EXERCISE 8.3

Prove the following (from Q.1 to Q.7):

1. sin θ/(1+cos θ) + (1+ cos θ)/sin θ = 2cosec θ

Solution:

L.H.S=

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$$

Taking the L.C.M of the denominators,

We get,

$$=\frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta).\sin\theta}$$

$$=\frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta).\sin\theta}$$

Since, $\sin^2\theta + \cos^2\theta = 1$

$$=\frac{1+1+2\cos\theta}{(1+\cos\theta).\sin\theta}$$

$$=\frac{2+2\cos\theta}{(1+\cos\theta).\sin\theta}$$

$$=\frac{2(1+\cos\theta)}{(1+\cos\theta).\sin\theta}$$

Since, 1/sin θ = cosec θ

$$=\frac{2}{\sin\theta}=2\operatorname{cosec}\theta$$

R.H.S

Hence proved.

2. tan A/(1+secA) - tan A/(1-secA) = 2cosec A

Solution:

L.H.S:

$$\frac{\tan A}{1+\sec A}-\frac{\tan A}{1-\sec A}$$

Taking LCM of the denominators,

$$=\frac{\tan A(1-\sec A)-\tan A(1+\sec A)}{(1+\sec A)(1-\sec A)}$$

Since, $(1 + \sec A)(1 - \sec A) = 1 - \sec^2 A$)

$$=\frac{\tan A(1-\sec A-1-\sec A)}{1-\sec^2 A}$$

$$=\frac{\tan A(-2\sec A)}{1-\sec^2 A}$$

$$=\frac{2\tan A.\sec A}{\sec^2 A-1}$$

Since,

$\sec^2 A - \tan^2 A = 1$

$\sec^2 A - 1 = \tan^2 A$

$= \frac{2 \tan A . \sec A}{\tan^2 A}$

Since, sec A = (1/cos A) and tan A = (sin A/ cos A)

$= \frac{2 \sec A}{\tan A} = \frac{2 \cos A}{\cos A \sin A}$

$= \frac{2}{\sin A}$

$= 2 \operatorname{cosec} A \ (\because \frac{1}{\sin A} = \operatorname{cosec} A)$

= R.H.S

Hence proved.

3. If tan A = ¾, then sinA cosA = 12/25

Solution:

According to the question,

tan A = ¾

We know,

tan A = perpendicular/ base

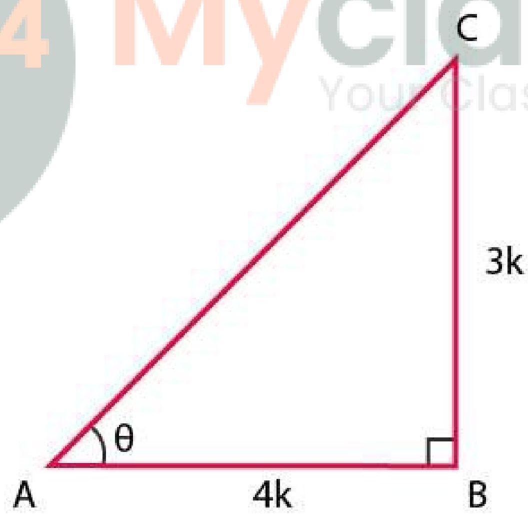
So,

tan A = 3k/4k

Where,

Perpendicular = 3k

Base = 4k



Using Pythagoras Theorem,

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$(\text{hypotenuse})^2 = (3k)^2 + (4k)^2 = 9k^2 + 16k^2 = 25k^2$

hypotenuse = 5k

To find sin A and cos A,

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3k}{5k} = \frac{3}{5}$$
$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4k}{5k} = \frac{4}{5}$$
Multiplying sin A and cos A,
$$\sin A \cos A = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$$
Hence, proved.

4. (sin α + cos α) (tan α + cot α) = sec α + cosec α
Solution:

L.H.S:
(sin α + cos α) (tan α + cot α)
As we know,
$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$
$$= (\sin\alpha + \cos\alpha)\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$$
$$= (\sin\alpha + \cos\alpha)\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha}\right)$$
$[\because \sin^2\alpha + \cos^2\alpha = 1]$
$$= (\sin\alpha + \cos\alpha)\left(\frac{1}{\sin\alpha\cos\alpha}\right)$$
$$= \frac{\sin\alpha}{\sin\alpha\cos\alpha} + \frac{\cos\alpha}{\sin\alpha\cos\alpha}$$
$$= \frac{1}{\cos\alpha} + \frac{1}{\sin\alpha}$$
$$= \sec\alpha + \operatorname{cosec}\alpha \left[\because \frac{1}{\cos\alpha} = \sec\alpha \text{ and } \frac{1}{\sin\alpha} = \operatorname{cosec}\alpha\right]$$
= R.H.S
Hence, proved.

5. (√3+1) (3 – cot 30°) = $\tan^3$ 60° – 2 sin 60°
Solution:

L.H.S: (√3 + 1) (3 – cot 30°)
= (√3 + 1) (3 – √3) [∵cos 30° = √3]
= (√3 + 1) √3 (√3 - 1) [∵(3 – √3) = √3 (√3 - 1)]
= $((\sqrt{3})^2 - 1)$ √3 [∵ (√3+1)(√3-1) = $((\sqrt{3})^2 - 1)$]
= (3-1) √3
= 2√3
Similarly solving R.H.S: $\tan^3$ 60° - 2 sin 60°
Since, tan 60º = √3 and sin 60º = √3/2,
We get,

$(\sqrt{3})^3 - 2.(\sqrt{3}/2) = 3\sqrt{3} - \sqrt{3}$
$= 2\sqrt{3}$
Therefore, L.H.S = R.H.S
Hence, proved.

6. 1 + (cot² α/1+cosec α = cosec α
Solution:

L.H.S:
Since,
$$\cot^2\alpha = \frac{\cos^2\alpha}{\sin^2\alpha} \text{ and } \operatorname{cosec}\alpha = \frac{1}{\sin\alpha}]$$
We get,
$$1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = 1 + \frac{\cos^2\alpha/\sin^2\alpha}{1+1/\sin\alpha}$$
$$= 1 + \frac{\cos^2\alpha/\sin^2\alpha}{\frac{\sin\alpha+1}{\sin\alpha}}$$
$$= 1 + \frac{\cos^2\alpha}{\sin\alpha(1+\sin\alpha)}$$
$$= \frac{\sin\alpha+\sin^2\alpha+\cos^2\alpha}{\sin\alpha+\sin^2\alpha}$$

And, we know that,
$$\sin^2\alpha + \cos^2\alpha = 1$$
$$= \frac{1+\sin\alpha}{\sin\alpha(1+\sin\alpha)}$$
Since,
$$\frac{1}{\sin\alpha} = \operatorname{cosec}\alpha]$$
$$= \frac{1}{\sin\alpha} = \operatorname{cosec}\alpha$$
= R.H.S

7. tan θ + tan (90° – θ) = sec θ sec (90° – θ)
Solution:

L.H.S=
Since, tan (90° - θ) = cot θ
tan θ + tan (90° - θ)= tan θ + cot θ

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

Since,

$\sin^2\theta + \cos^2\theta = 1$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$

$= \sec\theta \operatorname{cosec}\theta$

Since,

$\operatorname{cosec}\theta = \sec(90° - \theta)$

$= \sec\theta \sec(90° - \theta)$

= R.H.S

Hence, proved.