gives: $(dP/dV)_{a}d_{ia}b_{ati}c = -\gamma P_0/V_0$

For heat absorption ($dQ > 0$) during expansion ($dV > 0$): $P(\gamma/(\gamma-1)) + V(C_v/nR)(dP/dV) > 0$

This simplifies to: $(dP/dV) > -\gamma P_0/V_0$

Therefore, if $f'(V_0) > -\gamma P_0/V_0$, the gas absorbs heat.

Since the adiabatic slope is steeper (more negative), any process with a less steep slope will involve heat absorption.

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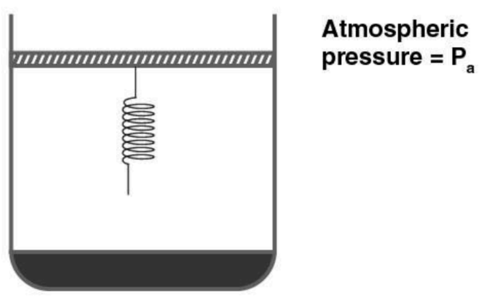
27. Consider one mole of perfect gas in a cylinder of unit cross section with a piston attached. A spring is attached to the piston and to the bottom of the cylinder. Initially the spring is unstretched and the gas is in equilibrium. A certain amount of heat Q is supplied to the gas causing an increase of volume from V_0 to V_1.



a) What is the initial pressure of the system?

b) What is the final pressure of the system?

c) Using the first law of thermodynamics, write down the relation between Q, P_a, V, V_0, and k.

Solution:

a) Initial pressure: Initially, spring is unstretched, so gas pressure balances atmospheric pressure: $P_i = P_a$

b) Final pressure: When volume increases by $(V_1 - V_0)$, spring compresses by same amount. Spring force = $k(V_1 - V_0)$ Final pressure: $Pf = P_a + k(V_1 - V_0)$

c) First law relation: $dQ = dU + dW$

Internal energy change: $dU = C_v\Delta T = (3/2)R(T_1 - T_0)$

Using $PV = RT$ for one mole: $T_1 - T_0 = (P_1V_1 - P_0V_0)/R = ([P_a + k(V_1 - V_0)]V_1 - P_aV_0)/R$

Work done by gas: $dW = \int PdV = \int[P_a + k(V - V_0)]dV$ $dW = P_a(V_1 - V_0) + (k/2)(V_1 - V_0)^2$

Therefore, the complete relation is: $Q = C_v(T_1 - T_0) + P_a(V_1 - V_0) + (k/2)(V_1 - V_0)^2$

Substituting the temperature relation: $Q = (3/2)([P_a + k(V_1 - V_0)]V_1 - P_aV_0) + P_a(V_1 - V_0) + (k/2)(V_1 - V_0)^2$