

$$\{\ln(mn) = \ln n + \ln m\} \left\{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\right\} \{\ln e = 1\}$$

Now differentiating both sides by x , we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x} \right)$$

$$\frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x} \right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x} \right) \times \sin 2x \sin 3x \sin 4x$$

27. Question

Find $\frac{dy}{dx}$, when:

$$y = \frac{x^3 \sin x}{e^x}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 3 \ln x + \ln \sin x - x$$

$$\{\ln(mn) = \ln n + \ln m\} \left\{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\right\} \{\ln e = 1\}$$

Now differentiating both sides by x , we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{\sin x} \times \cos x + \frac{3}{x} - 1 \right)$$

$$\frac{dy}{dx} = \left(\frac{\cos x}{\sin x} + \frac{3}{x} - 1 \right) \times y$$

$$\frac{dy}{dx} = \left(\cot x + \frac{3}{x} - 1 \right) \times \frac{x^3 \sin x}{e^x}$$

28. Question

Find $\frac{dy}{dx}$, when:

$$y = \frac{e^x \log x}{x^2}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = x + \ln(\ln x) - 2 \ln x$$

$$\{\ln(mn) = \ln n + \ln m\} \left\{ \ln\left(\frac{m}{n}\right) = \ln m - \ln n \right\} \{\ln e = 1\}$$

Now differentiating both sides by x , we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(1 + \frac{1}{x \ln x} - \frac{2}{x}\right)$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x \ln x} - \frac{2}{x}\right) \times y$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x \ln x} - \frac{2}{x}\right) \times \frac{e^x \log x}{x^2}$$

29. Question

Find $\frac{dy}{dx}$, when:

$$y = \frac{x \cos^{-1} x}{\sqrt{1-x^2}}$$

**Answer**

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \ln \cos^{-1} x + \ln(x) - \frac{1}{2} \ln(1 - x^2)$$

$$\{\ln(mn) = \ln n + \ln m\} \left\{ \ln\left(\frac{m}{n}\right) = \ln m - \ln n \right\} \{\ln e = 1\}$$

Now differentiating both sides by x , we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(-\frac{1}{\sqrt{1-x^2}} + \frac{1}{x} + \frac{2x}{2(1-x^2)}\right)$$

$$\frac{dy}{dx} = \left(-\frac{1}{\sqrt{1-x^2}} + \frac{1}{x} + \frac{2x}{2(1-x^2)}\right) \times y$$

$$\frac{dy}{dx} = \left(-\frac{1}{\sqrt{1-x^2}} + \frac{1}{x} + \frac{x}{(1-x^2)}\right) \times \frac{x \cos^{-1} x}{\sqrt{1-x^2}}$$

30. Question

Find $\frac{dy}{dx}$, when:

$$y = (1+x)(1+x^2)(1+x^4)(1+x^6)$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \ln(1+x) + \ln(1+x^2) + \ln(1+x^4) + \ln(1+x^6)$$

$$\{\ln(mn) = \ln n + \ln m\} \left\{ \ln\left(\frac{m}{n}\right) = \ln m - \ln n \right\} \{\ln e = 1\}$$

Now differentiating both sides by x , we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6} \right)$$

$$\frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6} \right) \times y$$

$$\frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6} \right) \times (1+x)(1+x^2)(1+x^4)(1+x^6)$$

31. Question

Find $\frac{dy}{dx}$, when:

$$y = x^x - 2^{\sin x}$$

Answer

simply taking log both sides would not help more.

For that let us assume $u = x^x$ and $v = 2^{\sin x}$

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$$u = x^x$$

Take log both sides

$$\ln u = x \ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x \left(\frac{1}{x} \right) + \ln x$$

$$\frac{du}{dx} = (1 + \ln x) \times u$$

$$\frac{du}{dx} = (1 + \ln x) \times x^x \dots\dots\dots(1)$$

$$v = 2^{\sin x}$$

Take log both sides,

$$\ln v = \sin x \cdot \ln 2$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \sin x(0) + \ln 2 \cdot \cos x$$

$$\frac{dv}{dx} = \ln 2 \cdot \cos x \times v$$

$$\frac{dv}{dx} = \ln 2 \cdot \cos x \times 2^{\sin x} \dots\dots(2)$$

$$\frac{dy}{dx} = (1 + \ln x) \times x^x - \ln 2 \cdot \cos x \times 2^{\sin x}$$



32. Question

Find $\frac{dy}{dx}$, when:

$$y = (\log x)^x + x^{\log x}$$

Answer

simply taking log both sides would not help more.

For that let us assume $u = (\ln x)^x$ and $v = x^{\ln x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\ln x)^x$$

Take log both sides

$$\ln u = x \ln(\ln x)$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x \left(\frac{1}{x} \times \frac{1}{\ln x} \right) + \ln(\ln x)$$

$$\frac{du}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x) \right) \times u$$

$$\frac{du}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x) \right) \times (\ln x)^x \dots\dots\dots(1)$$

$$v = x^{\ln x}$$

Take log both sides,

$$\ln v = \ln x \cdot \ln x$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = 2 \cdot \ln x \times \frac{1}{x}$$

$$\frac{dv}{dx} = \frac{2 \cdot \ln x}{x} \times v$$

$$\frac{dv}{dx} = \frac{2 \cdot \ln x}{x} \times x^{\ln x} \dots\dots(2)$$

$$\frac{dy}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x) \right) \times (\ln x)^x + \frac{2 \cdot \ln x}{x} \times x^{\ln x}$$

33. Question

Find $\frac{dy}{dx}$, when:

$$y = x^{\sin x} + (\sin x)^{\cos x}$$

Answer

simply taking log both sides would not help more.

For that let us assume $u = x^{\sin x}$ and $v = \sin x^{\cos x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\sin x}$$

Take log both sides

$$\ln u = \sin x \ln(x)$$

Differentiate



$$\frac{1}{u} \times \frac{du}{dx} = \sin x \left(\frac{1}{x} \right) + \ln(x) \times \cos x$$

$$\frac{du}{dx} = \left(\frac{\sin x}{x} + \ln(x) \times \cos x \right) \times u$$

$$\frac{du}{dx} = \left(\frac{\sin x}{x} + \ln(x) \times \cos x \right) \times (x^{\sin x} + (\sin x)^{\cos x}) \dots\dots\dots(1)$$

$$v = (\sin x)^{\cos x}$$

Take log both sides,

$$\ln v = \cos x \ln(\sin x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \cos x \left(\frac{1}{\sin x} \times \cos x \right)$$

$$\frac{dv}{dx} = \frac{\cos^2 x}{\sin x} \times v$$

$$\frac{dv}{dx} = \frac{\cos^2 x}{\sin x} \times (\sin x)^{\cos x} \dots\dots(2)$$

$$\frac{dy}{dx} = \frac{\cos^2 x}{\sin x} \times (\sin x)^{\cos x} + \left(\frac{\sin x}{x} + \ln(x) \times \cos x \right) \times (x^{\sin x} + (\sin x)^{\cos x})$$

34. Question

Find $\frac{dy}{dx}$, when:

$$y = (x \cos x)^x + (x \sin x)^{\frac{1}{x}}$$

Answer

simply taking log both sides would not help more.

For that let us assume $u = (x \cdot \cos x)^x$ and $v = (x \cdot \sin x)^{\frac{1}{x}}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x \cdot \cos x)^x$$

Take log both sides

$$\ln u = x (\ln(x) + \ln \cos x)$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x \left(\frac{1}{x} - \frac{\sin x}{\cos x} \right) + (\ln(x) + \ln \cos x)$$

$$\frac{du}{dx} = \left(x \left(\frac{1}{x} - \frac{\sin x}{\cos x} \right) + (\ln(x) + \ln \cos x) \right) \times u$$

$$\frac{du}{dx} = \left(x \left(\frac{1}{x} - \frac{\sin x}{\cos x} \right) + (\ln(x) + \ln \cos x) \right) \times ((x \cdot \cos x)^x) \dots\dots\dots(1)$$

$$v = (x \cdot \sin x)^{\frac{1}{x}}$$

Take log both sides,

$$\ln v = \frac{1}{x} \times (\ln x + \ln \sin x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \frac{1}{x} \left(\frac{1}{x} + \frac{\cos x}{\sin x} \right) - \frac{1}{x^2} \times (\ln x + \ln \sin x)$$

$$\frac{dv}{dx} = \left(\frac{1}{x} \left(\frac{1}{x} + \frac{\cos x}{\sin x} \right) - \frac{1}{x^2} \times (\ln x + \ln \sin x) \right) \times v$$

$$\frac{dv}{dx} = \left(\frac{1}{x} \left(\frac{1}{x} + \frac{\cos x}{\sin x} \right) - \frac{1}{x^2} \times (\ln x + \ln \sin x) \right) \times (x \cdot \sin x)^{\frac{1}{x}} \dots\dots(2)$$

$$\begin{aligned} \frac{dy}{dx} &= \left(\frac{1}{x} \left(\frac{1}{x} + \frac{\cos x}{\sin x} \right) - \frac{1}{x^2} \times (\ln x + \ln \sin x) \right) \times (x \cdot \sin x)^{\frac{1}{x}} \\ &\quad + \left(x \left(\frac{1}{x} - \frac{\sin x}{\cos x} \right) + (\ln(x) + \ln \cos x) \right) \times ((x \cdot \cos x)^x) \end{aligned}$$

35. Question

Find $\frac{dy}{dx}$, when:

$$y = (\sin x)^x + \sin^{-1} \sqrt{x}$$

Answer

simply taking log both sides would not help more.

For that let us assume $u = (\sin x)^x$ and $v = \sin^{-1} \sqrt{x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\sin x)^x$$

Take log both sides

$$\ln u = x \cdot \ln \sin x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x \left(\frac{\cos x}{\sin x} \right) + \ln \sin x$$

$$\frac{du}{dx} = \left(x \left(\frac{\cos x}{\sin x} \right) + \ln \sin x \right) \times u$$

$$\frac{du}{dx} = \left(x \left(\frac{\cos x}{\sin x} \right) + \ln \sin x \right) \times ((\sin x)^x) \dots\dots\dots(1)$$

for v we do not have to take log just simply differentiate it,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2\sqrt{x}} \dots\dots\dots(2)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2\sqrt{x}} + \left(x \left(\frac{\cos x}{\sin x} \right) + \ln \sin x \right) \times ((\sin x)^x)$$

36. Question

Find $\frac{dy}{dx}$, when:

$$y = x^{x \cos x} + \left(\frac{x^2 + 1}{x^2 - 1} \right)$$

Answer

simply taking log both sides would not help more.

For that let us assume $u = (x)^{x \cdot \cos x}$ and $v = \frac{x^2 + 1}{x^2 - 1}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{x \cdot \cos x}$$

Take log both sides

$$\ln u = x \cdot \cos x \cdot \ln x$$



Here there are three terms to differentiate for this; we can take two term as one and then apply product rule, I am taking $x \cdot \ln x$ as a single term

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x \left(x \left(\frac{1}{x} \right) + \ln x \right) + x \cdot \ln x (-\sin x)$$

$$\frac{du}{dx} = (\cos x(1 + \ln x) - x \cdot \ln x \cdot \sin x) \times u$$

$$\frac{du}{dx} = (\cos x(1 + \ln x) - x \cdot \ln x \cdot \sin x) \times (x \cdot \cos x \cdot \ln x) \dots\dots\dots(1)$$

for v we do not have to take log just simply differentiate it,

$$\frac{dv}{dx} = \frac{(x^2 - 1)(2x) - (x^2 + 1)(2x)}{(x^2 - 1)^2}$$

$$\frac{dy}{dx} = \frac{2x(-2)}{(x^2 - 1)^2} \dots\dots\dots(2)$$

$$\frac{dy}{dx} = (\cos x(1 + \ln x) - x \cdot \ln x \cdot \sin x) \times (x \cdot \cos x \cdot \ln x) + \frac{2x(-2)}{(x^2 - 1)^2}$$

37. Question

Find $\frac{dy}{dx}$, when:



$$y = e^x \sin^3 x \cos^4 x$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = x + 3 \cdot \ln \sin x + 4 \ln \cos x$$

$$\{\ln(mn) = \ln n + \ln m\} \left\{ \ln \left(\frac{m}{n} \right) = \ln m - \ln n \right\} \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{3 \cdot \cos x}{\sin x} - \frac{4 \sin x}{\cos x} + 1 \right)$$

$$\frac{dy}{dx} = \left(\frac{3 \cdot \cos x}{\sin x} - \frac{4 \sin x}{\cos x} + 1 \right) \times y$$

$$\frac{dy}{dx} = \left(\frac{3 \cdot \cos x}{\sin x} - \frac{4 \sin x}{\cos x} + 1 \right) \times e^x \cdot \sin^3 x \cdot \cos^4 x$$

38. Question

Find $\frac{dy}{dx}$, when:

$$y = 2^x \cdot e^{3x} \sin 4x$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = x \cdot \ln 2 + 3x + \ln \sin 4x$$

$$\{\ln(mn) = \ln n + \ln m\} \left\{ \ln\left(\frac{m}{n}\right) = \ln m - \ln n \right\} \{\ln e = 1\}$$

Now differentiating both sides by x , we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4$$

$$\frac{dy}{dx} = \left(\ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4 \right) \times y$$

$$\frac{dy}{dx} = \left(\ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4 \right) \times e^{3x} \cdot \sin 4x \cdot 2^x$$

39. Question

Find $\frac{dy}{dx}$, when:

$$y = x^x \cdot e^{(2x+5)}$$

Answer

Here we need to take log both the sides to get that differentiation simple.

$$\ln y = x \cdot \ln x + 2x + 5$$

$$\{\ln(mn) = \ln n + \ln m\} \left\{ \ln\left(\frac{m}{n}\right) = \ln m - \ln n \right\} \{\ln e = 1\}$$

Now differentiating both sides by x , we get,

$$\frac{1}{y} \times \frac{dy}{dx} = 1 + \ln x + 2$$

$$\frac{dy}{dx} = (\ln x + 3) \times y$$

$$\frac{dy}{dx} = (\ln x + 3) \times x^x \cdot e^{2x+5}$$

40. Question

Find $\frac{dy}{dx}$, when:

$$y = (2x + 3)^5 (3x - 5)^7 (5x - 1)^3$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 5 \cdot \ln(2x + 3) + 7 \cdot \ln(3x - 5) + 3 \cdot \ln(5x - 1)$$

$$\{\ln(mn) = \ln n + \ln m\} \left\{ \ln\left(\frac{m}{n}\right) = \ln m - \ln n \right\} \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{5 \times 2}{2x + 3} + \frac{7 \times 3}{3x - 5} + \frac{3 \times 5}{5x - 1}$$

$$\frac{dy}{dx} = \left(\frac{10}{2x + 3} + \frac{21}{3x - 5} + \frac{15}{5x - 1} \right) \times y$$

$$\frac{dy}{dx} = \left(\frac{10}{2x + 3} + \frac{21}{3x - 5} + \frac{15}{5x - 1} \right) \times (2x + 3)^5 (3x - 5)^7 (5x - 1)^3$$

41. Question

Find $\frac{dy}{dx}$, when:

$$(\cos x)^y = (\cos y)^x$$

Answer

. So the equation given is implicit, we will just take log both sides

$$y \cdot \ln(\cos x) = x \cdot \ln(\cos y)$$

Now differentiate it with respect to x and consider $\frac{dy}{dx} = y'$

$$y \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x \cdot y' = x \left(\frac{-\sin y}{\cos y} \times y' \right) + \ln \cos y$$

Taking y' one side, we get

$$y'(\ln \cos x + x \cdot \tan x) = \ln \cos y + y \cdot \tan x$$

$$y' = \frac{\ln \cos y + y \cdot \tan x}{\ln \cos x + x \cdot \tan x}$$

42. Question

Find $\frac{dy}{dx}$, when:

$$(\tan x)^y = (\tan y)^x$$

Answer

. So the equation given is implicit, we will just take log both sides

$$y \cdot \ln(\tan x) = x \cdot \ln(\tan y)$$

Now differentiate it with respect to x and consider $\frac{dy}{dx} = y'$

$$y \left(\frac{\sec^2 x}{\tan x} \right) + \ln \tan x \cdot y' = x \left(\frac{\sec^2 y}{\tan y} \times y' \right) + \ln \tan y$$

Taking y' one side, we get

$$y' \left(\ln \tan x + \frac{x}{\sin y \cdot \cos y} \right) = \ln \tan y + \frac{y}{\sin x \cdot \cos x}$$

$$y' = \frac{\sin 2x \cdot \ln \tan y + 2y}{\sin 2y \cdot \ln \tan x + 2x}$$

43. Question

Find $\frac{dy}{dx}$, when:

$$y = ((\log x)^x + (x)^{\log x})$$

Answer

we can write this equation as,

$$y = e^{x \ln(\ln x)} + e^{\ln x \cdot \ln x}$$

Differentiate

$$y' = (\ln x)^x \left(x \left(\frac{1}{\ln x} \times \frac{1}{x} \right) + \ln(\ln x) \right) + x^{\ln x} \left(2 \cdot \frac{\ln x}{x} \right)$$

$$y' = (\ln x)^x \left(\frac{1}{\ln x} + \ln(\ln x) \right) + x^{\ln x} \left(\frac{2 \ln x}{x} \right)$$

44. Question

If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, prove that $(1-x^2) \frac{dy}{dx} = (xy + 1)$.

Answer

differentiate the given y to get the result,

$$\frac{dy}{dx} = \frac{\sqrt{1-x^2} \left(\frac{1}{\sqrt{1-x^2}} \right) - \sin^{-1} x \left(\frac{-x}{\sqrt{1-x^2}} \right)}{(\sqrt{1-x^2})^2}$$

$$\frac{dy}{dx} = \frac{1 + \frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}}}{1-x^2}$$

$$\frac{dy}{dx} (1-x^2) = 1 + xy$$

45. Question

If $y = \sqrt{x+y}$, prove that $\frac{dy}{dx} = \frac{1}{(2y-1)}$.

Answer

differentiate the given y to get the result,

$$\frac{dy}{dx} = \frac{1 + \frac{dy}{dx}}{2\sqrt{x+y}}$$

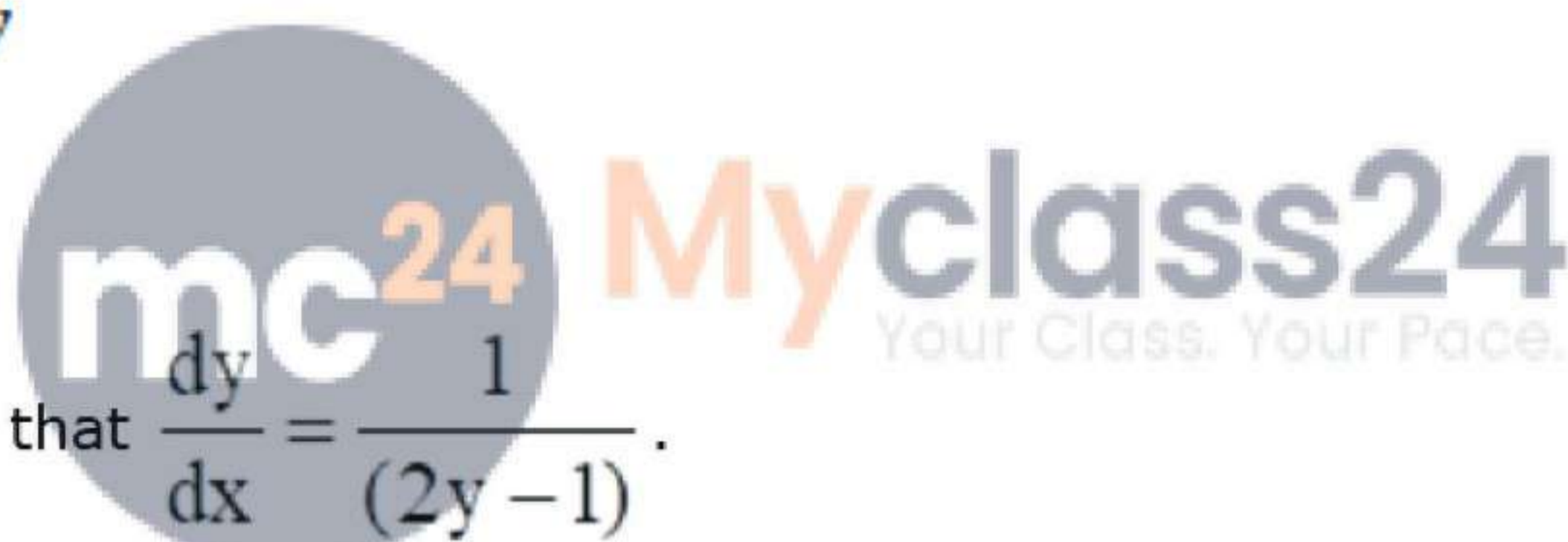
$$\text{let, } \frac{dy}{dx} = y'$$

$$y' = \frac{1+y'}{2\sqrt{x+y}} \text{ \{taking } y' \text{ one side\}}$$

$$y'(2\sqrt{x+y} - 1) = 1$$

$$\frac{dy}{dx} = \frac{1}{2y-1}$$

46. Question



If $x^a y^b = (x + y)^{(a+b)}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Answer

taking log both sides,

$$a \ln x + b \ln y = (a + b) \cdot \ln(x + y)$$

differentiating both sides,

$$\frac{a}{x} + \frac{b}{y} \times y' = \frac{a + b}{x + y} \times (1 + y')$$

Take y' one side,

$$y' \left(\frac{b}{y} - \frac{a + b}{x + y} \right) = \frac{a + b}{x + y} - \frac{a}{x}$$

$$y' = \frac{ax + bx - (ax + ay)}{x \cdot (x + y)} \times \frac{y \cdot (x + y)}{bx + by - (ay + by)}$$

$$y' = \frac{bx - ay}{x} \times \frac{y}{bx - ay}$$

$$y' = \frac{y}{x}$$



47. Question

If $(x^x + y^x) = 1$, show that $\frac{dy}{dx} = - \left\{ \frac{x^x (1 + \log x) + y^x (\log y)}{xy^{x-1}} \right\}$

Answer

differentiate both sides,

$$x^x (1 + \ln x) + y^x \left(\frac{x}{y} \times y' + \ln y \right) = 0$$

Taking y' one side,

$$y' = \left(\frac{x^x (1 + \ln x)}{y^x} - \ln y \right) \times \frac{y}{x}$$

$$y' = \frac{x^x (1 + \ln x) - y^x \cdot \ln y}{x \cdot y^{x-1}}$$

48. Question

If $y = e^{\sin x} + (\tan x)^x$, prove that $\frac{dy}{dx} = e^{\sin x} \cos x + (\tan x)^x [2x \operatorname{cosec} 2x + \log \tan x]$.

Answer

differentiate both sides,

$$y' = e^{\sin x} (\cos x) + (\tan x)^x \left(x \left(\frac{\sec^2 x}{\tan x} \right) + \ln \tan x \right)$$

$$y' = e^{\sin x} (\cos x) + (\tan x)^x (2x \operatorname{cosec} 2x + \ln \tan x)$$

49. Question

If $y = \log(x + \sqrt{1+x^2})$, prove that $\frac{dy}{dx} = \frac{1}{\log(x + \sqrt{1+x^2})} \cdot \frac{1}{\sqrt{1+x^2}}$.

Answer

differentiate both sides,

$$y' = \frac{1}{x + \sqrt{1+x^2}} \times \left(1 + \frac{x}{\sqrt{1+x^2}} \right)$$

$$y' = \frac{1}{x + \sqrt{1+x^2}} \times \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}}$$

$$y' = \frac{1}{\sqrt{1+x^2}}$$

50. Question

If $y = \log \sin \sqrt{x^2 + 1}$, prove that $\frac{dy}{dx} = \frac{x \cot \sqrt{x^2 + 1}}{\sqrt{x^2 + 1}}$.

Answer

differentiate both sides,

$$y' = \frac{1}{\sin(\sqrt{1+x^2})} \times \cos(\sqrt{1+x^2}) \times \frac{x}{\sqrt{1+x^2}}$$

$$y' = \frac{(\cot(\sqrt{1+x^2}) \cdot x)}{\sqrt{1+x^2}}$$

51. Question

If $y = \log \sqrt{\frac{1 - \cos x}{1 + \cos x}}$, show that $\frac{dy}{dx} = \operatorname{cosec} x$.

Answer

differentiate both sides,

$$y' = \sqrt{\frac{1 + \cos x}{1 - \cos x}} \times \frac{(1 + \cos x)(\sin x) - (1 - \cos x)(-\sin x)}{(1 + \cos x)^2}$$

$$y' = \sqrt{\frac{1 + \cos x}{1 - \cos x}} \times \frac{(\sin x + \sin x \cdot \cos x) + (\sin x - \sin x \cdot \cos x)}{(1 + \cos x)^2}$$

$$y' = \sqrt{\frac{1 + \cos x}{1 - \cos x}} \times \frac{2 \sin x}{(1 + \cos x)^2}$$

$$y' = \sqrt{\frac{2 \cos^2 \frac{x}{2}}{2 \sin^2 \frac{x}{2}}} \times \frac{4 \sin \frac{x}{2} \cdot \cos \frac{x}{2}}{(1 + \cos x)^2}$$

$$y' = 4 \cos \frac{x}{2} \times \frac{\cos \frac{x}{2}}{4 \cos^4 \frac{x}{2}}$$

$$y' = \frac{1}{\cos \frac{x}{2}}$$

$$y' = \sec^2 \frac{x}{2}$$

52. Question

If $y = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$, show that $\frac{dy}{dx} = \sec x$.

Answer

differentiate both sides,

$$y' = \frac{1}{\tan \left(\frac{\pi}{4} + \frac{x}{2} \right)} \times \sec^2 \left(\frac{\pi}{4} + \frac{x}{2} \right) \times \frac{1}{2}$$



$$y' = \frac{1}{2 \times \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}$$

$$y' = \frac{1}{\sin\left(\frac{\pi}{2} + x\right)}$$

$$y' = \sec x$$

53. Question

If $y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$, show that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$.

Answer

differentiate both sides,

$$y' = \frac{1}{2} \times \sqrt{\frac{1 + \sin 2x}{1 - \sin 2x}} \times \frac{(1 + \sin 2x)(-2 \cos 2x) - (1 - \sin 2x)(2 \cos 2x)}{(1 + \sin 2x)^2}$$

$$y' = \frac{1}{2} \times \sqrt{\frac{1}{1 - \sin^2 2x}} \times \frac{2 \cos 2x(-1 - \sin 2x - 1 + \sin 2x)}{1 + \sin 2x}$$

$$y' = \frac{1}{2} \times \frac{-4}{1 + \sin 2x}$$

$$y' = \frac{-2}{(\cos x + \sin x)^2}$$

$$y' = \frac{-1}{\left(\frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}\right)^2}$$

$$y' = \frac{-1}{\cos^2\left(\frac{\pi}{4} + x\right)}$$

$$\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} + x\right) = 0$$

54. Question

If $y = \log \sqrt{\frac{1 + \cos^2 x}{1 - e^{2x}}}$, show that $\frac{dy}{dx} = \frac{e^{2x}}{(1 - e^{2x})} - \frac{\sin x \cos x}{(1 + \cos^2 x)}$.

Answer

differentiate both sides,

$$y' = \sqrt{\frac{1 - e^{2x}}{1 + \cos^2 x}} \times \frac{(1 - e^{2x})(-2 \cos x \cdot \sin x) - (1 + \cos^2 x)(-2e^{2x})}{(1 - e^{2x})^2} \times \frac{1}{2}$$

$$\times \sqrt{\frac{1 - e^{2x}}{1 + \cos^2 x}}$$

$$y' = \sqrt{\frac{1 - e^{2x}}{\cos 2x}} \times \frac{(e^{2x} - 1)(\sin 2x) + 2e^{2x}(\cos 2x)}{(1 - e^{2x})^2} \times \frac{1}{2} \times \sqrt{\frac{1 - e^{2x}}{\cos 2x}}$$

$$y' = \frac{(e^{2x} - 1) \tan 2x + 2e^{2x}}{2 \cdot (1 - e^{2x})}$$

$$y' = \frac{e^{2x}}{(1 - e^{2x})} - \frac{\sin x \cdot \cos x}{(1 + \cos^2 x)}$$

55. Question

If $y = (x)^{\cos x} + (\sin x)^{\tan x}$, prove that $\frac{dy}{dx} = x^{\cos x} \left\{ \frac{\cos x}{x} - (\sin x) \log x \right\} + (\sin x)^{\tan x} \cdot \left\{ 1 + (\log \sin x) \sec^2 x \right\}$.

Answer

simply taking log both sides would not help more.

For that let us assume $u = (x)^{\cos x}$ and $v = (\sin x)^{\tan x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\cos x}$$

Take log both sides

$$\ln u = \cos x \cdot \ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x \left(\frac{1}{x} \right) + \ln x (-\sin x)$$

$$\frac{du}{dx} = \left(\frac{\cos x}{x} - \ln x \cdot \sin x \right) \times u$$

$$\frac{du}{dx} = \left(\frac{\cos x}{x} - \ln x \cdot \sin x \right) \times (x^{\cos x}) \dots\dots\dots(1)$$

$$v = (\sin x)^{\tan x}$$

Take log both sides,

$$\ln v = \tan x \times (\ln \sin x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \tan x \left(\frac{\cos x}{\sin x} \right) + \ln \sin x (\sec^2 x)$$

$$\frac{dv}{dx} = \left(\tan x \left(\frac{\cos x}{\sin x} \right) + \ln \sin x (\sec^2 x) \right) \times v$$

$$\frac{dv}{dx} = (1 + \ln \sin x (\sec^2 x)) \times (\sin x)^{\tan x} \dots\dots(2)$$

$$\frac{dy}{dx} = (1 + \ln \sin x (\sec^2 x)) \times (\sin x)^{\tan x} + \left(\frac{\cos x}{x} - \ln x \cdot \sin x \right) \times (x^{\cos x})$$

56. Question

If $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$, prove that $\frac{dy}{dx} = (\sin x)^{\cos x} [\cot x \cos x - \sin x (\log \sin x)] + (\cos x)^{\sin x} [\cos x (\log \cos x) - \sin x \tan x]$.

Answer

simply taking log both sides would not help more.

For that let us assume $u = (\sin x)^{\cos x}$ and $v = (\cos x)^{\sin x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\sin x)^{\cos x}$$

Take log both sides

$$\ln u = \cos x \cdot \ln \sin x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x \left(\frac{\cos x}{\sin x} \right) + \ln(\sin x) (-\sin x)$$

$$\frac{du}{dx} = \left(\frac{\cos^2 x}{\sin x} - \ln \sin x \cdot \sin x \right) \times u$$

$$\frac{du}{dx} = \left(\frac{\cos^2 x}{\sin x} - \ln \sin x \cdot \sin x \right) \times (\sin x^{\cos x}) \dots\dots\dots(1)$$

$$v = (\cos x)^{\sin x}$$

Take log both sides,

$$\ln v = \sin x \times (\ln \cos x)$$

Differentiate ,

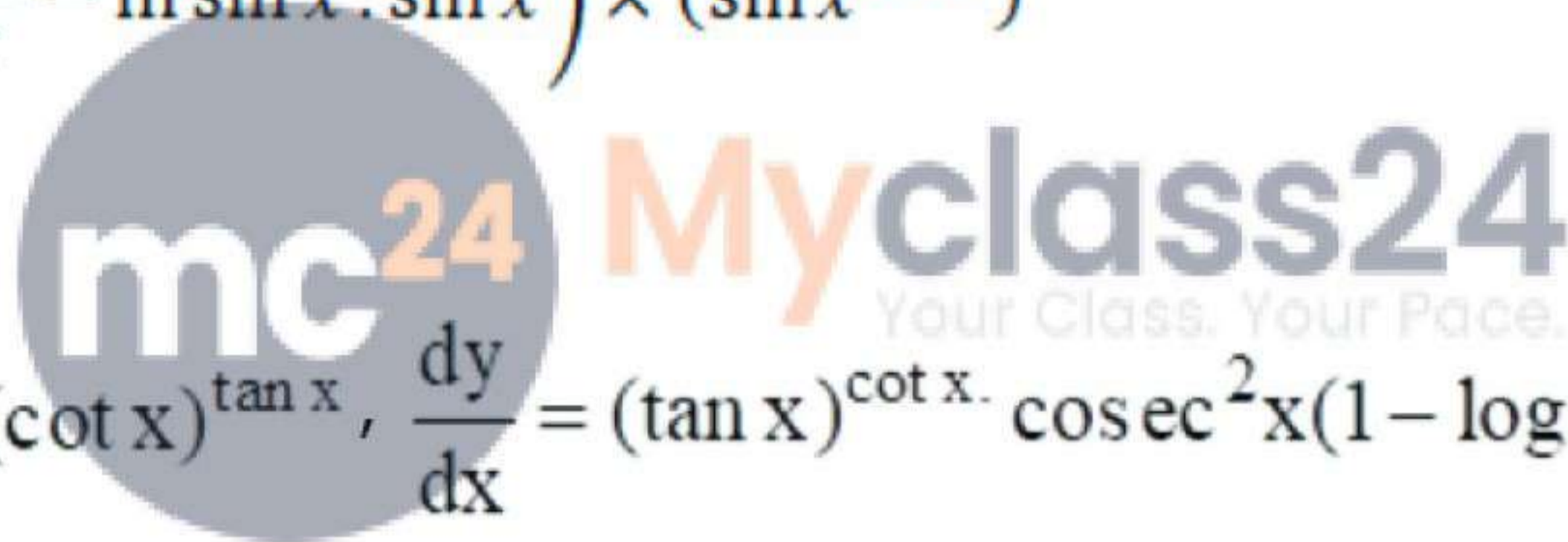
$$\frac{1}{v} \times \frac{dv}{dx} = \sin x \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x (\cos x)$$

$$\frac{dv}{dx} = \left(\sin x \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x (\cos x) \right) \times v$$

$$\frac{dv}{dx} = \left(\sin x \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x (\cos x) \right) \times (\cos x)^{\sin x} \dots\dots(2)$$

$$\begin{aligned} \frac{dy}{dx} &= \left(\sin x \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x (\cos x) \right) \times (\cos x)^{\sin x} \\ &\quad + \left(\frac{\cos^2 x}{\sin x} - \ln \sin x \cdot \sin x \right) \times (\sin x^{\cos x}) \end{aligned}$$

57. Question



If $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$, $\frac{dy}{dx} = (\tan x)^{\cot x} \cdot \operatorname{cosec}^2 x (1 - \log \tan x) + (\cot x)^{\tan x} \cdot \sec^2 x [\log(\cot x) - 1]$.

Answer

simply taking log both sides would not help more.

For that let us assume $u = (\tan x)^{\cot x}$ and $v = (\cot x)^{\tan x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\tan x)^{\cot x}$$

Take log both sides

$$\ln u = \cot x \cdot \ln \tan x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cot x \left(\frac{\sec^2 x}{\tan x} \right) + \ln(\tan x) (-\operatorname{cosec}^2 x)$$

$$\frac{du}{dx} = \left(\cot x \left(\frac{\sec^2 x}{\tan x} \right) - \ln(\tan x)(\operatorname{cosec}^2 x) \right) \times u$$

$$\frac{du}{dx} = (\operatorname{cosec}^2 x(1 - \ln(\tan x))) \times (\tan x)^{\cot x} \dots\dots\dots(1)$$

$$v = (\cot x)^{\tan x}$$

Take log both sides,

$$\ln v = \tan x \times (\ln \cot x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \tan x \left(\frac{-\operatorname{cosec}^2 x}{\cot x} \right) + \ln \cot x (\sec^2 x)$$

$$\frac{dv}{dx} = (\sec^2 x(\ln \cot x - 1)) \times v$$

$$\frac{dv}{dx} = (\sec^2 x(\ln \cot x - 1)) \times (\cot x)^{\tan x} \dots\dots(2)$$

$$\frac{dy}{dx} = (\sec^2 x(\ln \cot x - 1)) \times (\cot x)^{\tan x} + (\operatorname{cosec}^2 x(1 - \ln(\tan x))) \times (\tan x)^{\cot x}$$



58. Question

If $y = x^{\cos x} + (\cos x)^x$, prove that $\frac{dy}{dx} = x^{\cos x} \cdot \left\{ \frac{\cos x}{x} - (\sin x) \log x \right\} + (\cos x)^x [(\log \cos x) - x \tan x]$.

Answer

simply taking log both sides would not help more.

For that let us assume $u = (x)^{\cos x}$ and $v = (\cos x)^x$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\cos x}$$

Take log both sides

$$\ln u = \cos x \cdot \ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x \left(\frac{1}{x}\right) + \ln(x)(-\sin x)$$

$$\frac{du}{dx} = \left(\cos x \left(\frac{1}{x}\right) + \ln(x)(-\sin x)\right) \times u$$

$$\frac{du}{dx} = \left(\cos x \left(\frac{1}{x}\right) + \ln(x)(-\sin x)\right) \times (x)^{\cos x} \dots\dots\dots(1)$$

$$v = (\cos x)^x$$

Take log both sides,

$$\ln v = x \times (\ln \cos x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x. 1$$

$$\frac{dv}{dx} = \left(x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x. 1\right) \times v$$

$$\frac{dv}{dx} = \left(x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x. 1\right) \times (\cos x)^x \dots\dots(2)$$

$$\frac{dy}{dx} = \left(x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x. 1\right) \times (\cos x)^x + \left(\cos x \left(\frac{1}{x}\right) + \ln(x)(-\sin x)\right) \times (x)^{\cos x}$$

59. Question

If $y = x^{\log x} + (\log x)^x$, prove that $\frac{dy}{dx} = x^{(\log x)} \left\{ \frac{2 \log x}{x} \right\} + (\log x)^x \cdot \left\{ \frac{1}{\log x} + \log(\log x) \right\}$

Answer

simply taking log both sides would not help more.

For that let us assume $u = (x)^{\ln x}$ and $v = (\ln x)^x$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\ln x}$$

Take log both sides

$$\ln u = \ln x. \ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = 2 \ln x \left(\frac{1}{x} \right)$$

$$\frac{du}{dx} = \left(2 \ln x \left(\frac{1}{x} \right) \right) \times u$$

$$\frac{du}{dx} = \left(2 \ln x \left(\frac{1}{x} \right) \right) \times (x)^{\ln x} \dots\dots\dots(1)$$

$$v = (\ln x)^x$$

Take log both sides,

$$\ln v = x \times (\ln x)$$

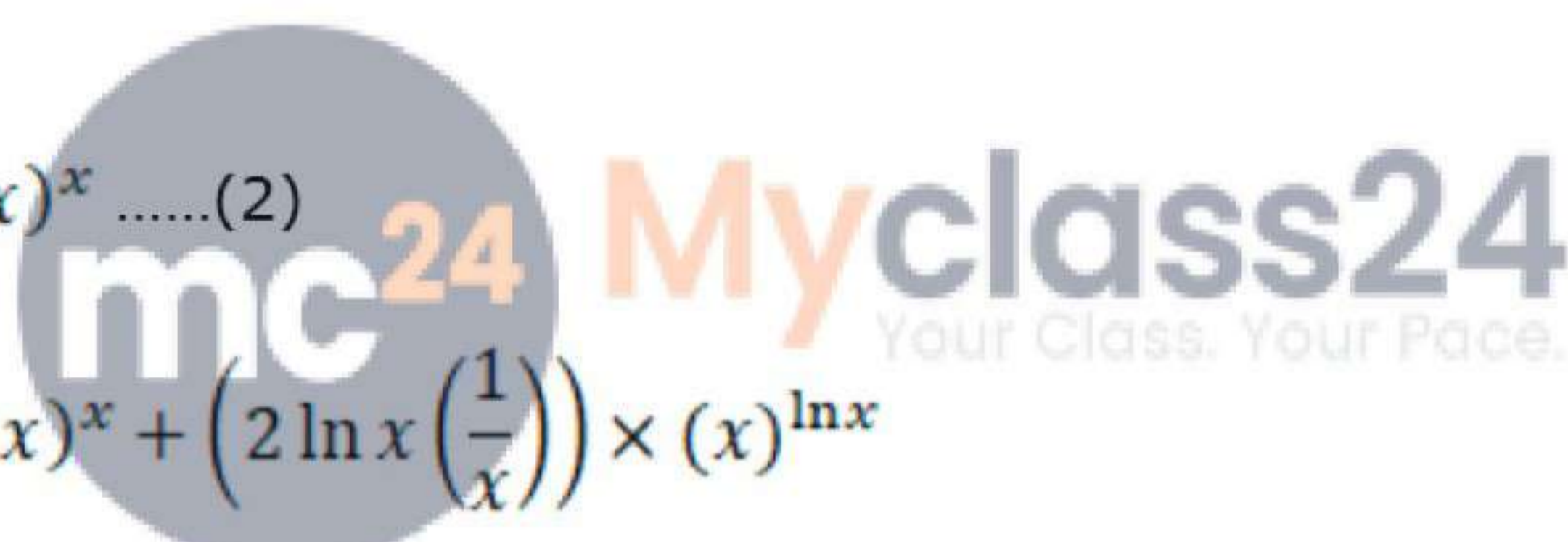
Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = x \left(\frac{1}{x} \right) + \ln x$$

$$\frac{dv}{dx} = (1 + \ln x) \times v$$

$$\frac{dv}{dx} = (1 + \ln x) \times (\ln x)^x \dots\dots(2)$$

$$\frac{dy}{dx} = (1 + \ln x) \times (\ln x)^x + \left(2 \ln x \left(\frac{1}{x} \right) \right) \times (x)^{\ln x}$$



60. Question

If $y = x^{(x^2-3)} + (x-3)^{x^2}$, find $\frac{dy}{dx}$.

Answer

equality is not given but we may assume that it is equal to 0.

We can also write this equation as

$$y = e^{(x^2-3)\ln x} + e^{x^2 \ln(x-3)} = 0$$

Now differentiating it,

$$y' = x^{x^2-3} \left(\frac{x^2-3}{x} + \ln x \cdot 2x \right) + (x-3)^{x^2} \cdot \left(\frac{x^2}{x-3} + \ln(x-3) \cdot 2x \right) = 0$$

$$y' = x^{x^2-3} \left(\frac{x^2-3}{x} + \ln x \cdot 2x \right) - (x-3)^{x^2} \cdot \left(\frac{x^2}{x-3} + \ln(x-3) \cdot 2x \right)$$

61. Question

$$\text{If } f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}, \text{ find } f'(0).$$

Answer

take log both the side,

$$\ln f(x) = (2+3x) \cdot \ln\left(\frac{3+x}{1+x}\right)$$

Now differentiate it,

$$\frac{1}{f(x)} \times f'(x) = (2+3x) \left(\frac{1+x}{3+x}\right) \left(\frac{1+x-(3+x)}{(1+x)^2}\right) + \ln\left(\frac{3+x}{1+x}\right) \cdot 3$$

$$f'(x) = \left(\frac{(2+3x)(-2)}{(3+x)(1+x)} + 3 \ln\left(\frac{3+x}{1+x}\right)\right) \times f(x)$$

To get $f'(0)$ we need to find $f(0)$,

Putting $x=0$ in f

$$f(0) = \left(\frac{3}{1}\right)^2$$

$$f(0) = 9$$

Now put $x=0$ in $f'(x)$,

$$f'(0) = \left(\left(\frac{2 \times (-2)}{3}\right) + 3 \ln 3\right) \times 9$$

$$f'(0) = 9 \left(3 \ln 3 - \frac{4}{3}\right)$$

62. Question

$$\text{If } y = (\sin x)^x + \sin^{-1} \sqrt{x}, \text{ find } \frac{dy}{dx}.$$

Answer

we can write this equation as,

$$y = e^{x \ln(\sin x)} + \sin^{-1} \sqrt{x}$$

Differentiate it,



$$y' = (\sin x)^x \left(\frac{x \times \cos x}{\sin x} + \ln(\sin x) \right) + \frac{1}{\sqrt{1 - \sqrt{x}^2}} \times \frac{1}{2\sqrt{x}}$$

$$y' = (\sin x)^x (x \cdot \cot x + \ln \sin x) + \frac{1}{2\sqrt{x} \cdot \sqrt{1 - x}}$$

63. Question

If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

Answer

simply differentiate both sides,

$$2(x^2 + y^2)(2x + 2y \cdot y') = x \cdot y' + y$$

Take y' one side

$$4x^3 + 4x^2 \cdot y \cdot y' + 4y^2 \cdot x + 4y^3 \cdot y' = x \cdot y' + y$$

$$y'(4x^2 \cdot y + 4y^3 - x) = y - 4x^3 - 4y^2 x$$

$$y' = \frac{y - 4x^3 - 4y^2 x}{4x^2 \cdot y + 4y^3 - x}$$



64. Question

$y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$, find $\frac{dy}{dx}$.

Answer

we can write this as,

$$y = e^{\cot x \cdot \ln x} + \frac{2x^2 - 3}{x^2 + x + 2}$$

Differentiate ,

$$y' = x^{\cot x} \left(\frac{\cot x}{x} + \ln x (-\operatorname{cosec}^2 x) \right) + \frac{(x^2 + x + 2)(4x) - (2x^2 - 3)(2x + 1)}{(x^2 + x + 2)^2}$$

$$y' = x^{\cot x} \left(\frac{\cot x}{x} + \ln x (-\operatorname{cosec}^2 x) \right) + \frac{2x^2 + 14x + 3}{(x^2 + x + 2)^2}$$

65. Question

Find , when:

$$\text{If } y = \tan^{-1} \frac{a}{x} + \log \sqrt{\frac{x-a}{x+a}}, \text{ prove that } \frac{dy}{dx} = \frac{2a^3}{(x^4 - a^4)}.$$

Answer

Differentiate it,

$$y' = \frac{1}{1 + \frac{a^2}{x^2}} \times \left(-\frac{a}{x^2}\right) + \sqrt{\frac{x+a}{x-a}} \times \frac{1}{2} \times \sqrt{\frac{x+a}{x-a}} \times \frac{(x+a) - (x-a)}{(x+a)^2}$$

$$y' = \frac{-a}{x^2 + a^2} + \frac{x-a}{2(x+a)} \times \frac{2a}{(x-a)^2}$$

$$y' = -\frac{a}{(x^2 + a^2)} + \frac{a}{x^2 - a^2}$$

$$y' = \frac{ax^2 + a^3 - ax^2 + a^3}{x^4 - a^4}$$

$$y' = \frac{2a^3}{x^4 - a^4}$$



66. Question

$$\text{If } x^m y^n = (x+y)^{m+n}, \text{ prove that } \frac{dy}{dx} = \frac{y}{x}.$$

Answer

taking log both sides,

$$m \ln x + n \ln y = (m+n) \cdot \ln(x+y)$$

differentiating both sides,

$$\frac{m}{x} + \frac{n}{y} \times y' = \frac{m+n}{x+y} \times (1+y')$$

Take y' one side,

$$y' \left(\frac{n}{y} - \frac{m+n}{x+y} \right) = \frac{m+n}{x+y} - \frac{m}{x}$$

$$y' = \frac{mx + nx - (mx + my)}{x \cdot (x+y)} \times \frac{y \cdot (x+y)}{nx + ny - (my + ny)}$$

$$y' = \frac{nx - my}{x} \times \frac{y}{nx - my}$$

$$y' = \frac{y}{x}$$

Exercise 10G

1. Question

If $y = (\sin x)^{(\sin x)^{(\sin x)^{(\sin x)^{\dots\infty}}}}$, prove that $\frac{dy}{dx} = \frac{y^2 \cot x}{(1 - y \log \sin x)}$.

Answer

Given : $y = (\sin x)^{(\sin x)^{(\sin x)^{(\sin x)^{\dots\infty}}}}$

To prove : $\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y \log \sin x}$

Formula used : $\log a = \log b^m$

$$\log a = m \log b$$

$$\frac{d(\log y)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

$$\frac{d(\sin x)}{dx} = \cos x$$



If u and v are functions of x , then $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

$$y = (\sin x)^y$$

taking log on both sides

$$\log y = \log (\sin x)^y$$

$$\log y = y \log (\sin x)$$

Differentiating both sides with respect to x

$$\frac{d(\log y)}{dx} = \frac{d[y \log(\sin x)]}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{dy}{dx} \log(\sin x) + y \frac{d \log(\sin x)}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{dy}{dx} \log(\sin x) + y \frac{1}{\sin x} \times \frac{d(\sin x)}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{dy}{dx} \log(\sin x) + y \frac{\cos x}{\sin x}$$

$$\left(\frac{1}{y} - \log \sin x\right) \frac{dy}{dx} = y \cot x$$

$$\frac{1 - y \log \sin x}{y} \frac{dy}{dx} = y \cot x$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y \log \sin x}$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y \log \sin x}$$

2. Question

If $y = (\cos x)^{(\cos x)^{(\cos x) \dots \infty}}$, prove that $\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \log \cos x}$.

Answer

Given : $y = (\cos x)^{(\cos x)^{(\cos x)^{(\cos x) \dots \infty}}}$

To prove : $\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \log \cos x}$

Formula used : $\log a = \log b^m$

$$\log a = m \log b$$

$$\frac{d(\log y)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

If u and v are functions of x , then $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

Given that $y = (\cos x)^y$

taking log on both sides

$$\log y = \log (\cos x)^y$$

$$\log y = y \log (\cos x)$$

Differentiating both sides with respect to x

$$\frac{d(\log y)}{dx} = \frac{d[y \log(\cos x)]}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{dy}{dx} \log(\cos x) + y \frac{d \log(\cos x)}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{dy}{dx} \log(\cos x) + y \frac{1}{\cos x} \times \frac{d(\cos x)}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{dy}{dx} \log(\cos x) + y \frac{-\sin x}{\cos x}$$

$$\left(\frac{1}{y} - \log \cos x\right) \frac{dy}{dx} = -y \tan x$$

$$\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \log \cos x}$$

$$\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \log \cos x}$$

3. Question

If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$, prove that $\frac{dy}{dx} = \frac{1}{(2y-1)}$.

Answer

Given : $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$

To prove : $\frac{dy}{dx} = \frac{1}{2y-1}$

Formula used : $\log a = \log b^m$

$$\log a = m \log b$$

$$\frac{d(\log y)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

$$\frac{dx}{dx} = 1$$

If u and v are functions of x , then $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$$

$$y = \sqrt{x + y}$$

squaring on both sides

$$y^2 = x + y$$

Differentiating with respect to x

$$2y \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$(2y - 1) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y-1}$$

$$\frac{dy}{dx} = \frac{1}{2y-1}$$

4. Question

If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \infty}}}$, prove that $\frac{dy}{dx} = \frac{\sin x}{(1-2y)}$.

Answer

Given : $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \infty}}}$

To prove : $\frac{dy}{dx} = \frac{\sin x}{2y-1}$

Formula used : $\log a = \log b^m$

$$\log a = m \log b$$

$$\frac{d(\log y)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

If u and v are functions of x, then $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)).g'(x)$

$$y = \sqrt{\cos x + y}$$

squaring on both sides

$$y^2 = \cos x + y$$

Differentiating with respect to x

$$2y \frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$(2y - 1) \frac{dy}{dx} = -\sin x$$

$$\frac{dy}{dx} = \frac{-\sin x}{2y-1} = \frac{\sin x}{1-2y}$$

$$\frac{dy}{dx} = \frac{\sin x}{1-2y}$$

$$\frac{dy}{dx} = \frac{\sin x}{1-2y}$$

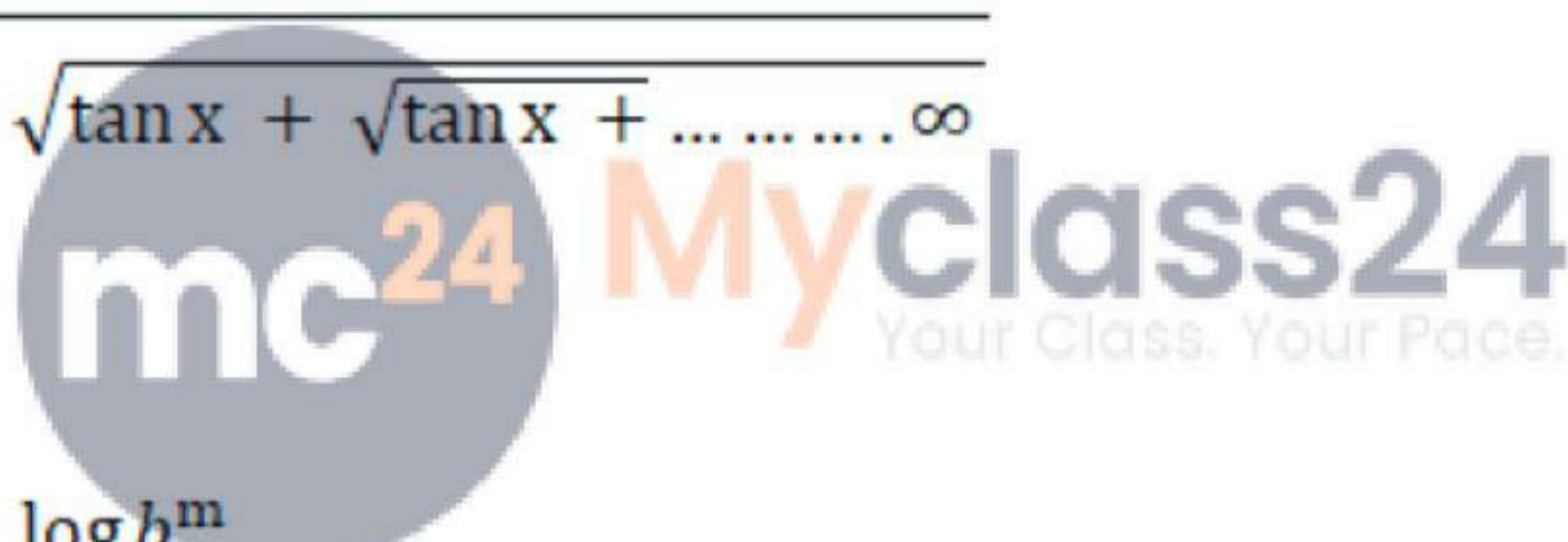
5. Question

If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots \infty}}}$, prove that $\frac{dy}{dx} = \frac{\sec^2 x}{(2y-1)}$.

Answer

Given : $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots \infty}}}$

To prove : $\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$



Formula used : $\log a = \log b^m$

$$\log a = m \log b$$

$$\frac{d(\log y)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

If u and v are functions of x, then $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)).g'(x)$

$$y = \sqrt{\tan x + y}$$

squaring on both sides

$$y^2 = \tan x + y$$

Differentiating with respect to x

$$2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$(2y - 1) \frac{dy}{dx} = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1} = \frac{\sec^2 x}{2y-1}$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$$

6. Question

If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \infty}}}$, show that $(2y - 1) \frac{dy}{dx} = \frac{1}{x}$.

Answer

Given : $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \infty}}}$

To show : $(2y - 1) \frac{dy}{dx} = \frac{1}{x}$

Formula used : $\log a = \log b^m$

$$\log a = m \log b$$

$$\frac{d(\log y)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

If u and v are functions of x, then $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

$$y = \sqrt{\log x + y}$$

squaring on both sides

$$y^2 = \log x + y$$

Differentiating with respect to x

$$2y \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$(2y - 1) \frac{dy}{dx} = \frac{1}{x}$$

$$(2y - 1) \frac{dy}{dx} = \frac{1}{x}$$

7. Question

If $y = a^{x^{a^{x^{\dots\infty}}}}$, prove that $\frac{dy}{dx} = \frac{y^2 (\log y)}{x[1 - y(\log x)(\log y)]}$.

Answer

Given : $y = a^{x^{a^{x^{\dots\infty}}}}$

To show : $\frac{dy}{dx} = \frac{y^2 (\log y)}{x[1 - y(\log x)(\log y)]}$

Formula used : $\log a = \log b^m$

$$\log a = m \log b$$

$$\frac{d(\log y)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

If u and v are functions of x, then $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

$$y = a^{x^y}$$

taking log on both sides

$$\log y = \log a^{x^y}$$

$$\log y = x^y \cdot \log a$$

taking log on both sides

$$\log(\log y) = \log(x^y \cdot \log a)$$

$$\log(\log y) = y \cdot \log x + \log(\log a)$$

Differentiating both sides with respect to x

$$\frac{d(\log [\log y])}{dx} = \frac{d(y \cdot \log x)}{dx} + 0 \text{ (as differentiation of } \log(\log a) \text{ [constant] is zero)}$$

$$\frac{1}{\log y} \frac{d \log y}{dx} = \log x \frac{dy}{dx} + y \cdot \frac{d \log x}{dx}$$

$$\frac{1}{\log y} \cdot \frac{1}{y} \frac{dy}{dx} = \log x \frac{dy}{dx} + y \cdot \frac{1}{x}$$

$$\left(\frac{1}{\log y} \cdot \frac{1}{y} - \log x\right) \frac{dy}{dx} = \frac{y}{x}$$

$$\left(\frac{1-y(\log x)(\log y)}{y(\log y)}\right) \frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{y^2(\log y)}{x[1-y(\log x)(\log y)]}$$

$$\frac{dy}{dx} = \frac{y^2(\log y)}{x[1-y(\log x)(\log y)]}$$

8. Question

If $y = x + \frac{1}{x + \frac{1}{x + \frac{1}{x + \dots \infty}}}$, prove that $\frac{dy}{dx} = \frac{y}{(2y-x)}$.

Answer

Given : $y = x + \frac{1}{x + \frac{1}{x + \frac{1}{x + \dots \infty}}}$

To show : $\frac{dy}{dx} = \frac{y}{(2y-x)}$



Formula used : $\log a = \log b^m$

$$\log a = m \log b$$

$$\frac{d(\log y)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

If u and v are functions of x, then $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

$$y = x + \frac{1}{y}$$

$$y^2 = xy + 1$$

Differentiating with respect to x

$$\frac{d(y^2)}{dx} = \frac{d(xy)}{dx} + 0 \text{ (as differentiation of constant is zero)}$$

$$2y \cdot \frac{dy}{dx} = x \cdot \frac{dy}{dx} + y$$

$$(2y - x) \frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{(2y-x)}$$

$$\frac{dy}{dx} = \frac{y}{(2y-x)}$$

Exercise 10H

1. Question

Differentiate x^6 with respect to $(1/\sqrt{x})$.

Answer

Given : Let $u = x^6$ and $v = \frac{1}{\sqrt{x}}$

To differentiate : x^6 with respect to $(1/\sqrt{x})$.

Formula used : $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

Let $u = x^6$ and $v = \frac{1}{\sqrt{x}}$

mc24 Myclass24
Your Class. Your Pace.

Differentiating u with respect to x

$$\frac{du}{dx} = 6x^5$$

Differentiating v with respect to x

$$\frac{dv}{dx} = \frac{-1}{2} x^{-\frac{3}{2}}$$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

$$\frac{du}{dv} = \frac{6x^5}{\frac{-1}{2} x^{-\frac{3}{2}}}$$

$$\frac{du}{dv} = -12x^{5+\frac{3}{2}}$$

$$\frac{du}{dv} = -12x^{\frac{13}{2}}$$

Ans. $-12x^{13/2}$

2. Question

Differentiate $\log x$ with respect to $\cot x$.

Answer

Given : Let $u = \log x$ and $v = \cot x$

To differentiate : $\log x$ with respect to $\cot x$

Formula used : $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

$$\frac{d(\log x)}{dx} = \frac{1}{x}$$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)).g'(x)$

Let $u = \log x$ and $v = \cot x$

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{1}{x}$$

Differentiating v with respect to x

$$\frac{dv}{dx} = -\operatorname{cosec}^2 x$$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

$$\frac{du}{dv} = \frac{\frac{1}{x}}{-\operatorname{cosec}^2 x}$$

$$\frac{du}{dv} = \frac{-1}{x \operatorname{cosec}^2 x}$$



3. Question

Differentiate $e^{\sin x}$ with respect to $\cos x$.

Answer

Given : Let $u = e^{\sin x}$ and $v = \cos x$

To differentiate : $e^{\sin x}$ with respect to $\cos x$

Formula used : $\frac{d(e^x)}{dx} = e^x$

$$\frac{d(\cos x)}{dx} = -\sin x$$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)).g'(x)$

Let $u = e^{\sin x}$ and $v = \cos x$

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d(e^{\sin x})}{dx} = \cos x \cdot e^{\sin x}$$

Differentiating v with respect to x

$$\frac{dv}{dx} = -\sin x$$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

$$\frac{du}{dv} = \frac{\cos x \cdot e^{\sin x}}{-\sin x}$$

$$\frac{du}{dv} = -e^{\sin x} \cdot \cot x$$

Ans. $-e^{\sin x} \cot x$

4. Question

Differentiate $\tan^{-1} \sqrt{\frac{1-x^2}{1+x^2}}$ with respect to $\cos^{-1} x^2$.

Answer

Given : Let $u = \tan^{-1} \sqrt{\frac{1-x^2}{1+x^2}}$ and $v = \cos^{-1} x^2$.

To differentiate : $\tan^{-1} \sqrt{\frac{1-x^2}{1+x^2}}$ with respect to $\cos^{-1} x^2$.

Formula used : $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)).g'(x)$

$$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu - u dv}{v^2}$$

$$\text{Let } u = \tan^{-1} \sqrt{\frac{1-x^2}{1+x^2}} \text{ and } v = \cos^{-1} x^2.$$

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d\left(\tan^{-1} \sqrt{\frac{1-x^2}{1+x^2}}\right)}{dx} = \frac{1}{1 + \frac{1-x^2}{1+x^2}} \cdot \frac{d\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)}{dx}$$

$$\frac{du}{dx} = \frac{1+x^2}{1+x^2+1-x^2} \cdot \frac{1}{2} \left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}} \cdot \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2}$$

$$\frac{du}{dx} = \frac{1+x^2}{2} \cdot \frac{1}{2} \left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}} \cdot \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2} = \frac{1+x^2}{2} \cdot \frac{1}{2} \left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$\frac{du}{dx} = \left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}} \cdot \frac{-x}{(1+x^2)} = \sqrt{\frac{1+x^2}{1-x^2}} \cdot \frac{-x}{(1+x^2)} = \frac{-x}{\sqrt{(1-x^2)(1+x^2)}} = \frac{-x}{\sqrt{1-x^4}}$$

$$\frac{du}{dx} = \frac{-x}{\sqrt{1-x^4}}$$

Differentiating v with respect to x

$$\frac{dv}{dx} = -\frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d(x^2)}{dx} = \frac{-2x}{\sqrt{1-x^4}}$$

$$\frac{dv}{dx} = \frac{-2x}{\sqrt{1-x^4}}$$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

$$\frac{du}{dv} = \frac{\frac{-x}{\sqrt{1-x^4}}}{\frac{-2x}{\sqrt{1-x^4}}} = \frac{1}{2}$$

$$\frac{du}{dv} = \frac{1}{2}$$

$$\text{Ans. } \frac{1}{2}$$

5. Question



Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Answer

Given : Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

To differentiate : $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

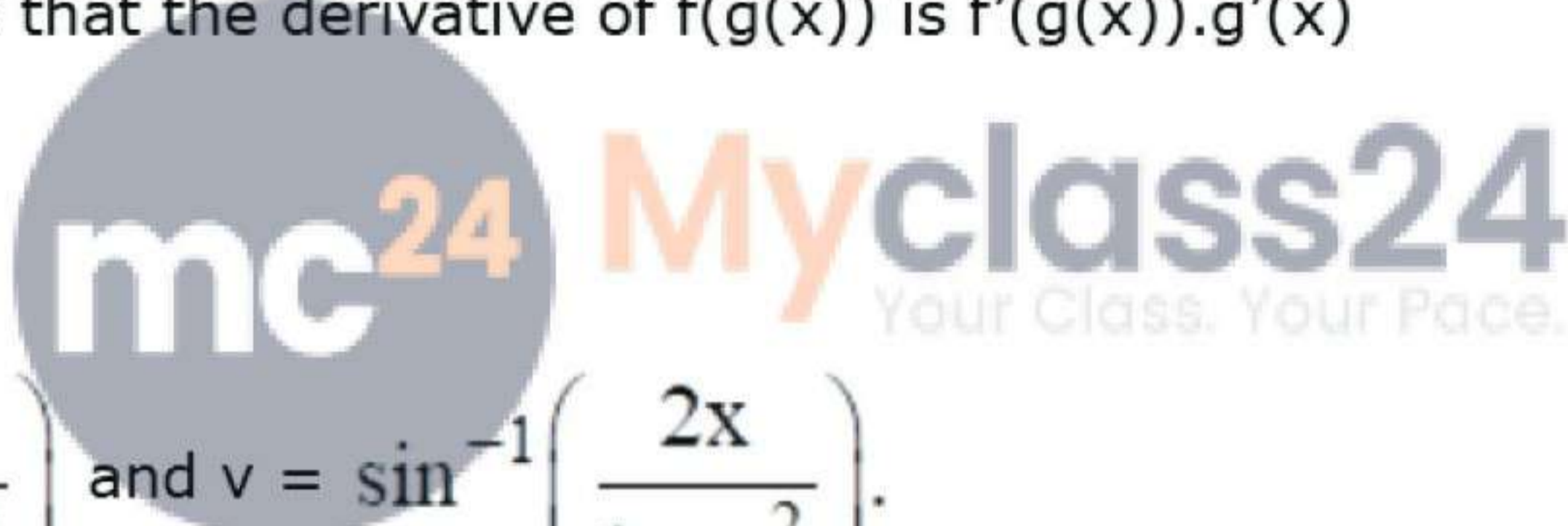
Formula used : $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$

$$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu - u dv}{v^2}$$



Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d\left(\tan^{-1}\frac{2x}{1-x^2}\right)}{dx} = \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2} \cdot \frac{d\left(\frac{2x}{1-x^2}\right)}{dx} = \frac{1}{1+\frac{4x^2}{1-x^2-2x^2}} \cdot \frac{2(1-x^2)+2x(2x)}{(1-x^2)^2}$$

$$\frac{du}{dx} = \frac{(1-x^2)^2}{1+x^4-2x^2+4x^2} \cdot \frac{2-2x^2+4x^2}{(1-x^2)^2} = \frac{(1-x^2)^2}{1+x^4+2x^2} \cdot \frac{2+2x^2}{(1-x^2)^2} = \frac{2(1+x^2)}{(1+x^2)^2} = \frac{2}{1+x^2}$$

$$\frac{du}{dx} = \frac{2}{1+x^2}$$

Differentiating v with respect to x

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{d\left(\frac{2x}{1+x^2}\right)}{dx} = \frac{1+x^2}{\sqrt{1+x^4+2x^2-4x^2}} \cdot \frac{2(1+x^2)-2x(2x)}{(1+x^2)^2}$$

$$\frac{dv}{dx} = \frac{1+x^2}{\sqrt{1+x^4-2x^2}} \cdot \frac{2+2x^2-4x^2}{(1+x^2)^2} = \frac{1+x^2}{\sqrt{(1-x^2)^2}} \cdot \frac{2-2x^2}{(1+x^2)^2} = \frac{1+x^2}{1-x^2} \cdot \frac{2(1-x^2)}{(1+x^2)^2} = \frac{2}{1+x^2}$$

$$\frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

$$\frac{du}{dv} = \frac{\frac{2}{(1+x^2)}}{\frac{2}{(1+x^2)}} = 1$$

$$\frac{du}{dv} = 1$$

Ans. 1

6. Question

Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}(2x^2 - 1)$.

Answer

Given : Let $u = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ and $v = \cos^{-1}(2x^2 - 1)$.

To differentiate : $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ with respect to $\cos^{-1}(2x^2 - 1)$

Formula used : $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$

$$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

The CHAIN RULE states that the derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu - u dv}{v^2}$$

Let $u = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$ and $v = \cos^{-1}(2x^2 - 1)$

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d\left(\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right)}{dx} = \frac{1}{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2} \cdot \frac{d\left(\frac{x}{\sqrt{1-x^2}}\right)}{dx} = \frac{1}{1+\frac{x^2}{1-x^2}} \cdot \frac{1(\sqrt{1-x^2}) + x\left(\frac{-2x}{2\sqrt{1-x^2}}\right)}{1-x^2}$$