

EXERCISE 5.4

1. Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$ verify that

(i) $(2A)^T = 2A^T$

(ii) $(A + B)^T = A^T + B^T$

(iii) $(A - B)^T = A^T - B^T$

(iv) $(AB)^T = B^T A^T$

Solution:

(i) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$$(2A)^T = 2A^T$$

Put the value of A

$$\Rightarrow \left(2 \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\right)^T = 2 \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2 \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

L.H.S = R.H.S

(ii) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$$(A + B)^T = A^T + B^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

L.H.S = R.H.S

Hence proved.

(iii) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$$(A - B)^T = A^T - B^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

L.H.S = R.H.S

(iv) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\begin{bmatrix} 2-6 & 0+12 \\ -7+10 & 0-20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2-6 & -7+10 \\ 0+12 & 0-20 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

So,

$$(AB)^T = B^T A^T$$

2. $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = [1 \ 0 \ 4]$ verify that $(AB)^T = B^T A^T$

Solution:

Given

$$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \text{ and } B = [1 \ 0 \ 4]$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} [1 \ 0 \ 4] \right)^T = [1 \ 0 \ 4]^T \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} [3 \ 5 \ 2]$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

L.H.S = R.H.S

So, $(AB)^T = B^T A^T$

3. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ find A^T, B^T and verify that

(i) $(A + B)^T = A^T + B^T$

(ii) $(AB)^T = B^T A^T$

(iii) $(2A)^T = 2 A^T$

Solution:

(i) Given

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Consider,

$$(A + B)^T = A^T + B^T$$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\left(\begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

L.H.S = R.H.S

So, $(A + B)^T = A^T + B^T$

(ii) Given

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Consider,

$$(AB)^T = B^T A^T$$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\begin{bmatrix} 1-2+0 & 2-1+0 & 3-3+0 \\ 2+2+0 & 4+1+3 & 6+3+3 \\ 1+4+0 & 2+2+1 & 3+6+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}^T$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 4 & 8 & 12 \\ 5 & 5 & 10 \end{bmatrix}^T = \begin{bmatrix} 1-2+0 & 2+2+0 & 1+4+0 \\ 2-1+0 & 4+1+3 & 2+2+1 \\ 3-3+0 & 6+3+3 & 3+6+1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}$$

L.H.S = R.H.S

So, $(AB)^T = B^T A^T$

(iii) Given

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Consider,

$$(2A)^T = 2A^T$$

$$\Rightarrow \left(2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \right)^T = 2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & 0 \\ 4 & 2 & 6 \\ 2 & 4 & 2 \end{bmatrix}^T = 2 \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix}$$

L.H.S = R.H.S

So,

$$(2A)^T = 2A^T$$

4. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = [1 \ 3 \ -6]$, verify that $(AB)^T = B^T A^T$

Solution:

Given

$$A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = [1 \ 3 \ -6]$$

Consider,

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} [1 \ 3 \ -6] \right)^T = [1 \ 3 \ -6]^T \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} -2 & -6 & -12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}^T [-2 \ 4 \ 5]$$

$$\Rightarrow \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ -12 & -24 & -30 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ -12 & -24 & -30 \end{bmatrix}$$

L.H.S = R.H.S

So,

$$(AB)^T = B^T A^T$$

5. If $A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$, find $(AB)^T$

Solution:

Given

$$A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$$

Now we have to find $(AB)^T$

$$\begin{aligned} &\Rightarrow \left(\begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix} \right)^T \\ &\Rightarrow \begin{bmatrix} 6 - 4 - 2 & 8 + 8 - 1 \\ -3 - 0 + 4 & -4 + 0 + 2 \end{bmatrix}^T \\ &\Rightarrow \begin{bmatrix} 0 & 15 \\ 1 & -2 \end{bmatrix}^T \\ &\Rightarrow \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix} \end{aligned}$$

So,

$$(AB)^T = \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

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