Corresponding Venn diagram –

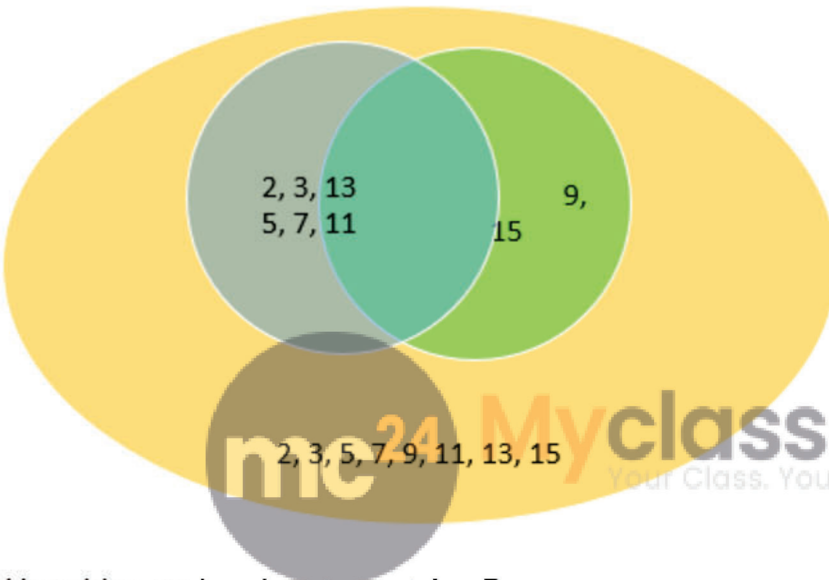
Q. 5. Let $A = \{2, 3, 5, 7, 11, 13\}$, $B = \{5, 7, 9, 11, 15\}$ be subsets of $U = \{2, 3, 5, 7, 9, 11, 13, 15\}$.

Using Venn diagrams, verify that:

(i) $(A \cup B') = (A' \cap B')$

(ii) $(A \cap B)' = (A' \cup B')$

Answer :



(i) Here blue region denotes set A – B

The green region denotes set B – A

The overlapping region denotes $A \cap B$, and the orange region denotes the universal set U.

From the Venn diagram we get $(A \cup B') = \{2, 3, 5, 7, 11, 13\}$ (B' is the set excluding those elements present in set B i.e. A - B region)

$A' = \{9, 15\}$ and $B' = \{2, 3, 13\}$

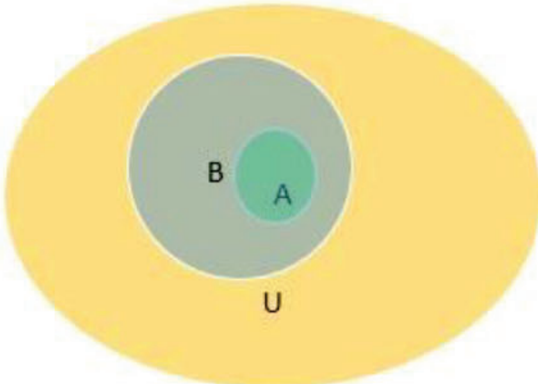
Therefore $A' \cap B' = \{\}$

Therefore $(A \cup B') \neq (A' \cap B')$ [Verified]

(ii) From the Venn diagram we get (A ∩ B)' = {2, 3, 9, 13, 15} (elements except those present in A ∩ B)

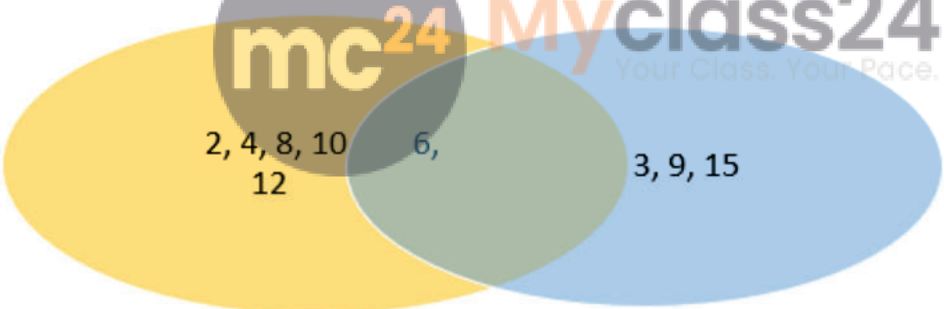
(A' ∪ B') = {2, 3, 9, 13, 15}

Therefore, (A ∩ B)' = (A' ∪ B') [Verified]



Q. 6. Using Venn diagrams, show that (A – B), A ∩ B) and (B – A) are disjoint sets, taking A = {2, 4, 6, 8, 10, 12} and B = {3, 6, 9, 12, 15, }.

Answer :



A - B is denoted by the yellow region only

B - A is denoted by the blue region only

AB is denoted by the common region (blue +yellow)

There is no intersection between these three regions

Hence the three sets are disjoint sets.

Exercise 1G

Q. 1. If A and B are two sets such that n(A) = 37, n(B) = 26 and n (A ∪ B) = 51, find n (A ∩ B).

Answer : Given:

$n(A) = 37$

$n(B) = 26$

$n(A \cup B) = 51$

To Find: $n(A \cap B)$

We know that,

$|A \cup B| = |A| + |B| - |A \cap B|$ (where A and B are two finite sets)

Therefore,

$n(A \cup B) = n(A) + n(B) – n(A \cap B)$

$51 = 37 + 26 – n(A \cap B)$

$n(A \cap B) = 63 – 51 = 12$

Therefore,

$n(A \cap B) = 12$

Q. 3. If A and B are two sets such that $n(A) = 24$, $n(B) = 22$ and $n(A \cap B) = 8$, find:

(i) $n(A \cup B)$

(ii) $n(A – B)$

(iii) $n(B – A)$

Answer : Given:

$n(A) = 24$, $n(B) =22$ and $n(A \cap B) = 8$

To Find:

(i) $n(A \cup B)$

$n(A \cup B) = n(A) + n(B) – n(A \cap B)$

$= 24 + 22 – 8$

$= 38$

Therefore,

$n(A \cup B) = 38$

(ii) $n(A - B)$

We know that,

n(A - B) = n(A) – n(A ∩ B)

= 24 – 8

= 16

Therefore,

n(A - B) = 16

(iii) n(B - A)

We know that,

n(B - A) = n(B) – n(A ∩ B)

= 22 – 8

= 14

Therefore,

n(B - A) = 14

Q. 4. If A and B are two sets such that n(A –B) = 24, n(B – A) = 19 and n(A ∩ B) = 11, find:

(i) n(A)

(ii) n(B)

(iii) n(A ∪ B)

Answer : Given:

n(A - B) = 24, n(B - A) =19 and n(A ∩ B) = 11

To Find:

(i) n(A)

We know that,

n(A) = n(A - B) + n(A ∩ B)

= 24 + 11

= 35

Therefore, n(A) = 35 …(1)

(ii) n(B)

We know that,

n(B) = n(B - A) + n(A ∩ B)

= 19 + 11

= 30

Therefore,

n(B) = 30 …(2)

(iii) n(A ∪ B)

We know that,

n(A ∪ B) = n(A) + n(B) – n(A ∩ B) {From (1) & (2) n(A) =35

and n(B) =30}

= 35 + 30 - 11

= 54

Therefore,

n(A ∪ B) = 54

Q. 5. In a committee, 50 people speak Hindi, 20 speak English and 10 speak both Hindi and English. How many speak at least one of these two languages?

Answer : Given:

People who speak Hindi = 50

People who speak English = 20

People who speak both English and Hindi = 10

To Find: People who speak at least one of these two languages

Let us consider,

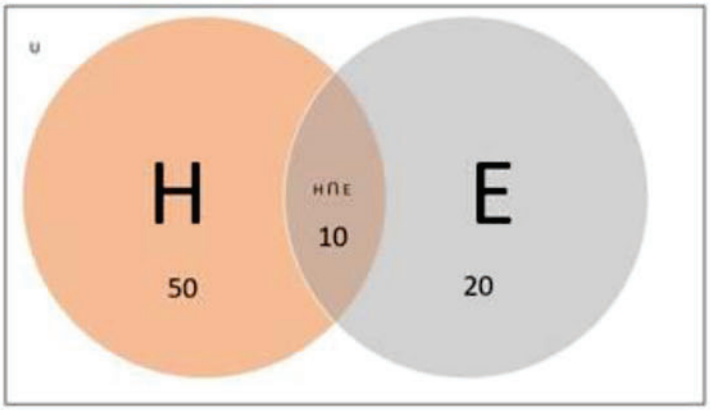
People who speak Hindi = n(H) = 50

People who speak English = n(E) = 20

People who speak both Hindi and English = n(H ∩ E) = 10

People who speak at least one of the two languages = n(H ∪ E)

Venn diagram:



Now, we know that,

n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

Therefore,

n(H ∪ E) = n(H) + n(E) – n(H ∩ E)

= 50 + 20 – 10

= 60

Thus, People who speak at least one of the two languages are 60.

Q. 6. In a group of 50 persons, 30 like tea, 25 like coffee and 16 like both. How many like

(i) either tea or coffee?

(ii) neither tea nor coffee?

Answer : Given:

In a group of 50 persons,

-30 like tea

-25 like coffee

-16 like both tea and coffee

To find:

(i) People who like either tea or coffee.

Let us consider,

Total number of people = n(X) = 50

People who like tea = n(T) = 30

People who like coffee = n(C) = 25

People who like both tea and coffee = n(T ∩ C) = 16

People who like either tea or coffee = n(T ∪ C)

Venn diagram:

Therefore,

n(T ∪ C) = n(T) + n(C) - n(T ∩ C)

= 30 + 25 – 16

= 39

Thus, People who like either tea or coffee = 39

(ii) People who like neither tea nor coffee.

People who like neither tea nor coffee = n(X) – n(T ∪ C)

= 50 – 39

= 11

Therefore, People who like neither tea nor coffee = 11

Q.7. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2, and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(i) Chemical C_1 but not chemical C_2
(ii) Chemical C_2 but not chemical C_1
(iii) Chemical C_1 or chemical C_2

Answer : Given:

Total number of individuals with skin disorder = 200

Individuals exposed to chemical C_1 = 120

Individuals exposed to chemical C_2 = 50

Individuals exposed to chemicals C_1 and C_2 both = 30

To Find:

(i) Individuals exposed to Chemical C_1 but not C_2

Let us consider,

Total number of individuals with skin disorder = n(C) = 200

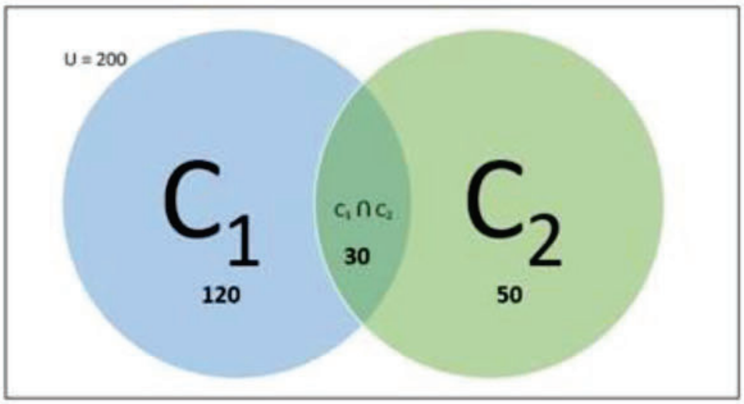
Individuals exposed to chemical $C_1 = n(C_1) = 120$

Individuals exposed to chemical $C_2 = n(C_2) = 50$

Individuals exposed to chemicals C_1 and C_2 both $= n(C_1 \cap C_2) = 30$

Individuals exposed to Chemical C_1 but not $C_2 = n(C_1 - C_2)$

Venn diagram:



Now,

$n(C_1 - C_2) = n(C_1) – n(C_1 \cap C_2)$

$= 120 – 30$

$= 90$

Therefore, number of individuals exposed to chemical C_1 but not $C_2 = 90$

(ii) Individuals exposed to Chemical C_2 but not C_1

Let us consider number of Individuals exposed to Chemical C_2 but not $C_1 = n(C_2 – C_1)$

Now,

$n(C_2 – C_1) = n(C_2) – n(C_1 \cap C_2)$

$= 50 – 30$

$= 20$

Therefore, number of individuals exposed to chemical C_2 but not $C_1 = 20$

(iii) Individuals exposed to Chemical C_1 or chemical C_2

Let us consider number of Individuals exposed to Chemical C_1 or chemical C_2 = $n(C_1 \cup C_2)$

Now,

$n(C_1 \cup C_2) = n(C_1) + n(C_2) - n(C_1 \cap C_2)$

= 120 + 50 – 30

= 140

Therefore, number of individuals exposed to chemical C_1 or C_2 = 140

Q. 8. In a class of a certain school, 50 students, offered mathematics, 42 offered biology and 24 offered both the subjects. Find the number of students offering

(i) mathematics only,

(ii) biology only,

(iii) any of the two subjects.

Answer : Given:

Number of students offered Mathematics = 50

Number of students offered Biology = 42

Number of students offered both Mathematics and Biology = 24

To Find:

(i) Number of students offered Mathematics only

Let us consider,

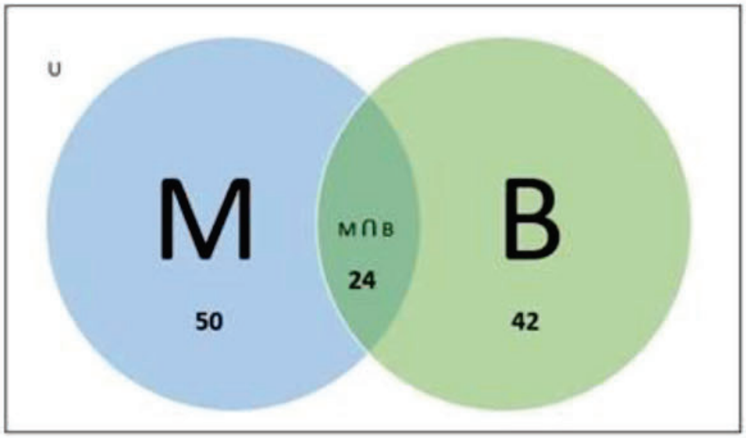
Number of students offered Mathematics = n(M) = 50

Number of students offered Biology = n(B) = 42

Number of students offered Mathematics & Biology both = $n(M \cap B)$ = 24

Number of students offered Mathematics only = n(M - B)

Venn diagram:



Now,

n(M - B) = n(M) – n(M ∩ B)

= 50 – 24

= 26

Therefore, Number of students offered Mathematics only= 26

(ii) Number of students offered Biology only

Number of students offered Biology only= n(B – M)

Now,

n(B – M) = n(B) – n(M ∩ B)

= 42 – 24

= 18

Therefore, Number of students offered Biology only = 18

(iii) Number of students offered any of two subjects

Number of students offered any of two subjects = n(M ∪ B)

Now,

n(M ∪ B) = n(M) + n(B) - n(M ∩ B)

= 50 + 42 – 24

= 140

Therefore, Number of students offered any of two subjects = 68

Q. 9. In an examination, 56% of the candidates failed in English and 48% failed in science. If 18% failed in both English and science, find the percentage of those who passed in both the subjects.

Answer : Given:

In an examination:

- 56% of candidates failed in English

- 48% of candidates failed in science

- 18% of candidates failed in both English and Science

To Find;

Percentage of students who passed in both subjects.

Let us consider,

Percentage of candidates who failed in English = n(E) = 56

Percentage of candidates who failed in Science = n(S) = 48

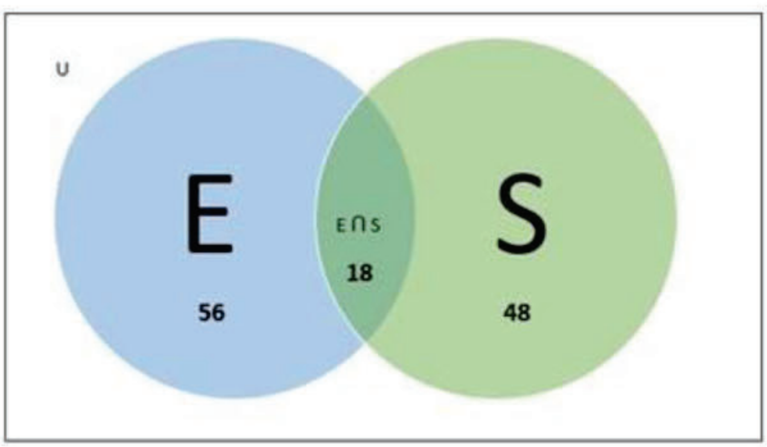
Percentage of candidates who failed in English and Science both

= n(E ∩ S) = 18

Percentage of candidates who failed in English only = n(E - S)

Percentage of candidates who failed in Science only = n(S - E)

Venn diagram:



Now,

n(E - S) = n(E) - n(E ∩ S)

= 56 – 18

= 38

n(S - E) = n(S) - n(E ∩ S)

= 48 – 18

= 30

Therefore,

Percentage of total candidates who failed =

n(E - S) + n(S - E) + n(E ∩ S)

= 38 + 30 + 18 = 86%

Now,

The percentage of candidates who passed in both English and

Science = 100 - 86 = 14%

Hence,

The percentage of candidates who passed in both English and

Science = 14%

Q. 10. In a group of 65 people, 40 like cricket and 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Answer : Given:

In a group of 65 people:

- 40 people like cricket
- 10 like both cricket and tennis

To Find:

- Number of people like tennis only
- Number of people like tennis

Let us consider,

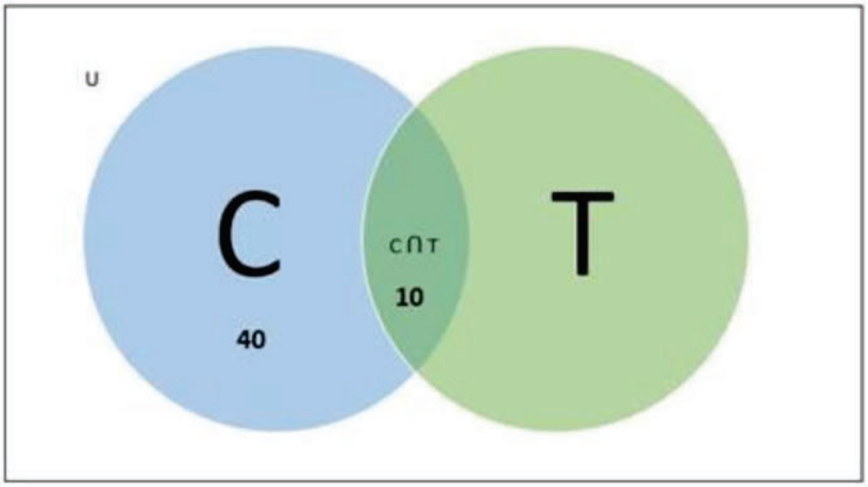
Number of people who like cricket = n(C) = 40

Number of people who like tennis = n(T)

Number of people who like cricket or tennis = n(C ∪ T) = 65

Number of people who like cricket and tennis both = n(C ∩ T) = 10

Venn diagram:



Now,

n(C ∪ T) = n(C) + n(T) - n(C ∩ T)

65 = 40 + n(T) – 10

n(T) = 65 – 40 + 10

= 35

Therefore, number of people who like tennis = 35

Now,

Number of people who like tennis only = n(T - C)

n(T - C) = n(T) - n(C ∩ T)

= 35 – 10

= 25

Therefore, the number of people who like tennis only = 25

Q. 11. A school awarded 42 medals in hockey, 18 in basketball and 23 in cricket. if these medals were bagged by a total of 65 students and only 4 students got medals in all the three sports, how many students received medals in exactly two of the three sports?

Answer : Given:

- Total number of students = 65

- Medals awarded in Hockey = 42

- Medals awarded n Basketball = 18

- Medals awarded in Cricket = 23

- 4 students got medals in all the three sports.

To Find:

Number of students who received medals in exactly two of the three sports.

Total number of medals = Medals awarded in Hockey + Medals awarded in Basketball + Medals awarded in Cricket

Total number of medals = 42 + 28 + 23

= 83

It is given that 4 students got medals in all the three sports.

Therefore, the number of medals received by those 4 students = 4 × 3 = 12

Now, the number of medals received by the rest of 61 students = 83 – 12 = 71

Among these 61 students, everyone at least received 1 medal.

Therefore, the number of extra medals = 71 – 1 × 61 = 10

Therefore, we can say that 10 students received more than one and less than three medals, or we can say that 10 students received medals in exactly two of three sports.

Q. 12. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read the newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, and 3 read all the three newspapers. Find

(i) The number of people who read at least one of the newspapers,
(ii) The number of people who read exactly one newspaper.

Answer : Given:

- Total number of people = 60

- Number of people who read newspaper H = 25

- Number of people who read newspaper T = 26

- Number of people who read newspaper I = 26

- Number of people who read newspaper H and I both = 9

- Number of people who read newspaper H and T both = 11

- Number of people who read newspaper T and I both = 8

- Number of people who read all three newspapers = 3

To Find:

(i) The number of people who read at least one of the newspapers

Let us consider,

Number of people who read newspaper H = n(H)= 25

Number of people who read newspaper T = n(T) = 26

Number of people who read newspaper I = n(I) =26

Number of people who read newspaper H and I both = n(H ∩ I) = 9

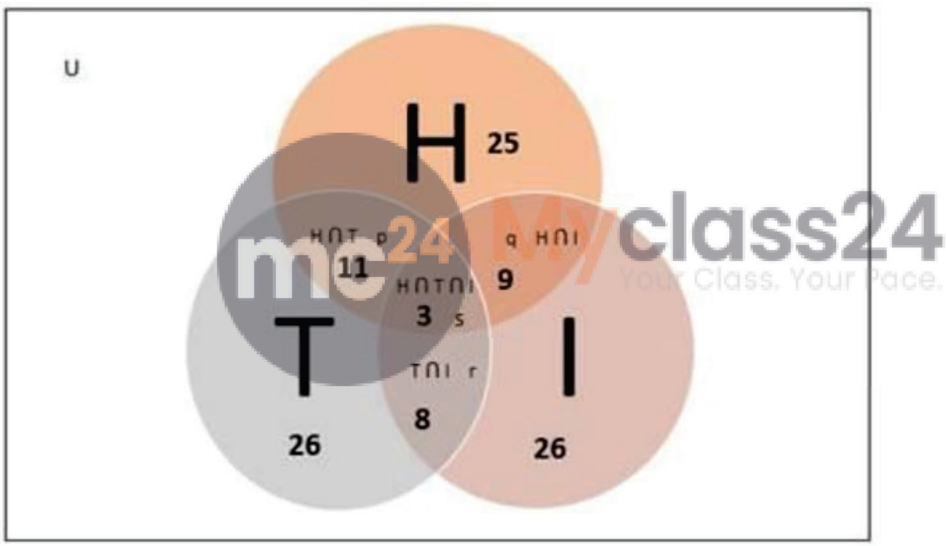
Number of people who read newspaper H and T both = n(H ∩ T) = 11

Number of people who read newspaper T and I both = n(T ∩ I) = 8

Number of people who read all three newspapers = n(H ∩ T ∩ I) = 3

Number of people who read at least one of the three newspapers= n(H ∪ T ∪ I)

Venn diagram:



We know that,

n(H ∪ T ∪ I) = n(H) + n(T) + n(I) – n(H ∩ I) – n(H ∩ T) – n(T ∩ I) + n(H ∩ T ∩ I)

= 25 + 26 + 26 – 9 – 11 – 8 + 3

= 52

Therefore,

Number of people who read at least one of the three newspapers = 52

(ii) The number of people who read exactly one newspaper

Number of people who read exactly one newspaper =

$n(H \cup T \cup I) – p – q – r – s$

Where,

p = Number of people who read newspaper H and T but not I

q = Number of people who read newspaper H and I but not T

r = Number of people who read newspaper T and I but not H

s = Number of people who read all three newspapers = 3

$p + s = n(H \cap T)$ …(1)

$q + s = n(H \cap I)$ …(2)

$r + s = n(T \cap I)$ …(3)

Adding (1), (2) and (3)

$p + s + q + s + r + s = n(H \cap T) + n(H \cap I) + n(T \cap I)$

$p + q + r + 3s = 9 + 11 + 8$

$p + q + r + s + 2s = 28$

$p + q + r + s = 28 - 2×3$

$p + q + r + s = 22$

Now,

$n(H \cup T \cup I) – p – q – r – s = n(H \cup T \cup I) – (p + q + r + s)$

= 52 -22

= 30

Hence, 30 people read exactly one newspaper.

Q. 13. In a survey of 100 students, the number of students studying the various languages is found as English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find

(i) how many students are studying Hindi?
(ii) how many students are studying English and Hindi both?

Answer : Given:

- Total number of students = 100

- Number of students studying English(E) only = 18

- Number of students learning English but not Hindi(H) = 23

- Number of students learning English and Sanskrit(S) = 8

- Number of students learning Sanskrit and Hindi = 8

- Number of students learning English = 26

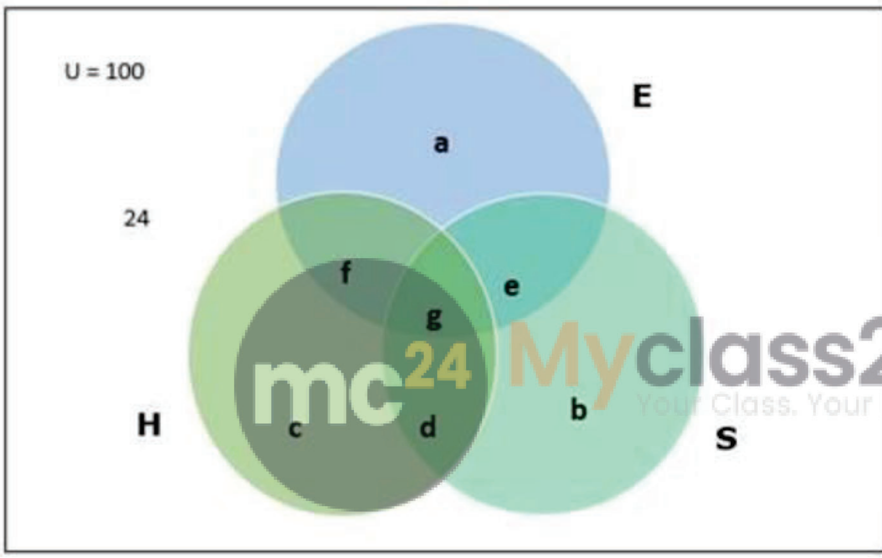
- Number of students learning Sanskrit = 48

- Number of students learning no language = 24

To Find:

(i) Number of students studying Hindi

Venn diagram:



From the above Venn diagram,

a = Number of students who study only English = 18

b = Number of students who study only Sanskrit

c = Number of students who study only Hindi

d = Number of students learning Hindi and Sanskrit but not English

e = Number of students learning English and Sanskrit but not Hindi

f = Number of students learning Hindi and English but not Sanskrit

g = Number of students learning all the three languages

e + g = Number of students learning English and Sanskrit = 8

= n(E ∩ S)

g + d = Number of students learning Hindi and Sanskrit = 8

= n(H ∩ S)

E = a + e + f + g = Number of students learning English

26 = 18 + 8 + f

f = 26 -26 = 0

Therefore, f = 0

Now,

Number of students learning English but not Hindi = a + e = 23

23 = 18 + e

Therefore, e = 5

Now, e + g = 8

5 + g = 8

Therefore, g = 3

S = b + e + d + g = Number of students studying Sanskrit

48 = b + 5 + 8 (Because, d + g = 8)

b = 48 – 13

Therefore, b = 35 (Number of students studying Sanskrit only)

Also, d + g = 8

d + 3 = 8

Therefore, d = 5

Now,

Number of students studying Hindi only = c

c = 100 – (a + e + b + d + f + g) – 24

= 100 – (18 + 5 + 35 + 5 + 0 + 3) -24

= 100 – 66 – 24

= 100 – 90 = 10

Number of students studying Hindi = c + f + g + d

= 10 + 0 + 3 + 5

= 18

Therefore, number of students studying Hindi = 18

(ii) Number of students studying English and Hindi both

Number of students studying English and Hindi both = f + g

= 0 + 3 = 3

Therefore, Number of students studying English and Hindi both = 3

Q. 14. In a town of 10,000 families, it was found that 40% of the families buy newspaper A, 20% buy newspaper B, 10% buy newspaper C, 5% buy A and B; 3% buy B and C, and 4% buy A and C. IF 2% buy all the three newspapers, find the number of families which buy

(i) A only,
(ii) B only,
(iii) none of A, B, and C.

Answer : Given:

Total number of families = 10000

Percentage of families that buy newspaper A = 40

Percentage of families that buy newspaper B = 20

Percentage of families that buy newspaper C = 10

Percentage of families that buy newspaper A and B = 5

Percentage of families that buy newspaper B and C = 3

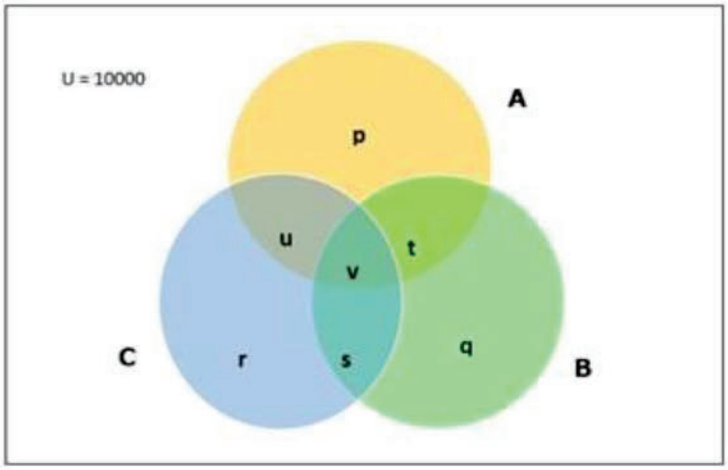
Percentage of families that buy newspaper A and C = 4

Percentage of families that buy all three newspapers = 2

To find:

(i) Number of families that buy newspaper A only

Consider the Venn Diagram below:



Number of families that buy newspaper A = n(A) = 40% of 10000

= 4000

Number of families that buy newspaper B = n(B) = 20% of 10000 = 2000

Number of families that buy newspaper C = n(C) = 10% of 10000

= 1000

Number of families that buy newspaper A and B = n(A ∩ B)

= 5% of 10000

= 500

Number of families that buy newspaper B and C = n(B ∩ C)

= 3% of 10000

= 300

Number of families that buy newspaper A and C = n(A ∩ C)

= 4% of 10000

= 400

Number of families that buys all three newspapers = n(A ∩ B ∩ C)=v

= 2% of 10000

= 200

We have,

n(A ∩ B) = v + t

500 = 200 + t

$t = 500 – 200 = 300$

$n(B \cap C) = v + s$

$300 = 200 + s$

$s = 300 – 200 = 100$

$n(A \cap C) = v + u$

$400 = 200 + u$

$u = 400 – 200 = 200$

p = Number of families that buy newspaper A only

We have,

$A = p + t + v + u$

$4000 = p + 300 + 200 + 200$

$p = 4000 – 700$

$p = 3300$

Therefore,

Number of families that buy newspaper A only = 3300

(ii) Number of families that buy newspaper B only

q = Number of families that buy newspaper B only

$B = q + s + v + t$

$2000 = q + 100 + 200 + 300$

$q = 2000 – 600 =1400$

Therefore,umber of families that buy newspaper B only = 1400

(iii) Number of families that buys none of the newspaper

Number of families that buy none of the newspaper =

$10000 – \{n(A) + n(B) + n(C) – n(A \cap B) – n(B \cap C) – n(A \cap C) + n(A \cap B \cap C)\}$

$= 10000 – (4000 + 2000 + 1000 – 500 – 300 – 400 + 200)$

$= 10000 – 6000$

$= 4000$

Therefore,

Number of families that buy none of the newspaper = 4000

Q. 15. A class has 175 students. The following description gives the number of students one or more of the subjects in this class: mathematics 100, physics 70, chemistry 46, mathematics and physics 30; mathematics and chemistry 28; physics and chemistry 23; mathematics, physics and chemistry 18. Find

(i) how many students are enrolled in mathematics alone, physics alone and chemistry alone,
(ii) The number of students who have not offered any of these subjects.

Answer : Given:

- Number of students in class = 175

- Number of students enrolled in Mathematics = 100

- Number of students enrolled in Physics = 70

- Number of students enrolled in Chemistry = 46

- Number of students enrolled in Mathematics and Physics = 30

- Number of students enrolled in Physics and Chemistry = 23

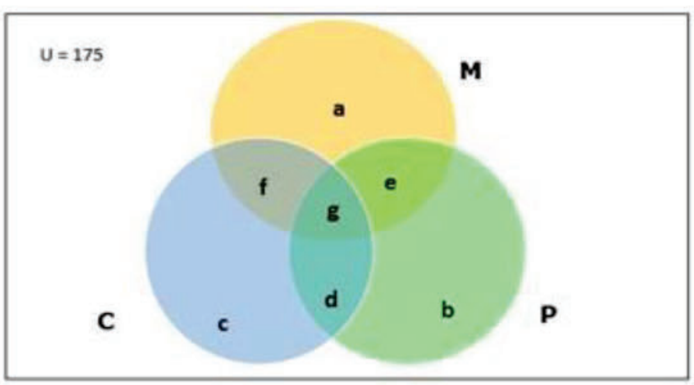
- Number of students enrolled in Mathematics and Physics = 28

- Number of students enrolled in all three subjects = 18

To find:

(i) Number of students enrolled in Mathematics alone, Physics alone and Chemistry alone

Venn diagram:



Number of students enrolled in Mathematics = 100 = n(M)

Number of students enrolled in Physics = 70 = n(P)

Number of students enrolled in Chemistry = 46 = n(C)

Number of students enrolled in Mathematics and Physics

= 30 = $n(M \cap P)$

Number of students enrolled in Mathematics and Chemistry

= 28 = $n(M \cap C)$

Number of students enrolled in Physics and Chemistry

= 23 = $n(P \cap C)$

Number of students enrolled in all the three subjects

= 18 = $n(M \cap P \cap C)$ = g

We have,

$n(M \cap P) = e + g$

30 = e + 18

e = 30 – 18 = 12

$n(M \cap C) = f + g$

28 = f + 18

f = 28 – 18 = 10

$n(P \cap C) = d + g$

23 = d + 18

d = 23 – 18 = 5

a = Number of students enrolled only in Mathematics

b = Number of students enrolled only in Physicsc = Number of students enrolled only in Chemistry

We have,

M = a + e + f + g

100 = a + 12 + 10 + 18

a = 100 – 40

a = 60

Therefore,

Number of students enrolled only in Mathematics = 60

P = b + e + d + g

70 = b + 12 + 5 + 18

b = 70 – 35

b = 35

Therefore,

Number of students enrolled only in Physics = 35

C = c + f + d + g

46 = c + 10 + 5 + 18

c = 46 – 33

c = 13

Therefore,

Number of students enrolled only in Chemistry = 13

(ii) Number of students who have not offered any of these subjects

Number of students who have not offered any of these subjects

= 175 – {n(M) + n(P) + n(C) – n(M ∩ P) – n(M ∩ C) – n(P ∩ C) + n(M ∩ P ∩ C)}

= 175 – (100 + 70 + 46 – 30 – 28 – 23 + 18)

= 175 – 153

= 22

Therefore,

Number of students who have not offered any of these subjects = 22

Exercise 1H

Q. 1. If a set A and n elements then find the number of elements in its power set P(A).

Answer : The power set of set A is a collection of all subsets of A.

For example: if the set A is {1, 2} then all possible subsets of A would be {} (empty set), {1}, {2}, {1, 2}

Hence powerset of A that is P(A) will be {ϕ, {1}, {2}, {1,2}}

Now if the number of elements in set A is n then the number of elements in power set of A P(A) is 2^n

Q. 2. If A = ϕ then write P(A).

Answer : The power set of set A is a collection of all subsets of A.

Here A = {ϕ}

Hence the subset of A will only be a null set ϕ

Hence P(A) = {ϕ}

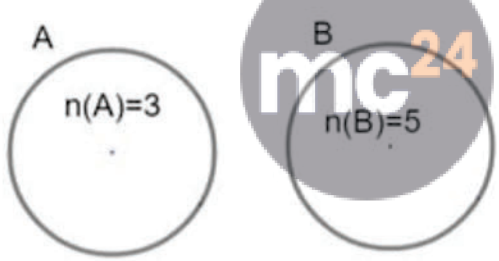
Q.3. If n(A) = 3 and n(B) = 5, find:

(i) The maximum number of elements in A ∪ B,
(ii) The minimum number of elements in A ∪ B.

Answer : Number of elements in set A n(A) = 3 and number of elements in set B n(B) = 5

The number of elements in A ∪ B is n(A ∪ B).

i) Now for elements in A ∪ B to be maximum, there should not be any intersection between both sets that is A and B both sets must be disjoint sets as shown.



Hence the number of elements in A ∪ B is n(A ∪ B) = n(A) + n(B)

⇒ n(A ∪ B) = 3 + 5

⇒ n(A ∪ B) = 8

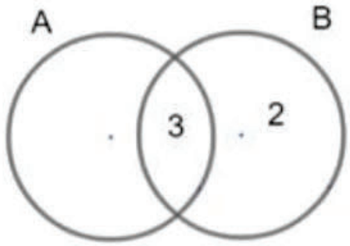
Hence maximum number of elements in A ∪ B is 8

ii) Now for a number of elements in A ∪ B to be minimum, there should be an ntersection between sets A and B so that some elements are common

The count will be minimum when all the elements from set A are also in set B the reverse are not possible because n(A) < n(B)

Hence if the 3 elements of A are in the intersection of A and B, then the number of elements only in B will be 2 because n(B) = 5

Visually it is represented as,



As seen from the figure the number of elements in $A \cup B$ is 5 hence minimum number of elements in $A \cup B = 5$

Q. 4. If A and B are two sets such than n(A) = 8, n(B) = 11 and n(A ∪ B) = 14 then find n(A ∩ B).

Answer : Given: $n(A) = 8$, $n(B) = 11$, $n(A \cup B) = 14$

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 14 = 8 + 11 - n(A \cap B)$

$\Rightarrow 14 = 19 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 19 - 14$

$\Rightarrow n(A \cap B) = 5$

Hence $n(A \cap B) = 5$

Q. 5. If A and B are two sets such that n(A) = 23, n(b) = 37 and n(A – B) = 8 then find n(A ∪ B).

Hint n(A) = n(A – B) + n(A ∩ B) n(A ∩ B) = (23 – 8) = 15.

Answer : Given: $n(A) = 23$, $n(B) = 37$, $n(A - B) = 8$

Using the hint

$n(A) = n(A - B) + n(A \cap B)$

$\Rightarrow 23 = 8 + n(A \cap B)$

$\Rightarrow n(A \cap B) = 23 - 8$

$\Rightarrow n(A \cap B) = 15$

Visualizing the hint given,

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cup B) = 23 + 37 - 15$

$\Rightarrow n(A \cup B) = 45$

Hence $n(A \cup B) = 45$

Q. 6. If A and B are two sets such than n(A) = 54, n(B) = 39 and n(B – A) = 13 then find n(A ∪ B).

Hint $n(B) = n(B - A) + n(A \cap B) \Rightarrow n(A \cap B) = (39 - 13) = 26$.

Answer : Given: $n(A) = 54$, $n(B) = 39$, $n(B - A) = 13$

Using the hint

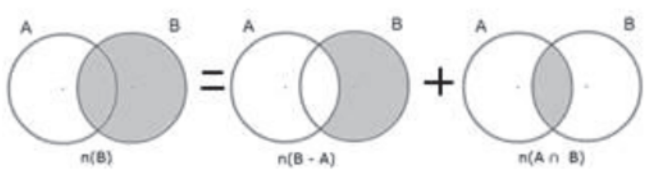
$n(B) = n(B - A) + n(A \cap B)$

$\Rightarrow 39 = 13 + n(A \cap B)$

$\Rightarrow n(A \cap B) = 39 - 13$

$\Rightarrow n(A \cap B) = 26$

Visualizing the hint given,



We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

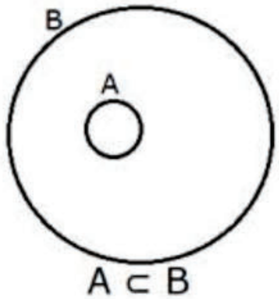
$\Rightarrow n(A \cup B) = 54 + 39 - 26$

$\Rightarrow n(A \cup B) = 67$

Hence $n(A \cup B) = 67$

Q. 7. If A ⊂ B, prove that B' ⊂ A'.

Answer : As $A \subset B$ the set A is inside set B



Hence A ∪ B = B

Taking compliment

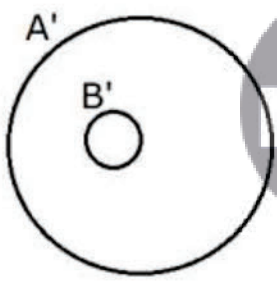
⇒ (A ∪ B)' = B.'

Using de-morgans law (A ∪ B)' = A' ∩ B.'

⇒ A' ∩ B' = B.'

A' ∩ B' = B' means that the set B' is inside the set A.'

Representing in Venn diagram,

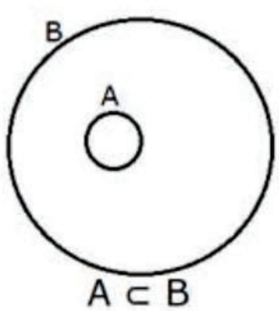


As seen from Venn diagram B' ⊂ A.'

Hence proved

Q. 8. If A ⊂ B, show that (B' – A') = ϕ.

Answer : As A ⊂ B the set A is inside set B



Hence A ∪ B = B

Taking compliment

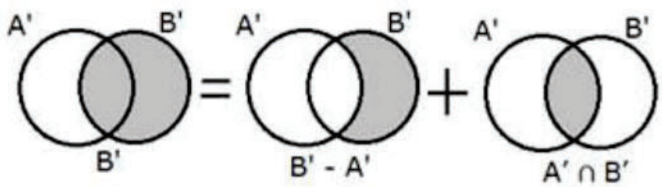
⇒ (A ∪ B)' = B'

Using De-Morgan's law (A ∪ B)' = A' ∩ B'

⇒ A' ∩ B' = B' ...(i)

Now we know that

B' = (B' – A') + (A' ∩ B')



Using (i)

⇒ B' = (B' – A') + B'

⇒ (B' – A') = B' – B'

⇒ (B' – A') = 0

⇒ (B' – A') = {ϕ}

Hence proved

Q. 9. Let A = {x : x = 6n ∈ N) and B = {x : x = 9n, n ∈ N}, find A ∩ B.

Answer : A = {x : x = 6n ∀ n ∈ N)

As x = 6n hence for n = 1, 2, 3, 4, 5, 6... x = 6, 12, 18, 24, 30, 36...

Hence A = {6, 12, 18, 24, 30, 36...}

B = {x : x = 9n ∀ n ∈ N)

As x = 9n hence for n = 1, 2, 3, 4... x = 9, 18, 27, 36...

Hence B = {9, 18, 27, 36...}

A ∩ B means common elements to both sets

The common elements are 18, 36, 54, ...

Hence A ∩ B = {18, 36, 54, ...}

All the elements are multiple of 18

Hence A ∩ B = {x: x = 18n ∀ n ∈ N}

Q. 10. If A = {5, 6, 7}, find P(A).

Answer : A = {5, 6, 7}

We have to find P(A) which is power set of A

The power set of set A is collection of all possible subsets of A

The possible subsets of A are {ϕ}, {5}, {6}, {7}, {5,6}, {5,7}, {6, 7}, {5, 6, 7}

Hence the power set P(A) will be

P(A) = {{ϕ}, {5}, {6}, {7}, {5,6}, {5,7}, {6, 7}, {5, 6, 7}}

Q. 11. If A = {3, {2}}, find P(A).

Answer : A = {3, {2}}

We have to find P(A) which is power set of A

The power set of set A is collection of all possible subsets of A

The possible subsets of A are {ϕ}, {3}, {{2}}, {3, {2}}

Hence the power set P(A) will be

P(A) = {{ϕ}, {3}, {{2}}, {3, {2}}}

Q. 12. Prove that A ∩ (A ∪ B)' = ϕ

Answer : LHS = A ∩ (A ∪ B)'

Using De-Morgan's law (A ∪ B)' = (A' ∩ B')

⇒ LHS = A ∩ (A' ∩ B')

⇒ LHS = (A ∩ A') ∩ (A ∩ B')

We know that A ∩ A' = ϕ

⇒ LHS = ϕ ∩ (A ∩ B')

We know that intersection of null set with any set is null set only

Let (A ∩ B') be any set X hence

⇒ LHS = ϕ ∩ X

⇒ LHS = ϕ

⇒ LHS = RHS

Hence proved

Q. 13. Find the symmetric difference A Δ B, when A = {1, 2, 3} and B = {3, 4, 5}.

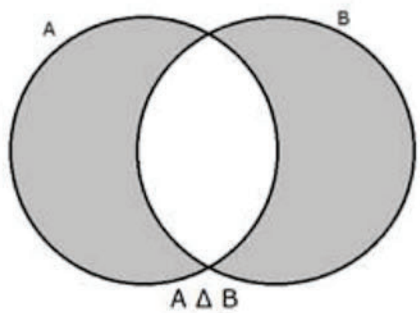
Answer : A = {1, 2, 3}

B = {3, 4, 5}

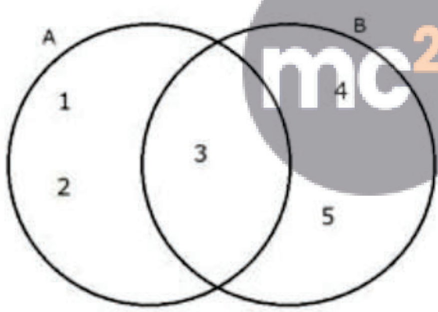
The symmetric difference A Δ B is given by

A Δ B = (A – B) ∪ (B – A)

Venn diagram representation:



Representing the given sets A and B through venn diagram



Hence as seen the elements in A Δ B are 1, 2, 4 and 5

Hence the symmetric difference A Δ B = {1, 2, 4, 5}

Q. 14. Prove that A – B = A ∩ B.'

Answer : Let x be some element in set A – B that is x ∈ (A – B)

Now if we prove that x ∈ (A ∩ B') then (A – B) = (A ∩ B')

x ∈ (A – B) means x ∈ A and x ∉ B

Now x ∉ B means x ∈ B.'

Hence we can say that x ∈ A and x ∈ B.'

Hence x ∈ A ∩ B.’

And as x ∈ A ∩ B’ and also x ∈ A – B we can conclude that

A – B = A ∩ B.’

Q. 15. If A = {x : x ϵ R, x < 5} and B = {x : x ϵ R, x > 4}, find A ∩ B.

Answer : A = {x: x ∈ R, x < 5}

As x takes all real values upto 5 hence the set A will contain all numbers from -∞ to 5

A = (-∞, 5)

B = {x: x ∈ R, x > 4}

As x takes all real values greater than 4 hence the set B will contain values from 4 to ∞

B = (4, -∞)

Hence their intersection or the common part between sets A and B would be values from 4 to 5

Hence A ∩ B = (4, 5)

Representing the sets on number line