

NCERT Solutions for Class-XII Math

Chapter-3 Exercise- Miscellaneous

1. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^nI + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in \mathbb{N}$

1. It is given that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

To show: P(n): $(aI + bA)^n = a^nI + na^{n-1}bA$, $n \in \mathbb{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1) : (aI + bA) = aI + ba^0A = aI + bA$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$P(k) : (aI + bA)^k = a^kI + ka^{k-1}bA$$

Now, we prove that the result is true for $n = k + 1$. Consider

$$\begin{aligned} (aI + bA)^{k+1} &= (aI + bA)^k (aI + bA) \\ &= (a^kI + ka^{k-1}bA)(aI + bA) \\ &= a^{k+1}I + ka^k bAI + a^k bIA + ka^{k-1}b^2A^2 \\ &= a^{k+1}I + (k+1)a^k bA + ka^{k-1}b^2A^2 \quad \dots(1) \end{aligned}$$

$$\text{Now, } A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

From (1), we have:

$$\begin{aligned} (aI + bA)^{k+1} &= a^{k+1}I + (k+1)a^k bA + 0 \\ &= a^{k+1}I + (k+1)a^k bA \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$(aI + bA)^n = a^nI + na^{n-1}bA \text{ where } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, n \in \mathbb{N}$$

2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbb{N}$

2. We will be proving the above equation by putting different values of n (i.e. $n=1, 2, 3 \dots n$)

For $n=1$,

$$\begin{aligned} A^1 &= \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} \\ &= \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{aligned}$$

For $n = 2$,

$$A^2 = A.A$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1.1+1.1+1.1 & 1.1+1.1+1.1 & 1.1+1.1+1.1 \\ 1.1+1.1+1.1 & 1.1+1.1+1.1 & 1.1+1.1+1.1 \\ 1.1+1.1+1.1 & 1.1+1.1+1.1 & 1.1+1.1+1.1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 3^{2-1} & 3^{2-1} & 3^{2-1} \\ 3^{2-1} & 3^{2-1} & 3^{2-1} \\ 3^{2-1} & 3^{2-1} & 3^{2-1} \end{bmatrix} \end{aligned}$$

For $n=3$,

$$A^3 = A^2.A$$

$$\begin{aligned} &= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3.1+3.1+3.1 & 3.1+3.1+3.1 & 3.1+3.1+3.1 \\ 3.1+3.1+3.1 & 3.1+3.1+3.1 & 3.1+3.1+3.1 \\ 3.1+3.1+3.1 & 3.1+3.1+3.1 & 3.1+3.1+3.1 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 3^2 & 3^2 & 3^2 \\ 3^2 & 3^2 & 3^2 \\ 3^2 & 3^2 & 3^2 \end{bmatrix} = \begin{bmatrix} 3^{3-1} & 3^{3-1} & 3^{3-1} \\ 3^{3-1} & 3^{3-1} & 3^{3-1} \\ 3^{3-1} & 3^{3-1} & 3^{3-1} \end{bmatrix}$$

For $n=4$,

$$A^4 = A^3 \cdot A$$

$$= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9.1+9.1+9.1 & 9.1+9.1+9.1 & 9.1+9.1+9.1 \\ 9.1+9.1+9.1 & 9.1+9.1+9.1 & 9.1+9.1+9.1 \\ 9.1+9.1+9.1 & 9.1+9.1+9.1 & 9.1+9.1+9.1 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 27 & 27 \\ 27 & 27 & 27 \\ 27 & 27 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} 3^3 & 3^3 & 3^3 \\ 3^3 & 3^3 & 3^3 \\ 3^3 & 3^3 & 3^3 \end{bmatrix} = \begin{bmatrix} 3^{4-1} & 3^{4-1} & 3^{4-1} \\ 3^{4-1} & 3^{4-1} & 3^{4-1} \\ 3^{4-1} & 3^{4-1} & 3^{4-1} \end{bmatrix}$$

And so on for other values of n .

If we notice each result then we will see that it is of same type that we are trying to prove.

So we can generalise the above results for all $n \in \mathbb{N}$

$$\therefore A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix} \quad (n=1, 2, 3 \dots n)$$

Hence Proved

3. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer

3. It is given that $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To prove : $P(n) : A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbb{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): A^1 = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}, n \in \mathbb{N}$$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$\begin{aligned} A^{k+1} &= A^k \cdot A \\ &= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3(1+2k) - 4k & -4(1+2k) + 4k \\ 3k + 1 - 2k & -4k - 1(1-2k) \end{bmatrix} \\ &= \begin{bmatrix} 3 + 6k - 4k & -4 - 8k + 4k \\ 3k + 1 - 2k & -4k - 1 + 2k \end{bmatrix} \\ &= \begin{bmatrix} 3 + 2k & -4 - 4k \\ 1 + k & -1 - 2k \end{bmatrix} \\ &= \begin{bmatrix} 1 + 2(k+1) & -4(k+1) \\ 1 + k & 1 - 2(k+1) \end{bmatrix} \end{aligned}$$

Therefore, the result is true for $n = k + 1$

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbb{N}$$

4. If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.
4. To prove: $AB - BA$ is a skew symmetric matrix.
Symmetric matrix: A symmetric matrix is a square matrix that is equal to its transpose. In simple words, matrix A is symmetric if

$$A = A'$$

where A' is the transpose of matrix A.

Skew Symmetric matrix: A skew symmetric matrix is a square matrix that is equal to minus of its transpose. In simple words, matrix A is skew symmetric if

$$A = -A'$$

Given: A and B are symmetric matrices i.e.

$$A = A' \quad \dots\dots (1)$$

$$B = B' \quad \dots\dots (2)$$

Now calculating the transpose of $AB - BA$,

$$\begin{aligned}(AB - BA)' &= (AB)' - (BA)' \\ &\text{(By property of transpose i.e. } (A - B)' = A' - B') \\ &= B'A' - A'B' \\ &\text{(By property of transpose i.e. } (AB)' = B'A') \\ &= BA - AB \\ &= -(AB - BA)\end{aligned}$$

Or we can say that: $(AB - BA) = - (AB - BA)'$

Clearly it satisfies the condition of skew symmetric matrix.

Hence $AB - BA$ is a skew symmetric matrix.

5. Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.
5. We suppose that A is a symmetric matrix, then $A' = A \dots(1)$

Consider

$$\begin{aligned}(B'AB)' &= \{B'(AB)\}' \\ &= (AB)'(B')' && [(AB)' = B'A'] \\ &= B'A'(B) && [(B')' = B] \\ &= B'(AB) \\ &= B'(AB) && [\text{Using (1)}]\end{aligned}$$

$$\therefore (B'AB)' = B'AB$$

Thus, if A is a symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew-symmetric matrix.

Then, $A' = -A$

Consider

$$\begin{aligned}(B'AB)' &= [B'(AB)]' = (AB)'(B')' \\ &= (B'A')B = B'(-A)B \\ &= -B'AB\end{aligned}$$

$$\therefore (B'AB)' = -B'AB$$

Thus, if A is a skew-symmetric matrix, then $B'AB$ is a skew -symmetric matrix.

Hence, if A is a symmetric or skew-symmetric matrix, then $B'AB$ is a symmetric or skew symmetric matrix accordingly.

6. Solve system of linear equations, using matrix method.

$$2x - y = -2$$

$$3x + 4y = 3$$

6. Given $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

Transpose of a matrix: If A be an $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. It is denoted by A' or A^T .

\therefore Transpose of A = $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Given equation

$A'A = I$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \cdot 0 + 2y \cdot 2y + z \cdot z & 0 \cdot x + 2y \cdot y + z \cdot (-z) & 0 \cdot x + 2y \cdot (-y) + z \cdot z \\ x \cdot 0 + y \cdot 2y + (-z) \cdot z & x \cdot x + y \cdot y + (-z) \cdot (-z) & x \cdot x + y \cdot (-y) + (-z) \cdot z \\ x \cdot 0 + (-y) \cdot (2y) + z \cdot z & x \cdot x + (-y) \cdot y + z \cdot (-z) & x \cdot x + (-y) \cdot (-y) + z \cdot z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4y^2 + z^2 & 2y^2 - z^2 & -2y^2 + z^2 \\ 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 - y^2 - z^2 \\ -2y^2 + z^2 & x^2 - y^2 - z^2 & x^2 + y^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As these matrices are equal to each other that means each element of matrix on L.H.S is equal each element of matrix on R.H.S.

\therefore On comparing elements on both sides we get

$$4y^2 + z^2 = 1 \quad \dots\dots (1)$$

$$2y^2 - z^2 = 0 \quad \dots\dots (2)$$

$$x^2 + y^2 + z^2 = 1 \quad \dots\dots (3)$$

$$x^2 - y^2 - z^2 = 0 \quad \dots\dots (4)$$

From equation (4) we get,

$$x^2 = y^2 + z^2 \quad \dots\dots (5)$$

Substituting this value in equation (3) we get,

$$2y^2 + 2z^2 = 1 \quad \dots\dots (6)$$

Subtracting equation (2) and (6) we get,

$$3z^2 = 1$$

$$z^2 = 1/3$$

$$z = \pm 1/3$$

Substituting value of z in equation (2) we get,

$$2y^2 = 1/3$$

$$y^2 = 1/6$$

$$y = \pm 1/6$$

Substituting values of y and z in equation (5) we get,

$$x^2 = \frac{1}{3} + \frac{1}{6}$$

$$x^2 = 1/2$$

$$x = \pm 1/2$$

Hence values of x, y, z are $\pm 1/2, \pm 1/6, \pm 1/3$ respectively.

7. For What values of x, $[1 \ 2 \ 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

7. We have

$$[1 \ 2 \ 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow [1+4+1 \quad 2+0 \quad 0+2+2] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [6 \ 2 \ 4] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [6(0) + 2(2) + 4(x)] = O$$

$$\Rightarrow [4 + 4x] = [0]$$

$$\therefore 4 + 4x = 0$$

$$\Rightarrow x = -1$$

Thus, the required value of x is -1 .

8. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$

8. To prove: $A^2 - 5A + 7I = 0$

$$\text{Given: } A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\text{L.H.S: } A^2 - 5A + 7I$$

$$\text{R.H.S} = 0$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Calculating value of A^2 :

$$A^2 = A.A$$

$$\begin{aligned} &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3.3+1.(-1) & 3.1+1.2 \\ (-1).3+2.(-1) & (-1).1+2.2 \end{bmatrix} \\ &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \end{aligned}$$

Substituting value in L.H.S we get,

$$\begin{aligned} &= A^2 - 5A + 7I \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5-(-5)+0 & 3-10+7 \end{bmatrix} \end{aligned}$$

\Rightarrow

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0 = \text{R.H.S}$$

\therefore L.H.S = R.H.S

9. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

9. We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow [x+0-2 \quad 0-10+0 \quad 2x-5-3] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow [x-2 \quad -10 \quad 2x-8] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow [x(x-2) - 40 + 2x - 8] = 0$$

$$\Rightarrow [x^2 - 2x - 40 + 2x - 8] = [0]$$

$$\Rightarrow [x^2 - 48] = [0]$$

$$\therefore x^2 - 48 = 0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm\sqrt{48}$$

10. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	Product		
I	10000	2000	18000
II	6000	20000	8000

- (a) If unit sale prices of x, y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.
- (b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.
10. (a) Given the unit sale prices of x, y and z as Rs 2.50, Rs 1.50 and Rs 1.00 respectively.

Unit sale prices can be represented in form of matrix as: $\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

Calculating total revenue in market I:

Number of products in the form of matrix: $[10000 \quad 2000 \quad 18000]$

So the total revenue is given by:

$$= [10000 \quad 2000 \quad 18000] \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$= [10000.(2.50) + 2000.(1.50) + 18000.(1.00)]$$

$$= [25000 + 3000 + 18000]$$

$$= [46000]$$

∴ Total revenue in market is **Rs 46000**.

Calculating total revenue in market II:

Number of products in the form of matrix: $[6000 \quad 20000 \quad 8000]$

So the total revenue is given by:

$$= [6000 \quad 20000 \quad 8000] \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$= [6000.(2.50) + 20000.(1.50) + 8000.(1.00)]$$

$$= [15000 + 30000 + 8000]$$

$$= [53000]$$

∴ Total revenue in market is **Rs 53000**.

(b) Given the unit cost prices of x, y and z as Rs 2.00, Rs 1.00 and 50 paise respectively.

Calculating gross profit in market I:

Unit cost prices can be represented in form of matrix as:

$$\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

So the total cost of products in market I is given by:

$$= [10000 \quad 2000 \quad 18000] \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$$= [10000.(2.00) + 2000.(1.00) + 18000.(0.50)]$$

$$= [20000 + 2000 + 9000]$$

$$= [31000]$$

Since the total revenue in market I is Rs 46000, the gross profit in this market is given by:

$$(Rs 46000 - Rs 31000)$$

$$= \mathbf{Rs 15000}.$$

Calculating gross profit in market II:

The total cost of products in market II is given by:

$$\begin{aligned}
&= [6000 \quad 20000 \quad 8000] \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} \\
&= [6000.(2.00) + 20000.(1.00) + 8000.(0.50)] \\
&= [12000 + 20000 + 4000] \\
&= [36000]
\end{aligned}$$

Since the total revenue in market II is Rs 53000, the gross profit in this market is given by:

$$\begin{aligned}
&(\text{Rs } 53000 - \text{Rs } 36000) \\
&= \text{Rs } 17000.
\end{aligned}$$

11. Find the matrix X so that $x \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

11. It is given that:

$$x \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let $x = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Therefore, we have:

$$\begin{aligned}
\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} &= \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} &= \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}
\end{aligned}$$

Equating the corresponding elements of the two matrices, we have:

$$a + 4c = -7, \quad 2a + 5c = -8, \quad 3a + 6c = -9$$

$$b + 4d = 2, \quad 2b + 5d = 4, \quad 3b + 6d = 6$$

Now, $a + 4c = -7 \Rightarrow a = -7 - 4c$

$$\therefore 2a + 5c = -8 \Rightarrow -14 - 8c + 5c = -8$$

$$\Rightarrow -3c = 6$$

$$\Rightarrow c = -2$$

$$\therefore a = -7 - 4(-2) = -7 + 8 = 1$$

Now, $b + 4d = 2 \Rightarrow b = 2 - 4d$

$$\begin{aligned} \therefore 2b + 5d = 4 &\Rightarrow 4 - 8d + 5d = 4 \\ &\Rightarrow -3d = 0 \\ &\Rightarrow d = 0 \end{aligned}$$

$$\therefore b = 2 - 4(0) = 2$$

Thus, $a = 1, b = 2, c = -2, d = 0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

12. If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^n A$. Further, prove that $(AB)^n = A^n B^n$ for all $n \in \mathbb{N}$

12. To prove: $AB^n = B^n A$

Given A and B are square matrices of same order such that $AB = BA$.

We have to prove it using mathematical induction.

Steps involved in mathematical induction are-

1. Prove the equation for $n=1$
2. Assume the equation to be true for $n=k$, where $k \in \mathbb{N}$
3. Finally prove the equation for $n=k+1$

Let $P(n): AB^n = B^n A$

For $n=1$,

$$\text{L.H.S: } AB^1 = AB$$

$$\text{R.H.S: } B^1 A = B A = AB$$

So, L.H.S=R.H.S

$\therefore P(n)$ is true for $n=1$.

Now assuming $P(n)$ to be true for $n=k$, where $k \in \mathbb{N}$

$$P(k): AB^k = B^k A \quad \dots\dots (1)$$

Now proving for $n=k+1$, i.e. $P(k+1)$ is also true

$$\text{L.H.S} = AB^{k+1}$$

$$= AB^{k+1}$$

$$= (AB^k).B$$

$$= (B^k A).B$$

$\dots\dots$ from (1)

$$= B^k(A.B)$$

$$= B^k(BA)$$

$(\because AB = BA)$

$$= B^{k+1} A$$

$$\text{R.H.S} = B^{k+1} A$$

$$= B^{k+1} A$$

\therefore L.H.S=R.H.S

All conditions are proved. Hence $P(k+1)$ is true.

\therefore By mathematical induction we have proved that $AB^n = B^n A$.

Now, to prove: $(AB)^n = A^n B^n$ for all $n \in \mathbb{N}$

For $n=1$,

$$\text{L.H.S} = (AB)^n = (AB)^1 = AB$$

$$\text{R.H.S} = A^n B^n = A^1 B^1 = AB$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

\therefore It is true for $n=1$

Assuming it to be true for $n=k$ then,

$$(AB)^k = A^k B^k \quad \dots\dots (2)$$

Now proving for $n=k+1$,

$$\begin{aligned} \text{L.H.S} &= (AB)^{k+1} \\ &= (AB)^k (AB)^1 \\ &= (A^k B^k) AB \\ &= A^k (B^k \cdot A) B \\ &= A^k (A \cdot B^k) B && (AB^n = B^n A) \\ &= (A^k A) (B^k B) \\ &= A^{k+1} B^{k+1} \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= A^{k+1} B^{k+1} \\ &= A^{k+1} B^{k+1} \end{aligned}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

All conditions are proved. Hence $P(k+1)$ is true.

\therefore By mathematical induction we have proved that $(AB)^n = A^n B^n$ for all $n \in \mathbb{N}$

Hence proved.

13. Choose the correct answer in the following questions:

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

A. $1 + \alpha^2 + \beta\gamma = 0$

B. $1 - \alpha^2 + \beta\gamma = 0$

C. $1 - \alpha^2 - \beta\gamma = 0$

D. $1 + \alpha^2 - \beta\gamma = 0$

13. $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$$\begin{aligned} \therefore A^2 &= A \cdot A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \\ &= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} \\ &= \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^2 = I \Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have;

$$\alpha^2 + \beta\gamma = 1$$

$$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$

$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$$

14. If the matrix A is both symmetric and skew symmetric, then
- A is a diagonal matrix
 - A is a zero matrix
 - A is a square matrix
 - None of these Answer

14. The correct option is **B**.

Explanation. Given A is both symmetric and skew symmetric matrix then, $A = A'$ and also $A = -A'$

$$\Rightarrow A' = -A'$$

$$\Rightarrow 2A' = 0$$

$$\Rightarrow A' = 0$$

Clearly it is observed that transpose of A is a null matrix or zero matrix then matrix A must also be a zero matrix.

Hence A is a zero matrix.

15. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to
- A
 - I - A
 - I
 - 3A

15. (C)

$$\begin{aligned} (I + A)^3 - 7A &= I^3 + A^3 + 3I^2A + 3A^2I - 7A \\ &= I + A^3 + 3A + 3A^3 - 7A \\ &= 1 + A^2 \cdot A + 3A + 3A - 7A \quad [A^2 = A] \\ &= 1 + A \cdot A - A \\ &= I + A^2 - A \\ &= I + A - A \\ &= I \end{aligned}$$

$$\therefore (I + A)^3 - 7A = I$$