Exercise 23(C)

Solution 1:

(i)

$$2\sin A = 1$$
$$\sin A = \frac{1}{2}$$
$$\sin A = \sin 30^0$$
$$A = 30^0$$

(ii)

$$2\cos 2A = 1$$
$$\cos 2A = \frac{1}{2}$$
$$\cos 2A = \cos 60^0$$
$$2A = 60^0$$
$$A = 30^0$$

(iii)

$$\sin 3A = \frac{\sqrt{3}}{2}$$
$$\sin 3A = \sin 60^0$$
$$3A = 60^0$$
$$A = 20^0$$

(iv)

$$\sec 2A = 2$$
$$\sec 2A = \sec 60^0$$
$$2A = 60^0$$
$$A = 30^0$$

(V)

$$\sqrt{3}\tan A = 1$$
$$\tan A = \frac{1}{\sqrt{3}}$$
$$\tan A = \tan 30^0$$
$$A = 30^0$$

(vi)

$$\tan 3A = 1$$
$$\tan 3A = \tan 45^0$$
$$3A = 45^0$$
$$A = 15^0$$

(vii)

$$2\sin 3A = 1$$
$$\sin 3A = \frac{1}{2}$$
$$\sin 3A = \sin 30^0$$
$$3A = 30^0$$
$$A = 10^0$$

(viii)

$$\sqrt{3}\cot 2A = 1$$
$$\cot 2A = \frac{1}{\sqrt{3}}$$
$$\cot 2A = \cot 60^0$$
$$2A = 60^0$$
$$A = 30^0$$

Solution 2:

(i)

$(\sin A - 1)(2\cos A - 1) = 0$

$(\sin A - 1) = 0$ *and* $2\cos A - 1 = 0$

$\sin A = 1$ *and* $\cos A = \frac{1}{2}$

$\sin A = \sin 90^0$ *and* $\cos A = \cos 60^0$

$A = 90^0$ *and* $A = 60^0$

(ii)

$(\tan A - 1)(\operatorname{cosec} 3A - 1) = 0$

$\tan A - 1 = 0$ *and* $\operatorname{cosec} 3A - 1 = 0$

$\tan A = 1$ *and* $\operatorname{cosec} 3A = 1$

$\tan A = \tan 45^0$ *and* $\operatorname{cosec} 3A = \operatorname{cosec} 90^0$

$A = 45^0$ $A = 30^0$

(iii)

$(\sec 2A - 1)(\operatorname{cosec} 3A - 1) = 0$

$\sec 2A - 1 = 0$ *and* $\operatorname{cosec} 3A - 1 = 0$

$\sec 2A = 1$ *and* $\operatorname{cosec} 3A = 1$

$\sec 2A = \sec 0^0$ *and* $\operatorname{cosec} 3A = \operatorname{cosec} 90^0$

$A = 0^0$ $A = 30^0$

(iv)

$\cos 3A.(2\sin 2A - 1) = 0$

$\cos 3A = 0$ *and* $2\sin 2A - 1 = 0$

$\cos 3A = \cos 90^0$ *and* $\sin 2A = \frac{1}{2}$

$3A = 90^0$ *and* $\sin 2A = \sin 30^0$

$A = 30^0$ $2A = 30^0 \Rightarrow A = 15^0$

(v)

$(\operatorname{cosec} 2A - 2)(\cot 3A - 1) = 0$

$\operatorname{cosec} 2A - 2 = 0$ *and* $\cot 3A - 1 = 0$

$\operatorname{cosec} 2A = 2$ *and* $\cot 3A = 1$

$\operatorname{cosec} 2A = \operatorname{cosec} 30^0$ *and* $\cot 3A = \cot 45^0$

$2A = 30^0$ *and* $3A = 45^0$

$A = 15^0$ *and* $A = 15^0$

Solution 3:

(i)

$$2\sin x^\circ - 1 = 0$$

$$\sin x^\circ = \frac{1}{2}$$

(ii)

$$\sin x^\circ = \frac{1}{2}$$

$$\sin x^\circ = \sin 30^0$$

$$x^0 = 30^0$$

(iii)

$$\cos x^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan x^\circ = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Solution 4:

(i)

$$4\cos^2 x^\circ - 1 = 0$$

$$4\cos^2 x^\circ = 1$$

$$\cos^2 x^\circ = \left(\frac{1}{2}\right)^2$$

$$\cos x^0 = \frac{1}{2}$$

$$\cos x^0 = \cos 60^0$$

$$x^0 = 60^0$$

(ii)

$$\sin^2 x^\circ + \cos^2 x^\circ = \sin^2 60^\circ + \cos^2 60^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

(iii)

$$\frac{1}{\cos^2 x^0} - \tan^2 x^0 = \frac{1}{\cos^2 60^0} - \tan^2 60^0$$

$$= \frac{1}{\left(\frac{1}{2}\right)^2} - \left(\sqrt{3}\right)^2$$

$$= 4 - 3$$

$$= 1$$

Solution 5:

$$4\sin^2\theta - 1 = 0$$

$$\sin^2\theta = \frac{1}{4}$$

$$\sin\theta = \frac{1}{2}$$

$$\sin\theta = \sin 30^0$$

$$\theta = 30^0$$

$$\cos^2\theta + \tan^2\theta = \cos^2 30^0 + \tan^2 30^0$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{3}{4} + \frac{1}{3}$$

$$= \frac{9+4}{12}$$

$$= \frac{13}{12}$$

Solution 6:

$$\sin 3A = 1$$

$$\sin 3A = \sin 90^0$$

$$3A = 90^0$$

$$A = 30^0$$

(i)

$$\sin A = \sin 30^0$$

$$\sin A = \frac{1}{2}$$

(ii)

$$\cos 2A = \cos 2(30^0)$$

$$= \cos 60^0$$

$$= \frac{1}{2}$$

(iii)

$$\tan^2 A - \frac{1}{\cos^2 A} = \tan^2 30^0 - \frac{1}{\cos^2 30^0}$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 - \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{3} - \frac{4}{3}$$

$$= \frac{-3}{3}$$

$$= -1$$

Solution 7:

(i)

$$2\cos 2A = \sqrt{3}$$

$$\cos 2A = \frac{\sqrt{3}}{2}$$

$$\cos 2A = \cos 30^0$$

$$2A = 30^0$$

$$A = 15^0$$

(ii)

$$\sin 3A = \sin 3(15^0)$$

$$= \sin 45^0$$

$$= \frac{1}{\sqrt{2}}$$

(iii)

$$\sin^2(75^\circ - A) + \cos^2(45^\circ + A) = \sin^2(75^\circ - 15^0) + \cos^2(45^\circ + 15^0)$$

$$= \sin^2 60^0 + \cos^2 60^0$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

Solution 8:

(i)

Given that $x = 30^0$

$$\sin x + \cos y = 1$$
$$\sin 30^0 + \cos y = 1$$
$$\cos y = 1 - \sin 30^0$$
$$\cos y = 1 - \frac{1}{2}$$
$$\cos y = \frac{1}{2}$$
$$\cos y = \cos 60^0$$
$$y = 60^0$$

(ii)

Given that $B = 90^0$

$$3 \tan A - 5 \cos B = \sqrt{3}$$
$$3 \tan A - 5 \cos 90^0 = \sqrt{3}$$
$$3 \tan A - 0 = \sqrt{3}$$
$$\tan A = \frac{\sqrt{3}}{3}$$
$$\tan A = \frac{1}{\sqrt{3}}$$
$$\tan A = \tan 30^0$$
$$A = 30^0$$

Solution 9:

(i)

$$\cos x^\circ = \frac{10}{20}$$

$$\cos x^\circ = \frac{1}{2}$$

(ii)

$$\cos x^\circ = \frac{1}{2}$$

$$\cos x^\circ = \cos 60^0$$

$$x^\circ = 60^0$$

(iii)

$$\frac{1}{\tan^2 x^0} - \frac{1}{\sin^2 x^0} = \frac{1}{\tan^2 60^0} - \frac{1}{\sin^2 60^0}$$

$$= \frac{1}{\left(\sqrt{3}\right)^2} - \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{3} - \frac{4}{3}$$

$$= -1$$

(iv)

$$\tan x^\circ = \tan 60^\circ$$

$$= \sqrt{3}$$

We know that $\tan x^\circ = \frac{AB}{BC}$

$$\Rightarrow \tan x^\circ = \frac{y}{10}$$

$$\Rightarrow y = 10\tan x^\circ$$

$$\Rightarrow y = 10\tan 60^\circ$$

$$\Rightarrow y = 10\sqrt{3}$$

Solution 10:

(i)

$$\tan\theta^0 = \frac{5}{5} = 1$$

(ii)

$$\tan\theta^0 = 1$$
$$\tan\theta^0 = \tan 45^0$$
$$\theta^0 = 45^0$$

(iii)

$$\sin^2\theta^\circ - \cos^2\theta^0 = \sin^2 45^\circ - \cos^2 45^0$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= 0$$

(iv)

$$\sin\theta^0 = \frac{5}{x}$$
$$\sin 45^0 = \frac{5}{x}$$
$$x = \frac{5}{\sin 45^0}$$
$$x = \frac{5}{\frac{1}{\sqrt{2}}}$$
$$x = 5\sqrt{2}$$

Solution 11:

(i)

$$2\sin A\cos A - \cos A - 2\sin A + 1 = 0$$
$$2\sin A\cos A - \cos A = 2\sin A - 1$$
$$(2\sin A - 1)\cos A - (2\sin A - 1) = 0$$
$$(2\sin A - 1) = 0 \quad and \quad \cos A = 1$$
$$\sin A = \frac{1}{2} \quad and \quad \cos A = \cos 0^0$$
$$A = 30^0 \quad and \quad A = 0^0$$

(ii)

$$\tan A - 2\cos A\tan A + 2\cos A - 1 = 0$$
$$\tan A - 2\cos A\tan A = 1 - 2\cos A$$
$$\tan A(1 - 2\cos A) - (1 - 2\cos A) = 0$$
$$(1 - 2\cos A)(\tan A - 1) = 0$$
$$1 - 2\cos A = 0 \quad and \quad \tan A - 1 = 0$$
$$\cos A = \frac{1}{2} \quad and \quad \tan A = 1$$
$$A = 60^0 \quad and \quad A = 45^0$$

(iii)

$$2\cos^2 A - 3\cos A + 1 = 0$$
$$2\cos^2 A - \cos A - 2\cos A + 1 = 0$$
$$\cos A(2\cos A - 1) - (2\cos A - 1) = 0$$
$$(2\cos A - 1)(\cos A - 1) = 0$$
$$2\cos A - 1 = 0 \quad and \quad \cos A - 1 = 0$$
$$\cos A = \frac{1}{2} \quad and \quad \cos A = 1$$
$$A = 60^0 \quad and \quad A = 0^0$$

(iv)

$$2\tan 3A\cos 3A - \tan 3A + 1 = 2\cos 3A$$
$$2\tan 3A\cos 3A - \tan 3A = 2\cos 3A - 1$$
$$\tan 3A(2\cos 3A - 1) = 2\cos 3A - 1$$
$$(2\cos 3A - 1)(\tan 3A - 1) = 0$$
$$2\cos 3A - 1 = 0 \quad and \quad \tan 3A - 1 = 0$$
$$\cos 3A = \frac{1}{2} \quad and \quad \tan 3A = 1$$
$$3A = 60^0 \quad and \quad 3A = 45^0$$
$$A = 20^0 \quad and \quad A = 15^0$$

Solution 12:

(i)

$$2\cos 3x - 1 = 0$$

$$\cos 3x = \frac{1}{2}$$

$$3x = 60^0$$

$$x = 20^0$$

(ii)

$$\cos\frac{x}{3} - 1 = 0$$

$$\cos\frac{x}{3} = 1$$

$$\frac{x}{3} = 0^0$$

$$x = 0^0$$

(iii)

$$\sin\left(x + 10^\circ\right) = \frac{1}{2}$$

$$\sin\left(x + 10^\circ\right) = \sin 30^0$$

$$x + 10^\circ = 30^0$$

$$x = 20^0$$

(iv)

$$\cos\left(2x - 30^\circ\right) = 0$$

$$\cos\left(2x - 30^\circ\right) = \cos 90^0$$

$$2x - 30^\circ = 90^0$$

$$2x = 120^0$$

$$x = 60^0$$

(v)

$$2\cos(3x - 15^\circ) = 1$$

$$\cos(3x - 15^\circ) = \frac{1}{2}$$

$$\cos(3x - 15^\circ) = \cos 60^0$$

$$3x - 15^\circ = 60^0$$

$$3x = 75^0$$

$$x = 25^0$$

(vi)

$$\tan^2(x - 5^\circ) = 3$$

$$\tan(x - 5^\circ) = \sqrt{3}$$

$$\tan(x - 5^\circ) = \tan 60^0$$

$$x - 5^0 = 60^0$$

$$x = 65^0$$

(vii)

$$3\tan^2(2x - 20^\circ) = 1$$

$$\tan(2x - 20^\circ) = \frac{1}{\sqrt{3}}$$

$$\tan(2x - 20^\circ) = \tan 30^0$$

$$2x - 20^\circ = 30^0$$

$$2x = 50^0$$

$$x = 25^0$$

(viii)

$$\cos\left(\frac{x}{2}+10^{\circ}\right)=\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{x}{2}+10^{\circ}\right)=\cos 30^{0}$$

$$\frac{x}{2}+10^{\circ}=30^{0}$$

$$x=40^{0}$$

(ix)

$$\sin^2 x+\sin^2 30^{\circ}=1$$

$$\sin^2 x=1-\sin^2 30^{\circ}$$

$$\sin^2 x=1-\frac{1}{4}$$

$$\sin^2 x=\frac{3}{4}$$

$$\sin x=\frac{\sqrt{3}}{2}$$

$$x=60^{0}$$

(x)

$$\cos^2 30^{\circ}+\cos^2 x=1$$

$$\cos^2 x=1-\cos^2 30^{\circ}$$

$$\cos^2 x=1-\frac{3}{4}$$

$$\cos x=\frac{1}{2}$$

$$x=60^{0}$$

(xi)

$$\cos^2 30^\circ + \sin^2 2x = 1$$

$$\sin^2 2x = 1 - \cos^2 30^\circ$$

$$\sin^2 2x = 1 - \frac{3}{4}$$

$$\sin 2x = \frac{1}{2}$$

$$2x = 30^0$$

$$x = 15^0$$

(xii)

$$\sin^2 60^\circ + \cos^2(3x - 9^\circ) = 1$$

$$\cos^2(3x - 9^\circ) = 1 - \sin^2 60^\circ$$

$$\cos^2(3x - 9^\circ) = 1 - \frac{3}{4}$$

$$\cos^2(3x - 9^\circ) = \frac{1}{4}$$

$$\cos(3x - 9^\circ) = \frac{1}{2}$$

$$3x - 9^\circ = 60^0$$

$$3x = 69^0$$

$$x = 23^0$$

Solution 13:

(i)

$$4\cos^2 x = 3$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = 30^0$$

(ii)

$$\cos^2 x + \cot^2 x = \cos^2 30^0 + \cot^2 30^0$$

$$= \frac{3}{4} + 3$$

$$= \frac{15}{4}$$

$$= 3\frac{3}{4}$$

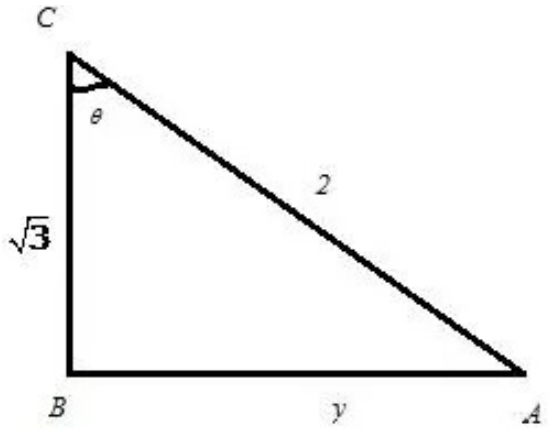
(iii)

$$\cos 3x = \cos 3(30^0) = \cos 90^0 = 0$$

(iv)

$$\sin 2x = \sin 2(30^0) = \sin 60^0 = \frac{\sqrt{3}}{2}$$

Solution 14:



(i)

From ΔABC

$$\sin x^\circ = \frac{\sqrt{3}}{2}$$

(ii)

$$\sin x^\circ = \frac{\sqrt{3}}{2}$$
$$\sin x^0 = \sin 60^0$$
$$x^0 = 60^0$$

(iii)

$$\tan x^\circ = \tan 60^\circ$$
$$= \sqrt{3}$$

(iv)

$$\cos x^\circ = \frac{y}{2}$$
$$\cos 60^0 = \frac{y}{2}$$
$$\frac{1}{2} = \frac{y}{2}$$
$$y = 1$$

Solution 15:

$$2\cos(A + B) = 1$$

$$\cos(A + B) = \frac{1}{2}$$

$$\cos(A + B) = \cos 60^0$$

$$A + B = 60^0 \quad \ldots\ldots\ldots(1)$$

$$2\sin(A - B) = 1$$

$$\sin(A - B) = \frac{1}{2}$$

$$A - B = 30^0 \quad \ldots\ldots\ldots(2)$$

Adding (1) and (2)

$$A + B + A - B = 60^0 + 30^0$$

$$2A = 90^0$$

$$A = 45^0$$

$$A + B = 60^0$$

$$B = 60^0 - A$$

$$B = 60^0 - 45^0$$

$$B = 15^0$$