Slope of AB = $\left(\frac{0+1}{5-6}\right) = \frac{1}{-1} = -1$

Slope of BC = $\left(\frac{3-0}{2-5}\right) = \frac{3}{-3} = -1$

Slope of CA = $\left(\frac{3+1}{2-6}\right) = \frac{4}{-4} = -1$

Therefore slopes of AB, BC and CA are equal, so Points A,B,C are collinear.

Q. 11. Using slopes, find the value of x for which the points A(5, 1), B(1, -1) and C(x, 4) are collinear.

Answer : For three points to be collinear, the slope of all pairs must be equal, that is the slope of AB = slope of BC = slope of CA

Given points are A(5, 1), B(1, -1) and C(x, 4)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of AB = $\left(\frac{-1-1}{1-5}\right) = \frac{-2}{-4} = \frac{1}{2} = 0.5$

The slope of BC = $\left(\frac{4+1}{x-1}\right) = \left(\frac{5}{x-1}\right)$

Slope of CA = $\left(\frac{4-1}{x-5}\right) = \left(\frac{3}{x-5}\right)$

The slope of all lines must be the same

$$\Rightarrow 0.5 = \left(\frac{5}{x-1}\right)$$

$$\Rightarrow 0.5x - 0.5 = 5$$

$$\Rightarrow 0.5x = 5.5$$

$\Rightarrow x = 11$

Note:- We can use any two points to get the value of “x”.

Q. 12. Using slopes show that the points A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3) taken in order, are the vertices of a rectangle.

Answer : A rectangle has all sides perpendicular to each other, so the product of slope of every adjacent line is equal to -1.

Given point in order are A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\text{Slope of AB} = \left(\frac{-4+1}{-2+4}\right) = \frac{-3}{2}$$

$$\text{Slope of BC} = \left(\frac{0+4}{4+2}\right) = \frac{4}{6} = \frac{2}{3}$$

$$\text{The slope of CD} = \left(\frac{3-0}{2-4}\right) = \frac{3}{-2}$$

$$\text{Slope of DA} = \left(\frac{3+1}{2+4}\right) = \frac{4}{6} = \frac{2}{3}$$

⇒slopeofAB × slopeofBC

$$\Rightarrow \frac{-3}{2} \times \frac{2}{3} = -1$$

Hence AB is perpendicular to BC

Slope of BC × slope of CD

$$\frac{2}{3} \times \frac{3}{-2} = -1$$

Hence BC is perpendicular to CD

Slope of CD × slope of DA

$$\Rightarrow \frac{3}{-2} \times \frac{2}{3} = -1$$

Hence CD is perpendicular to DA

Slope of DA × slope of AB

$$\Rightarrow \frac{2}{3} \times \frac{-3}{2} = -1$$

Hence DA is perpendicular to AB.

All angles are 90°.

So this is a rectangle ABCD.

Q. 13. Using slopes. Prove that the points A(-2, -1), B(1,0), C(4, 3) and D(1, 2) are the vertices of a parallelogram.

Answer : The property of parallelogram states that opposite sides are equal.

We have 4 sides as AB,BC,CD,DA

Given points are A(-2,-1),B(1,0),C(4,3) and D(1,2)

AB and CD are opposite sides, and BC and DA are the other two opposite sides.

So slopes of AB = CD and slopes BC = DA

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\text{Slope of AB} = \left(\frac{0+1}{1+2}\right) = \frac{1}{3}$$

The slope of BC = $\left(\frac{3-0}{4-1}\right) = \frac{3}{3} = 1$

The slope of CD = $\left(\frac{2-3}{1-4}\right) = \frac{-1}{-3} = \frac{1}{3}$

Slope of DA = $\left(\frac{2+1}{1+2}\right) = \frac{3}{3} = 1$

Therefore the Slope of AB = Slope of CD and

The slope of BC = Slope of DA

Also, the product of slope of two adjacent sides is not equal to -1, therefore it is not a rectangle.

Hence ABCD is a parallelogram.

Q. 14. If the three points A(h, k), B(x_1, y_1) and C(x_2, y_2) lie on a line then show that ($h - x_1$)($y_2 - y_1$) = ($k - y_1$)($x_2 - x$).

Answer : For the lines to be in a line, the slope of the adjacent lines should be the same.

Given points are A(h,k),B(x_1,y_1) and C(x_2,y_2)

So slope of AB = BC = CA

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of AB = $\left(\frac{y_1-k}{x_1-h}\right)$

Slope of BC = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

Slope of CA = $\left(\frac{y_2-k}{x_2-h}\right)$

$$\Rightarrow\left(\frac{y_1-k}{x_1-h}\right) = \left(\frac{y_2-y_1}{x_2-x_1}\right) = \left(\frac{y_2-k}{x_2-h}\right)$$

Now Cross multiplying the first two equality,

$$(y_1 - k)(x_2 - x_1) = (x_1 - h)(y_2 - y_1)$$

$$\Rightarrow (h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$$

Hence proved.

Q. 15 If the points A(a, 0), B(0, b) and P(x, y) are collinear, using slopes, prove that

$$\frac{x}{a} + \frac{y}{b} = 1$$

Answer : Given points are A(a,0),B(0,b) and P(x,y)

For three points to be collinear, the slope of all pairs must be equal, that is the slope of AB = slope of BP = slope of PA.

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of AB = $\left(\frac{b-0}{0-a}\right) = \frac{b}{-a}$

Slope of BP = $\left(\frac{y-b}{x-0}\right) = \frac{y-b}{x}$

Slope of PA = $\left(\frac{y-0}{x-a}\right) = \frac{y}{x-a}$

Now Slope of AB = BP = PA

$$\frac{b}{-a} = \frac{y-b}{x} = \frac{y}{x-a}$$

Using the first two equality

$$\Rightarrow \frac{b}{-a} = \frac{y-b}{x}$$

$$\Rightarrow bx = -a(y-b)$$

$$\Rightarrow bx = -ay + ab$$

Dividing the equation by “ab”, We get

$$\frac{x}{a} = -\frac{y}{b} + 1$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

Hence proved.

Q. 16. **A line passes through the points A(4, -6) and B(-2, -5). Show that the line AB makes an obtuse angle with the x-axis.**

Answer : For the line to make an obtuse angle with X-axis, the angle of the line should be greater than 90

For the angle to be greater than 90°, tanθ must be negative

Where tanθ is the slope of the line.

Given points are A(4, -6) and B(-2, -5)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

The slope of line AB is $\left(\frac{-5+6}{-2-4}\right) = \frac{1}{-6} = \frac{-1}{6}$

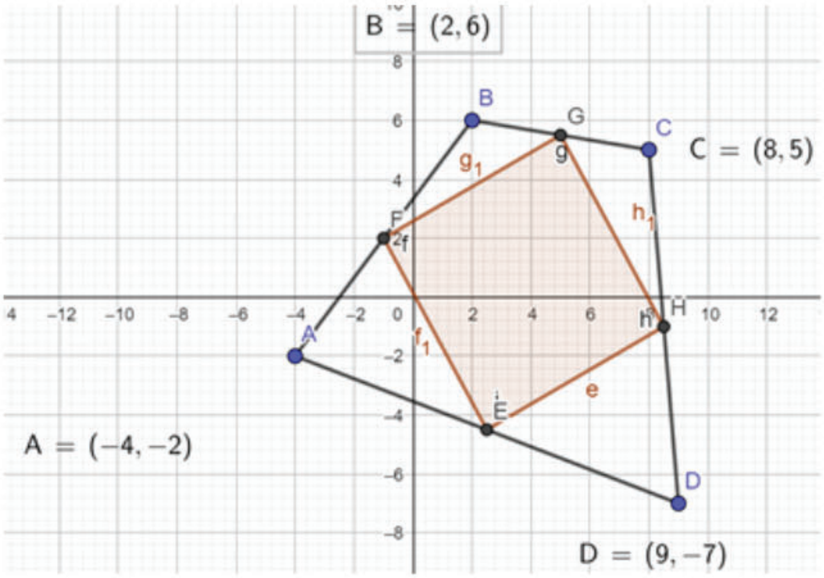
Which is less than 0, hence negative.

$\Rightarrow \tan\theta = \frac{-1}{6} \langle 0$,tanθ is negative in 2^{nd} quadrant whose angle is >90°.

So line AB makes obtuse angle(>90) with the X-axis.

Q. 17. The vertices of a quadrilateral are A(-4, -2), B(2, 6), C(8, 5) and D(9, -7). Using slopes, show that the midpoints of the sides of the quad. ABCD from a parallelogram.

Answer :



The vertices of the given quadrilateral are A(-4,-2) B(2, 6), C(8, 5) and D(9, -7)

The mid point of a line A(x_1,y_1) and B(x_2,y_2) is found out by $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Now midpoint of AB = $\left(\frac{-4+2}{2},\frac{-2+6}{2}\right) = (-1,2)$

The midpoint of BC = $\left(\frac{2+8}{2},\frac{6+5}{2}\right) = (5,5.5)$

The midpoint of CD = $\left(\frac{8+9}{2},\frac{5-7}{2}\right) = (8.5,-1)$

Midpoint of DA = $\left(\frac{-4+9}{2},\frac{-2-7}{2}\right) = (2.5,-4.5)$

So now we have four points

P(-1,2),Q(5,5.5),R(8.5,-1),S(2.5,-4.5)

$$\text{slope} = \left(\frac{y_2-y_1}{x_2-x_1}\right)$$

Slope of PQ = $\left(\frac{5.5-2}{5+1}\right) = \frac{3.5}{6} = \frac{7}{12}$

Slope of QR = $\left(\frac{-1-5.5}{8.5-5}\right) = \frac{-6.5}{3.5} = \frac{-1.3}{0.7} = \frac{-13}{7}$

Slope of RS = $\left(\frac{-4.5+1}{2.5-8.5}\right) = \frac{-3.5}{-6} = \frac{7}{12}$

Slope of SP = $\left(\frac{-4.5-2}{2.5+1}\right) = \frac{-6.5}{3.5} = \frac{-13}{7}$

Now we can observe that slope of PQ = RS and slope of QR = SP

Which shows that line PQ is parallel to RS and line QR is parallel to SP

Also, the product of two adjacent lines is not equal to -1

Therefore PQRS is a parallelogram.

Q. 18. Find the slope of the line which makes an angle of 30^0 with the positive direction of the y-axis, measured anticlockwise.

Answer : According to the given figure, the angle made by the line from X-axis is 90+30 = 120°

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

We also know that slope of a line is equal to tanθ, Where

θ = 120°

tan(120°) = tan(90°+30°) = -cot(30°) = -√3

Therefor the slope of the given line is -√3.

Q.19.

Find the angle between the lines whose slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$.

Answer : To find out the angle between two lines, the angle is equal to the difference in θ.

The slope of a line = $\tan\theta = \left(\frac{y_2-y_1}{x_2-x_1}\right)$

So slope of the first line = $\sqrt{3} = \tan\theta_1 \Rightarrow \tan\theta_1 = \sqrt{3}$

$\Rightarrow \theta_1 = \tan^{-1}(\sqrt{3})$

$\Rightarrow \theta_1 = 60°$

The slope of the second line = $\frac{1}{\sqrt{3}} = \tan\theta_2 \Rightarrow \theta_2 = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \theta_2 = 30°$

Now the difference between the two lines is θ_1-θ_2

= 60°-30°

= 30°

Q. 20. Find the angle between the lines whose slopes are

$\left(2-\sqrt{3}\right)$ **and** $\left(2+\sqrt{3}\right)$

Answer : We know that if slope of two lines are m1 and m2 respectively, then the angle between them is given by

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Here $m_2 = 2 + \sqrt{3}$ and $m_1 = 2 - \sqrt{3}$

$$\tan\theta = \frac{(2+\sqrt{3}) - (2-\sqrt{3})}{1 + (2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{2\sqrt{3}}{1 + \left(2^2 - \left(\sqrt{3}\right)^2\right)}$$

$$= \frac{2\sqrt{3}}{1 + 1} = \sqrt{3}$$

$$\tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\sqrt{3}\right)$$

$$\Rightarrow \theta = 60°$$

Where θ is the angle between two lines.

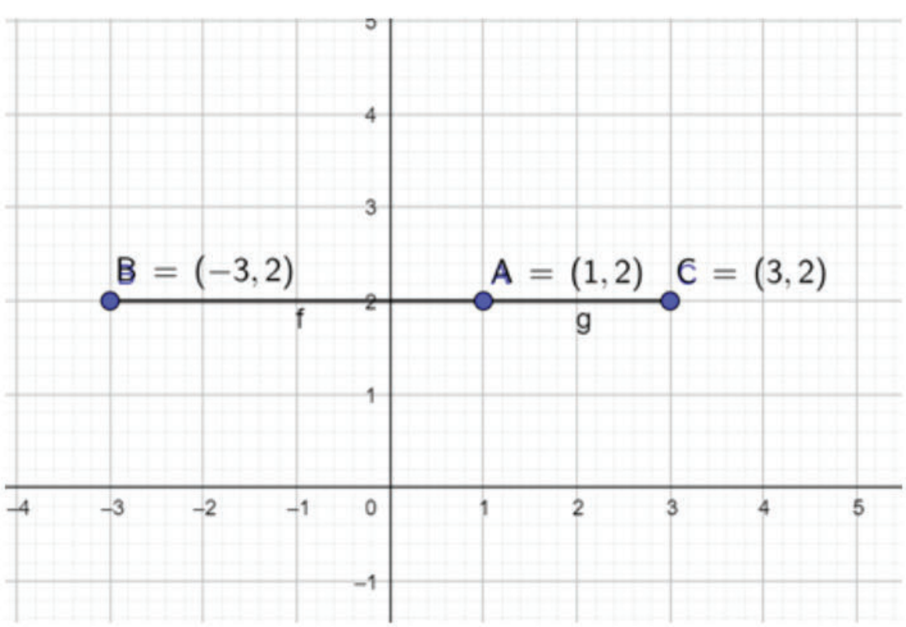
Q. 21. If A(1, 2), B(-3, 2) and C(3, 2) be the vertices of a ΔABC, show that

(i) $\tan A = 2$

(ii) $\tan B = \frac{2}{3}$

(iii) $\tan C = \frac{4}{7}$

Answer : Points A,B,C lie on a same line, therefore the slope of each line is same and hence it does not form a triangle.



Q. 22. If θ is the angle between the lines joining the points (0, 0) and B(2, 3), and the points C(2, -2) and D(3, 5), show that

$$\tan\theta = \frac{11}{23}.$$

Answer : The given points are A(0,0),B(2,3) and C(2,-2),D(3,5).

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

The slope of line AB is $\left(\frac{3-0}{2-0}\right) = \frac{3}{2} = m_1$

And the slope of line CD is $\left(\frac{5+2}{3-2}\right) = 7 = m_2$

We know that angle between two lines with their slopes as m_1 and m_2 is given by

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{7 - \frac{3}{2}}{1 + 7 \times \frac{3}{2}}$$

$$= \frac{\frac{14-3}{2}}{\frac{2+21}{2}}$$

$$= \frac{11}{23}$$

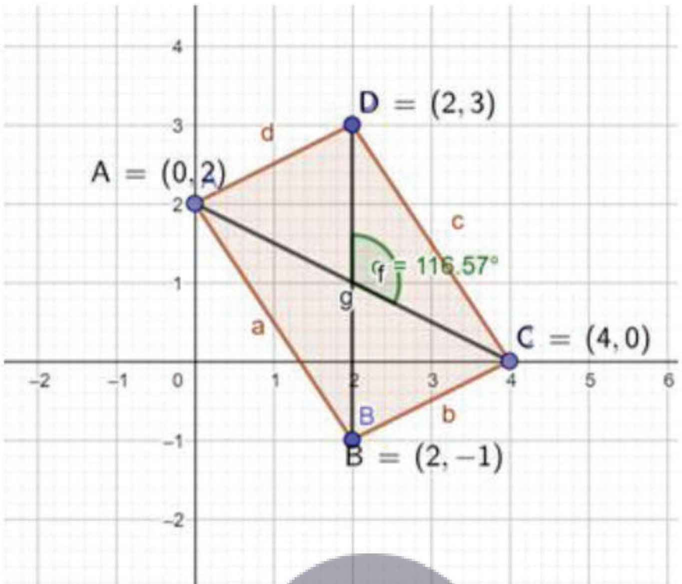
$$\Rightarrow \tan\theta = \frac{11}{23}$$

Hence proved.

Q. 23. If θ is the angle between the diagonals of a parallelogram ABCD whose vertices are A(0, 2), B(2,-1), C(4,

Answer : Given points of the parallelogram are A(0, 2), B(2,-1), C(4, 0) and D(2, 3)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$



The slope of diagonal AC = $\left(\frac{0-2}{4-0}\right) = \frac{-2}{4} = \frac{-1}{2} = m_1$

The slope of diagonal BD = $\left(\frac{3+1}{2-2}\right) = \frac{4}{0} = \infty = m_2$

So diagonal BD is perpendicular to X-axis. Hence it is parallel to Y-axis.

Product of slope of two diagonals is equal to -1.

$$m_1 \times m_2 = -1$$

$$\Rightarrow \left(\frac{-1}{2}\right) \times \tan\theta = -1$$

$$\Rightarrow \tan\theta = 2$$

Hence proved.

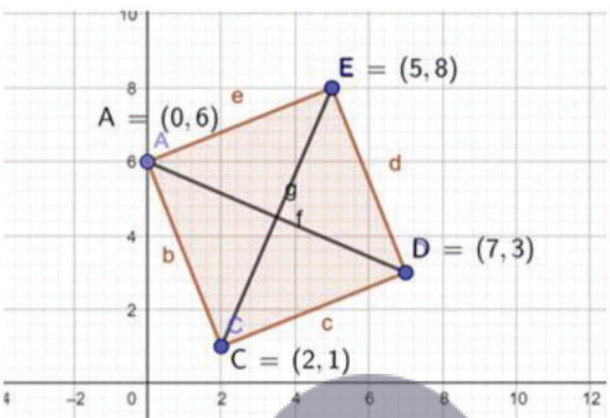
Q. 24. Show that the points A(0, 6), B(2, 1) and C(7, 3) are three corners of a square ABCD. Find (i) the slope of the diagonal BD and (ii) the coordinates of the fourth vertex D.

Answer : In a square, all sides are perpendicular to the adjacent side, so the product of slope of two adjacent sides is -1.

Let the position of point D(a,b).

Given points of the square are A(0, 6),B(2, 1),C(7, 3) and D(a,b).

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$



$$\text{The slope of line AB} = \left(\frac{1-6}{2-0}\right) = \frac{-5}{2} = m_1$$

$$\text{The slope of line BC} = \left(\frac{3-1}{7-2}\right) = \frac{2}{5} = m_2$$

$$\text{The slope of line CD} = \left(\frac{b-3}{a-7}\right) = m_3$$

$$\text{The slope of line DA} = \left(\frac{b-6}{a-0}\right) = \frac{b-6}{a} = m_4$$

$$\text{The slope of diagonal AC} = \left(\frac{3-6}{7-0}\right) = \frac{-3}{7}$$

The slope of diagonal BD = m_5

(i) We know that in a square, two diagonals are perpendicular to each other, therefore

The slope of diagonal AC×slope of diagonal BD = -1

$$m_5 \times \frac{-3}{7} = -1$$

$$\Rightarrow m_5 = \frac{7}{3}$$

So the slope of diagonal BD is 7/3.

(ii) We know that midpoint of diagonal AC = midpoint of diagonal BD

$O\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ and comparing x and y coordinates respectively.

$$\left(\frac{7+0}{2}, \frac{3+6}{2}\right) = \left(\frac{a+2}{2}, \frac{b+1}{2}\right)$$

$$\Rightarrow \left(\frac{7}{2}, \frac{9}{2}\right) = \left(\frac{a+2}{2}, \frac{b+1}{2}\right)$$

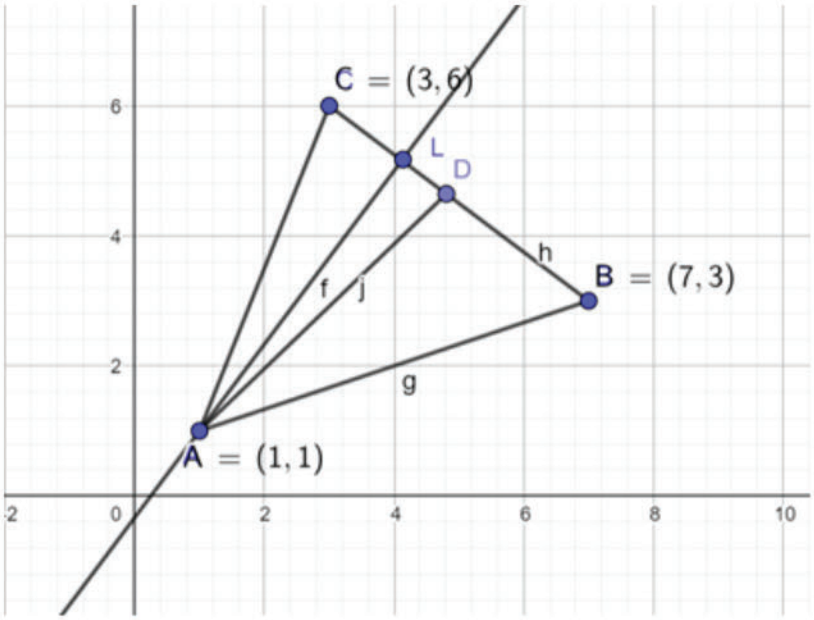
$$\Rightarrow \frac{7}{2} = \frac{a+2}{2} \& \frac{9}{2} = \frac{b+1}{2}$$

$$\Rightarrow a = 5 \& b = 8$$

So coordinate of the point D(5,8).

Q. 25. A(1, 1), B(7, 3) and C(3, 6) are the vertices of a ΔABC. If D is the midpoint of BC and AL ⊥ BC, find the slopes of (i) AD and (ii) AL.

Answer :



Given points are

A(1, 1), B(7, 3) and C(3, 6)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of line BC = $\left(\frac{3-6}{7-3}\right) = \frac{-3}{4}$

(i) As D is the midpoint of BC, coordinate of D are $D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{7+3}{2}, \frac{3+6}{2}\right) = \left(5, \frac{9}{2}\right)$$

Now the slope of AD = $\left(\frac{\frac{9}{2}-1}{5-1}\right) = \left(\frac{\frac{7}{2}}{4}\right) = \frac{3.5}{4}$

(ii) As AL is perpendicular to BC

The slope of AL×slope of BC = -1

Let slope of AL be m_1

$$\frac{-3}{4} \times m_1 = -1$$

$$\Rightarrow m_1 = \frac{4}{3}$$

So Slope of line AL is $\frac{4}{3}$.

Exercise 20C

Q. 1. Find the equation of a line parallel to the x - axis at a distance of

(i) 4 units above it

(ii) 5 units below it

Answer : (i) Equation of line parallel to x - axis is given by y = constant, as the y - coordinate of every point on the line parallel to x - axis is 4,i.e. constant. Now the point lies above x - axis means in positive direction of y - axis,

So, the equation of line is given as y = 4.

(ii) Equation of line parallel to x - axis is given by y = constant, as the y - coordinate of every point on the line parallel to x - axis is - 5 i.e. constant. Now the point lies below x - axis means in negative direction of y - axis,

So, the equation of line is given as y = - 5.

Q. 2. Find the equation of a line parallel to the y - axis at a distance of

(i) 6 units to its right

(ii) 3 units to its left

Answer : (i) Equation of line parallel to y - axis is given by x = constant, as the x - coordinate of every point on the line parallel to y - axis is 6 i.e. constant. Now the point lies to the right of y - axis means in the positive direction of x - axis,

So, required equation of line is x = 6.

(ii) Equation of line parallel to y - axis is given by x = constant, as the x - coordinate of every point on the line parallel to y - axis is - 3. Now point lies to the left of y - axis means in the negative direction of x - axis, So, required equation of line is given as x = - 3.

Q. 3. Find the equation of a line parallel to the x - axis and having intercept - 3 on the y - axis.

Answer: Equation of line parallel to x - axis is given by y = constant, as x - coordinate of every point on the line parallel to y - axis is - 3 i.e. constant.

So, the required equation of line is y = - 3.

Q. 4. Find the equation of a horizontal line passing through the point (4, - 2).

Answer : Equation of line parallel to x - axis (horizontal) is y = constant, as y - coordinate of every point on the line parallel to x - axis is - 2 i.e. constant. Therefore equation of the line parallel to x - axis and passing through (4, - 2) is y = - 2.

Q. 5. Find the equation of a vertical line passing through the point (- 5, 6).

Answer : Equation of line parallel to y - axis (vertical) is given by x = constant, as x - coordinate is constant for every point lying on line i.e. 6.

So, the required equation of line is given as x = 6.

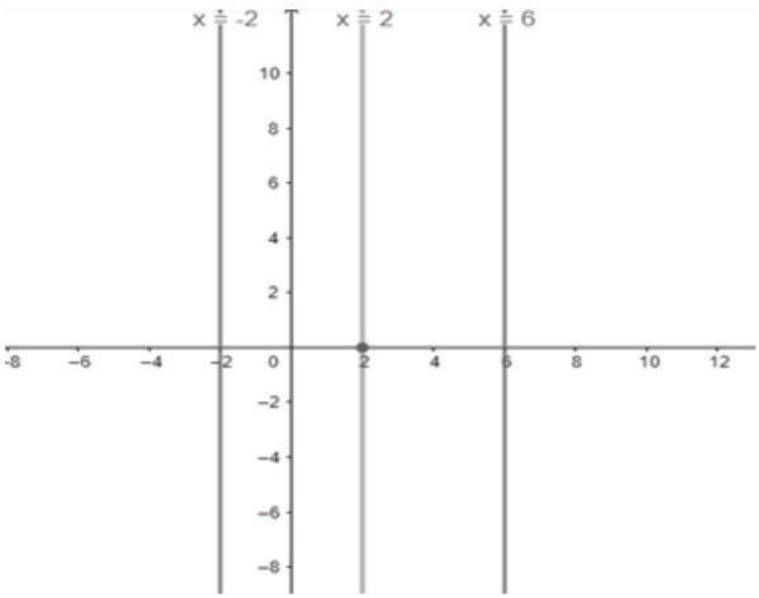
Q. 6. Find the equation of a line which is equidistant from the lines x = - 2 and x = 6.

Answer : For the equation of line equidistant from both lines, we will find point through which line passes and is equidistant from both line.

As any point lying on x = - 2 line is (- 2, 0) and on x = 6 is (6, 0), so mid - point is

$$(x, y) = \left(\frac{-2+6}{2}, \frac{0+0}{2}\right)$$

(x, y) = (2,0)



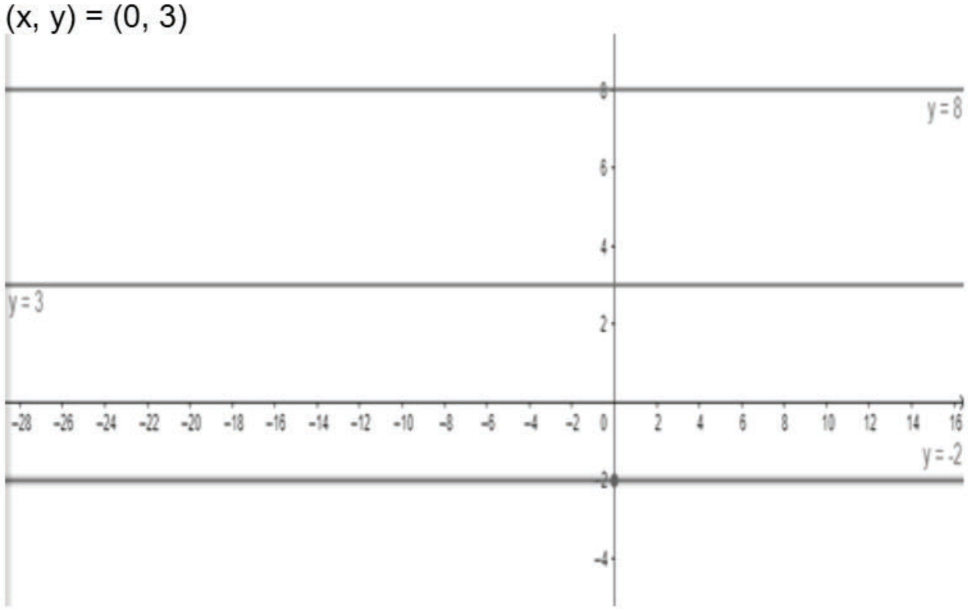
So, equation of line is x = 2.

Q. 7. Find the equation of a line which is equidistant from the lines y = 8 and y = -2.

Answer : For the equation of line equidistant from both lines, we will find point through which line passes and is equidistant from both line.
As any point lying on y = 8 line is (0, 8) and on y = - 2 is (0, - 2), so mid - point is

$$(x, y) = \left(\frac{0+0}{2}, \frac{8-2}{2}\right)$$

(x, y) = (0, 3)



So, equation of line is y = 3.

Q. 8 A. Find the equation of a line

whose slope is 4 and which passes through the point (5, - 7)

Answer : As slope is given m = 4 and passing through (5, - 7).using slope - intercept form of equation of line, we will find value of intercept first

y = mx + c ………..(1)

- 7 = 4(5) + c

- 7 = 20 + c

c = - 7 - 20

c = - 27

Putting the value of c in equation (1), we have

y = 4x + (- 27)

y = 4x - 27

4x - y - 27 = 0

So, the required equation of line is 4x - y - 27 = 0.

Q. 8 B. Find the equation of a line

whose slope is - 3 and which passes through the point (- 2, 3);

Answer : As slope is given m = - 3 and line is passing through point (- 2, 3).Using slope - intercept form of equation of line, we will find intercept first

y = mx + c ……………….(1)

3 = - 3(- 2) + c

3 = 6 + c

c = 3 - 6

c = - 3

Putting the value of c in equation (1), we have

y = - 3x + (- 3)

y = - 3x - 3

3x + y + 3 = 0

So, the required equation of line is 3x + y + 3 = 0.

Q. 8 C. Find the equation of a line

which makes an angle of $\frac{2\pi}{3}$ with the positive direction of the x – axis and passes through the point (0, 2)

Answer : We have given angle so we have to find slope first given by m = tanθ.

$$m = \tan\theta \Rightarrow \tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{3}\right)$$

$$m \Rightarrow -\tan\left(\frac{\pi}{3}\right) = -(\sqrt{3})$$ (tanx is negative in II quadrant)

$$m = -\sqrt{3}$$

Now the line is passing through the point (0, 2).Using the slope - intercept form of the equation of the line, we will find intercept

y = mx + c ……………….(1)

$$2 = -(\sqrt{3})(0) + c \Rightarrow c = 2$$

Putting the value of c in equation(1),we have

$$y = -(\sqrt{3})x + 2$$

$$-(\sqrt{3})x - y + 2 = 0$$

So, required equation of line is $-(\sqrt{3})x - y + 2 = 0.$

Q. 9. Find the equation of a line whose inclination with the x - axis is 30^0 and which passes through the point (0, 5).

Answer : As angle is given so we have to find slope first given by m = tanθ

m = tan30°

$$m = \frac{1}{\sqrt{3}}$$

Now the line is passing through the point (0, 5).using slope - intercept form of the equation of the line, we will find the intercept

y = mx + c ……………….(1)

$$5 = \frac{1}{\sqrt{3}}(0) + c \Rightarrow c = 5$$

Putting the value of c in equation (1),we have

$$y = \frac{1}{\sqrt{3}}x + 5$$

$$x - (\sqrt{3})y + 5\sqrt{3} = 0$$

So, required equation of line is $x - (\sqrt{3})y + 5\sqrt{3} = 0.$

Q. 10. Find the equation of a line whose inclination with the x - axis is 150^0 and which passes through the point (3, - 5).

Answer : As angle is given so we have to find slope first give by m = tanθ

m = tan150°

$m = \tan(180° - 30°) \Rightarrow -\tan 30° = -\frac{1}{\sqrt{3}}$ (tan (180° - θ) is in II quadrant, tanx is negative)

Now the line is passing through the point (3, - 5).Using the slope - intercept form of the equation of the line, we will find the intercept y = mx + c ……………….(1)

$$-5 = -\frac{1}{\sqrt{3}}(3) + c \Rightarrow c = -5 + \sqrt{3}$$

Putting the value of c in equation (1),we have

$$y = -\frac{1}{\sqrt{3}}x + \left(-5 + \sqrt{3}\right)$$

$$x + \left(\sqrt{3}\right)y + 5\sqrt{3} - 3 = 0$$

So, required equation of line is $x + \left(\sqrt{3}\right)y + 5\sqrt{3} - 3 = 0.$

Q. 11. Find the equation of a line passing through the origin and making an angle of 120^0 with the positive direction of the x - axis.

Answer : As angle is given so we have to find slope first give by m = tanθ

m = tan120°

$$m = \tan\left(180° - 60°\right) \Rightarrow -\tan 60° = -(\sqrt{3})$$

(tan (180° - θ) is in II quadrant, tanx is negative)

Now equation of line passing through origin is given as y = mx

$$y = -\left(\sqrt{3}\right)x$$

$$\left(\sqrt{3}\right)x + y = 0$$

So, required equation of line is $\left(\sqrt{3}\right)x + y = 0$

Q. 12. Find the equation of a line which cuts off intercept 5 on the x - axis and makes an angle of 600 with the positive direction of the x - axis.

Answer : As intercept is given i.e. c = 5 and angle given so first we will find slope of line.

m = tanθ

$$m = \tan 60° \Rightarrow \sqrt{3}$$

Now using slope intercept form of the equation of a line

y = mx + c

$$y = (\sqrt{3})x + 5$$

$$(\sqrt{3})x - y + 5 = 0$$

So, the required equation of line is $(\sqrt{3})x - y + 5 = 0$.

Q. 13. Find the equation of the line passing through the point P(4, - 5) and parallel to the line joining the points A(3, 7) and B(- 2, 4).

Answer :. As two points passing through a line parallel to the line are given, we will calculate slope using two points(slope of parallel lines is equal).

$$m = \frac{y_{2-y_1}}{x_2 - x_1} \Rightarrow \frac{4-7}{-2-3} = \frac{-3}{-5}$$

$$m = \frac{3}{5}$$

Now using the slope - intercept form, we will find intercept for a line passing through (4, - 5)

y = mx + c ………………….(1)

$$-5 = \frac{3}{5}(4) + c \Rightarrow -5 - \frac{12}{5} = c$$

$$c = \frac{-25-12}{5} \Rightarrow c = -\frac{37}{5}$$

Putting value in equation (1)

$$y = \frac{3}{5}(x) + \left(\frac{-37}{5}\right) \Rightarrow 3x - 5y - 37 = 0$$

So, the required equation of line is $3x - 5y - 37 = 0$.

Q. 14. Find the equation of the line passing through the point P(- 3, 5) and perpendicular to the line passing through the points A(2, 5) and B(- 3, 6)

Answer : As two points passing through line perpendicular to the line are given, we will calculate slope using two points. Let slopes of the two lines be m1 and m2.

$$m_1 = \frac{y_{2-y_1}}{x_2 - x_1} \Rightarrow \frac{6-5}{-3-2} = -\frac{1}{5}$$

$$m_1 = -\frac{1}{5}$$

Now the slope of the equation can be found using

$m_1 m_2 = -1$ where m_1, m_2 are slopes of two perpendicular lines

$$\frac{-1}{5}.m_2 = -1 \Rightarrow m_2 = 5$$

Using slope - intercept form we will find intercept for line passing through (- 3, 5)

y = mx + c ………………….(1)

5 = 5(- 3) + c

c = 5 + 15

c = 20

Putting value in equation (1)

y = 5x + 20

5x - y + 20 = 0

So, the required equation of line 5x - y + 20 = 0.

Q. 15 A. Find the slope and the equation of the line passing through the points:

(i) (3, - 2) and (- 5, - 7)

Answer : Slope of equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-7-(-2)}{-5-3} = \frac{-5}{-8}$$

$$m = \frac{5}{8}$$

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

$$y-(-2) = \frac{5}{8}(x-3) \Rightarrow 8(y+2) = 5(x-3)$$

8y + 16 = 5x - 15

5x - 8y - 16 - 15 = 0

5x - 8y - 31 = 0

So, required equation of line is 5x - 8y - 31 = 0.

Q. 15 B. Find the slope and the equation of the line passing through the points:

(- 1, 1) and (2, - 4)

Answer : The slope of the equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-4-1}{2-(-1)} = \frac{-5}{3}$$

$$m = -\frac{5}{3}$$

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

$$y - 1 = \frac{-5}{3}(x - (-1)) \Rightarrow 3(y-1) = -5(x+1)$$

3y - 3 + 5x + 5 = 0

5x + 3y + 2 = 0

So, required equation of line is 5x - 8y - 31 = 0.

Q. 15 C

Find the slope and the equation of the line passing through the points:

(5, 3) and (- 5, - 3)

Answer : The slope of the equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-3-3}{-5-5} = \frac{-6}{-10}$$

$$m = \frac{3}{5}$$

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

$$y - 3 = \frac{3}{5}(x - 5) \Rightarrow 5(y - 3) = 3(x - 5)$$

3x - 15 - 5y + 15 = 0

3x - 5y = 0

So, required equation of line is 3x - 5y = 0.

Q. 15 D. Find the slope and the equation of the line passing through the points:

(a, b) and (- a, b)

Answer : The slope of the equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{b - b}{-a - a} = 0$$

m = 0 (Horizontal line)

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

y - b = 0(x - a)

y = b

So, required equation of line is y = b.

Q. 16. Find the angle which the line joining the points $(1, \sqrt{3})$ and $(\sqrt{2}, \sqrt{6})$ makes with the x - axis.

Answer : To find angle, we will find slope using two points

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{(\sqrt{6}) - (\sqrt{3})}{(\sqrt{2}) - 1} = \frac{(\sqrt{3})((\sqrt{2}) - 1)}{((\sqrt{2}) - 1)}$$

$m = \sqrt{3}$

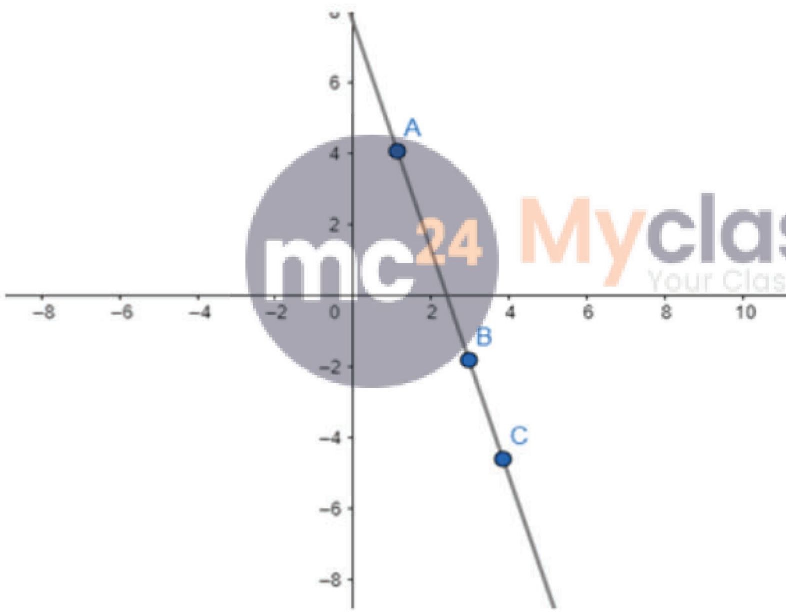
Now as we have m = tanθ

$$\tan\theta = \left(\sqrt{3}\right) \Rightarrow \theta = 60^\circ$$

So, angle line makes with the positive x - axis is 60°.

Q. 17. Prove that the points A(1, 4), B(3, - 2) and C(4, - 5) are collinear. Also, find the equation of the line on which these points lie.

Answer : If two lines having the same slope pass through a common point, then two lines will coincide. Hence, if A, B and C are three points in the XY - plane, then they will lie on a line, i.e., three points are collinear if and only if slope of AB = slope of BC.



Slope of AB = slope of BC

$$\frac{-2-4}{3-1} = \frac{-5-(-2)}{4-3} \Rightarrow \frac{-6}{2} = \frac{-3}{1}$$

- 3 = - 3

Hence verified, i.e. points are collinear. Now using two point form of the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

y - 4 = - 3(x - 1)

y - 4 + 3x - 3 = 0

3x + y - 7 = 0

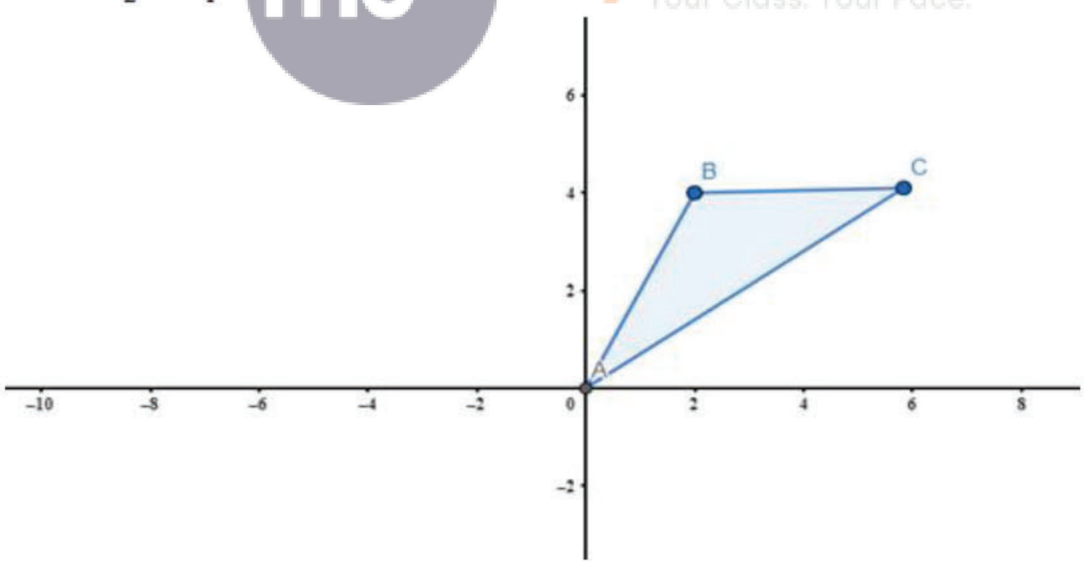
So, required equation of line is 3x + y - 7.

Q. 18. If A(0, 0), b(2, 4) and C(6, 4) are the vertices of a ΔABC, find the equations of its sides.

Answer : Using two point form equation of lines AB, BC and AC can be find. Now A is origin so the lines passing through A (origin) are simply y = mx so we have to find slope of AB and AC.

For line AB,

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{4 - 0}{2 - 0} = \frac{4}{2}$$



m = 2

So, the equation of line AB is y = 2x.

For line AC,

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{4-0}{6-0} = \frac{4}{6}$$

$$m = \frac{2}{3}$$

Now using y = mx

$$y = \frac{2}{3}x \Rightarrow 2x - 3y = 0$$

So, the equation of line AC is 2x - 3y = 0.

Now for line BC, the y coordinate of both is same means horizontal line (parallel to the x - axis) then the equation of line BC is given as

y = 4

So, the required equations of lines for AB: y = 2x

AC: 2x - 3y = 0

BC: y = 4

Q. 19. If A (- 1, 6), B(- 3, - 9) and C(5, - 8) are the vertices of a ΔABC, find the equations of its medians.

Answer : <u>Construction: -</u> Draw median from vertices A, B and C on lines BC,AC and AC respectively .Let the mid - points of lines BC,AC and AB be L,M and N respectively.

Now find the coordinate of L, M and N using mid - point theorem.

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{coordinates of L} = \left(\frac{-3+5}{2}, \frac{-9+(-8)}{2}\right) \Rightarrow \left(1, \frac{-17}{2}\right)$$