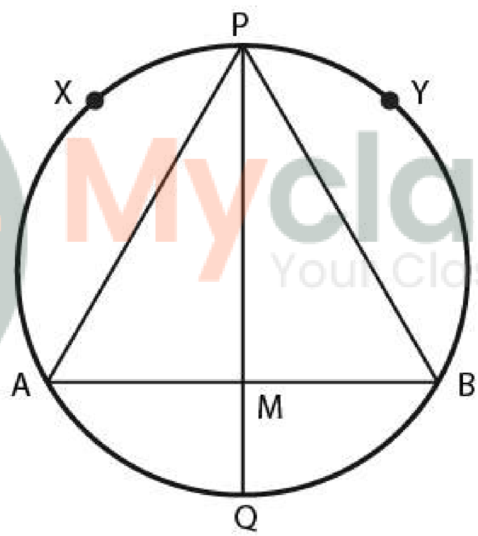
EXERCISE 10.3

If arcs AXB and CYD of a circle are congruent, find the ratio of AB and CD. Solution:

According to the question,
We have,
AXB ≅ CYD.
We know that,
If two arcs of a circle are congruent, then their corresponding arcs are also equal.
So, we have chord AB = chord CD.
Hence, we get,
AB/CD = 1
AB/CD = 1/1
AB : CD = 1:1

1. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that arc PXA ∠ Arc PYB.
Solution:



According to the question,
We have,
PQ is the perpendicular bisect of AB,
So, we get,
AM = BM …eq.(1)
In △APM and △BPM,
From eq.(1),
AM = BM
∠AMP = ∠BMP = 90º
PM = PM [Common side]
Therefore, △APM ≅ △BPM [By SAS congruence rule]
So, AP = BP [CPCT]
Hence, arc PXA ≅ Arc PYB

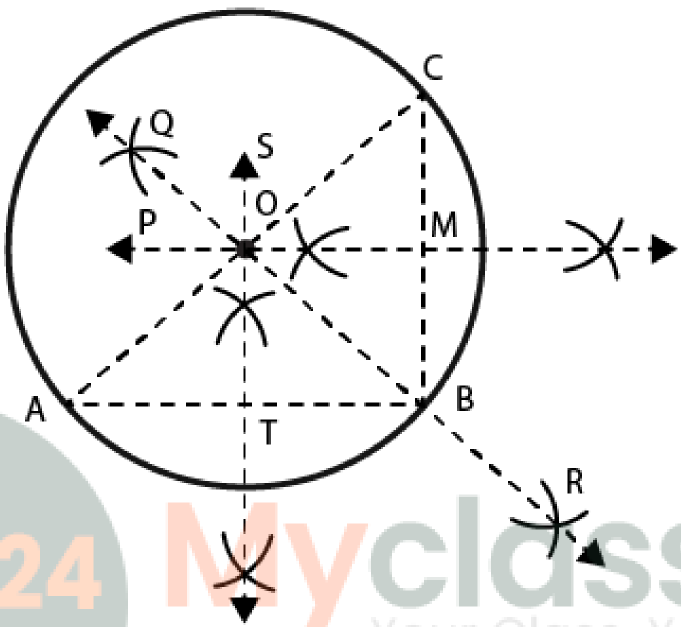
Therefore, if two chords of a circle are equal, then their corresponding arcs are congruent.

2. A, B and C are three points on a circle. Prove that the perpendicular bisectors of AB, BC and CA are concurrent.

Solution:

According to the question,

Three non-collinear points A, B and C are on a circle.



To prove: Perpendicular bisectors of AB, BC and CA are concurrent.

Construction: Join AB, BC and CA.

Draw:

ST, perpendicular bisector of AB,

PM, perpendicular bisector of BC

And, QR perpendicular bisector of CA

As point A, B and C are not collinear, ST, PM and QR are not parallel and will intersect.

Proof:

O lies on ST, the ⊥ bisector of AB

OA = OB … (1)

Similarly, O lies on PM, the ⊥ bisector of BC

OB = OC … (2)

And, O lies on QR, the ⊥ bisector of CA

OC = OA … (3)

From (1), (2) and (3),

OA = OB = OC

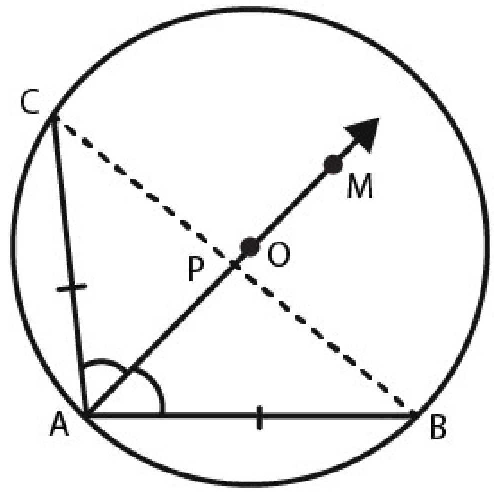
Let OA = OB = OC = r

Draw circle, with centre O and radius r, passing through A, B and C.

Hence, O is the only point equidistance from A, B and C.

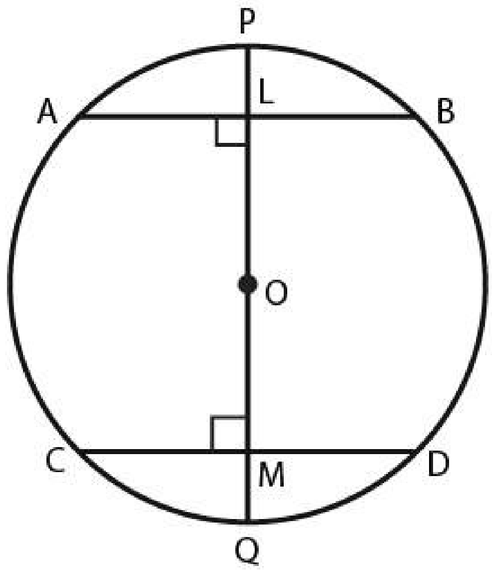
Therefore, the perpendicular bisectors of AB, BC and CA are concurrent.

3. AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the centre of the circle.
Solution:



According to the question,
We have,
AB and AC are two chords which are equal with centre O.
AM is the bisector of ∠BAC.
To prove: AM passes through O.
Construction: Join BC.
Let AM intersect BC at P.
Proof: In DBAP and DCAP
AB = AC [Given]
∠BAP = ∠CAP [Given]
And AP = BP [Common side]
△BAP ≅ △CAP [By SAS]
∠BPA = ∠CPA [CPCT]
We know that,
CP = PB
But, since ∠BPA and ∠CPA are linear pair angles,
We have,
∠BPA+∠CPA =180º
∠BPA = ∠CPA = 90º
Then,
AP is perpendicular bisector of the chord BC, which will pass through the centre O on being produced.
Therefore, AM passes through O.

4. If a line segment joining mid-points of two chords of a circle passes through the centre of the circle, prove that the two chords are parallel.
Solution:
Consider AB and CD to be the chords of the circle with center O.

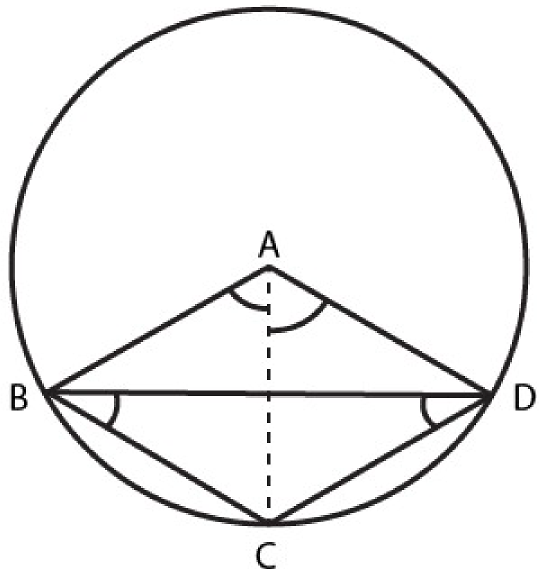


Let L be the midpoint of AB.
Let M be the midpoint of CD.
Let PQ be the line passing through these midpoints and the center of the circle.
Then, PQ is the diameter of the circle.
We know that,
Line joining center to the midpoint of a chord is always perpendicular to the chord.
Since M is the midpoint of CD,
We have, OM ⊥ CD
⇒ OMD = 90°
Similarly, L is the midpoint of AB,
OL ⊥ AB
⇒ OLA = 90°
But, we know,
∠OLA and ∠OMD are alternate angles.
So, AB ∥ CD.
Hence, proved.

5. ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that ∠CBD + ∠CDB = ½ ∠BAD

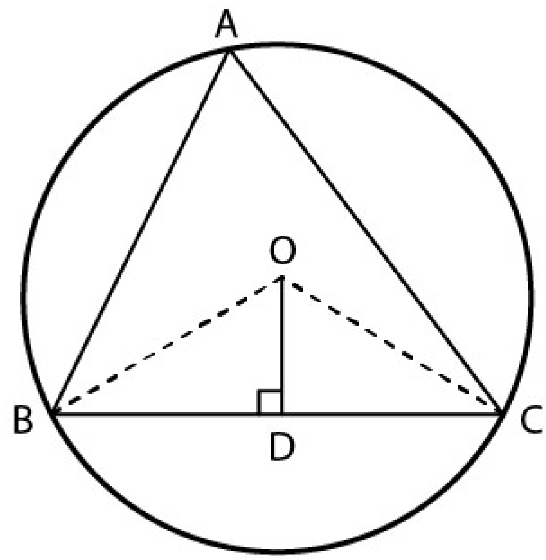
Solution:

According to the question,
We have,
A quadrilateral ABCD such that A is the centre of the circle passing through B, C and D.
Construction:
Join CA and BD.



We know that,
In a circle, angle subtended by an arc at the center is twice the angle subtended by it at any other point in the remaining part of the circle
So,
The arc DC subtends ∠DAC at the center and ∠CAB at point B in the remaining part of the circle,
We get,
∠DAC = 2∠CBD …(1)
Similarly,
The arc BC subtends ∠CAB at the center and ∠CDB at point D in the remaining part of the circle,
We get,
∠CAB = 2∠CDB …(2)
From equations (1) and (2),
We have:
DAC + ∠CAB = 2∠CDB + 2∠CBD
⇒ ∠BAD = 2(∠CDB + ∠CBD)
⇒ (∠CDB + ∠CBD) = ½ (∠BAD)

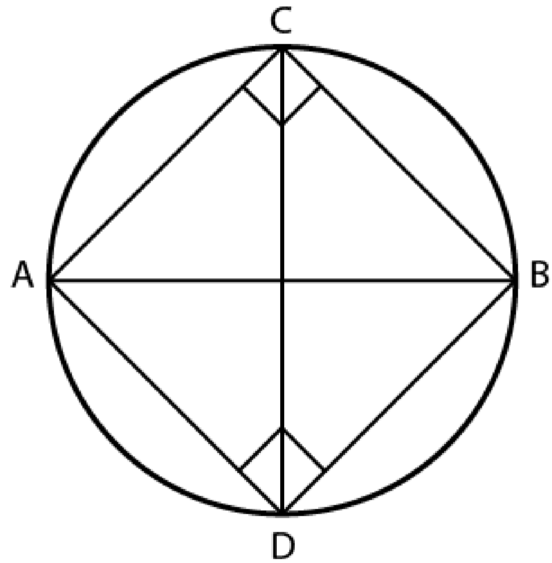
6. O is the circumcentre of the triangle ABC and D is the mid-point of the base BC. Prove that ∠BOD = ∠A.
Solution:
According to the question,
We have,
O is the circumcenter of the triangle ABC and D is the midpoint of BC.



To prove: ∠BOD = ∠A
Construction: Join OB and OC.
In ΔOBD and ΔCD:
OD = OD (common)
DB = DC (D is the midpoint of BC)
OB = OC (radius of the circle)
By SSS congruence rule,
We get,
ΔOBD ≅ ΔOCD.
∠BOD = ∠COD (By CPCT)
Let ∠BOD = ∠COD = x
We know that,
Angle subtended by an arc at the center of the circle is twice the angle subtended by it at any other point in the remaining part of the circle.
So, we have,
2∠BAC = ∠BOC
⇒ 2∠BAC = ∠BOD + ∠DOC
⇒ 2∠BAC = x + x
⇒ 2∠BAC = 2x
⇒ ∠BAC = x
⇒ ∠BAC = ∠BOD
Hence, proved.

7. On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that ∠BAC = ∠BDC.
Solution:
According to the question,
We have,

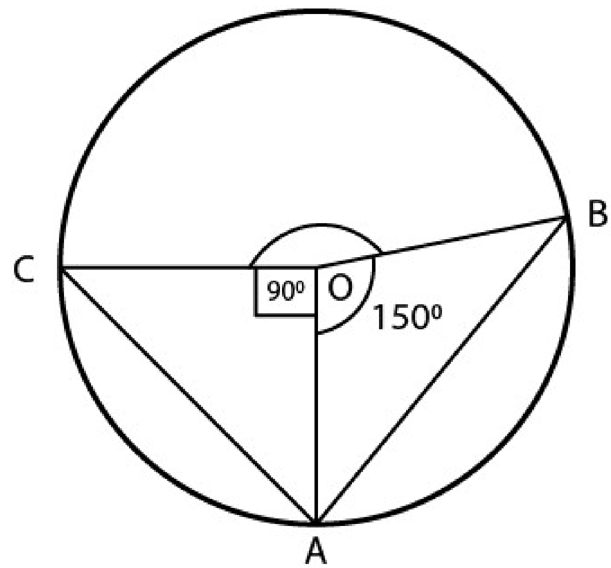
ACB and ADB are two right triangles.



To Prove: ∠BAC = ∠BDC
We know that,
ACB and ADB are right angled triangles,
Then,
∠C + ∠D = 90° + 90°
∠C + ∠D = 180°
Therefore ADBC is a cyclic quadrilateral as sum of opposite angles of a cyclic quadrilateral = 180°
We also have,
∠BAC and ∠BDC lie in the same segment BC and angles in the same segment of a circle are equal.
∴ ∠BAC = ∠BDC.
Hence Proved.

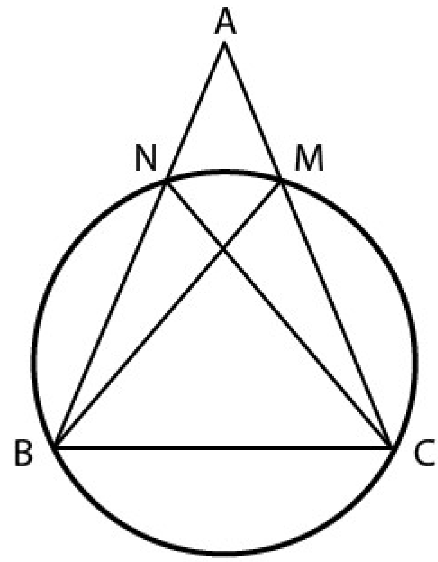
8. Two chords AB and AC of a circle subtends angles equal to 90º and 150º, respectively at the centre. Find ∠BAC, if AB and AC lie on the opposite sides of the centre.
Solution:

According to the question,
We have,
In ΔAOB,
OA = OB (radius of the circle)



Since angle opposite to equal sides are equal, we get,
∠OBA = ∠OAB
We know that,
According to angle sum property, sum of all angles of a triangle = 180º
Using the angle sum property in ΔAOB, we get,
∠OAB + ∠AOB +∠OBA = 180°
⇒ ∠OAB +90° + ∠OAB = 180°
⇒ 2∠OAB = 180° - 90°
⇒ 2∠OAB = 90°
⇒ ∠OAB = 45°
Now, in ΔAOC,
OA = OC (radius of the circle)
Since, angle opposite to equal sides are equal
∴ ∠OCA = ∠OAC
Using the angle sum property in ΔAOB, sum of all angles of the triangle is 180°, we have:
∠OAC + ∠AOC +∠OCA = 180°
⇒ ∠OAC +150° + ∠OAC = 180°
⇒ 2∠OAC = 180° - 150°
⇒ 2∠OAC = 30°
⇒ ∠OAC = 15°
Now, ∠BAC = ∠OAB + ∠OAC
= 45° + 15°
= 60°
∴ ∠BAC = 60°

9. If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.
Solution:



According to the question,
BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC.
So, we have,
$\angle BMC = \angle BNC = 90^o$
We know that,
If a line segment joining two points subtends equal angles on the same side of the line containing the segment, then the four points are concyclic.
Considering the question,
Since BC joins the two points, B and C, subtending equal angles, ∠BMC and ∠BNC, at M and N on the same side BC containing the segment, then B, C, M and N are concyclic.
Hence, we get that,
B, C, M and N are concyclic.