

• Curve is $y = \frac{x}{\pi} + 2 \sin^2 x$

• $x = 0$ and

• $x = \pi$

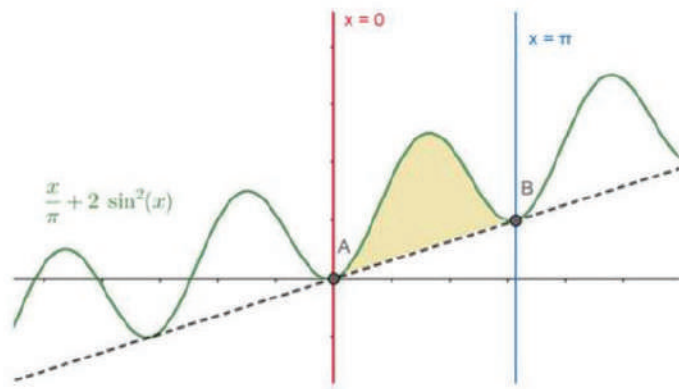
The given curve $y = \frac{x}{\pi} + 2 \sin^2 x$ is similar to $y = \sin^2 x$.

Now consider the y values for some random x values between 0 and π for the function $y = \sin^2 x$.

x	y
0	0
$\frac{\pi}{6}$	$\frac{1}{4}$
$\frac{\pi}{4}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{3}{4}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{3}{4}$
π	0



From the table we can clearly draw the graph for $y = \frac{x}{\pi} + 2 \sin^2 x$



The required area under the curve is given by:

$$\text{Area} = \int_0^{\pi} \left[\frac{x}{\pi} + 2 \sin^2 x \right] dx$$

[using the property $\cos 2x = 1 - 2\sin^2 x$]

$$\begin{aligned} &= \frac{1}{\pi} \int_0^{\pi} [x] dx + 2 \int_0^{\pi} \frac{1 - \cos 2x}{2} dx \\ &= \frac{1}{\pi} \left(\frac{x^2}{2} \right)_0^{\pi} + 2 \left\{ \frac{1}{2} [x]_0^{\pi} - \frac{1}{2} \left[\frac{\sin 2x}{2} \right]_0^{\pi} \right\} \end{aligned}$$

[using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int \cos x dx = \sin x$]

$$\begin{aligned} &= \frac{1}{\pi} \left[\frac{\pi^2}{2} - \frac{0^2}{2} \right] + 2 \left\{ \frac{1}{2} [\pi - 0] - \frac{1}{4} [\sin 2(\pi) - \sin 2(0)] \right\} \\ &= \frac{1}{\pi} \left[\frac{\pi^2}{2} \right] + \left\{ [\pi - 0] - \frac{1}{2} [\sin 2(\pi)] + 0 \right\} \\ &= \left[\frac{\pi}{2} \right] + \left\{ [\pi] - \frac{1}{2} [\sin 2(\pi)] \right\} \\ &= \left[\frac{3\pi}{2} \right] - \frac{1}{2} [0] \end{aligned}$$

[as $\sin \pi = 0$, then $\sin 2\pi = 0$]

$$= \frac{3\pi}{2}$$

Hence the required area of the curve $y = \frac{x}{\pi} + 2 \sin^2 x$ from $x = 0$ to $x = \pi$ is $= \frac{3\pi}{2}$ sq. units.

24. Question

Find the area of bounded by the curve $y = \cos x$, the x-axis and the ordinates $x = 0$ and $x = 2\pi$.

Answer

Given

- Curve is $y = \cos x$
- X-axis
- $x = 0$ and
- $x = 2\pi$

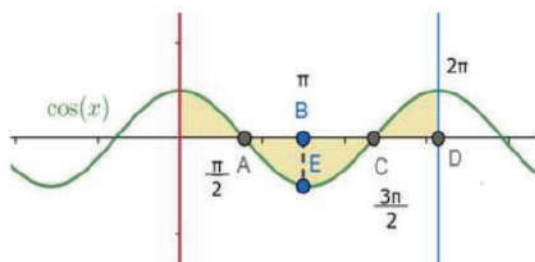
The given curve is $y = \cos x$.

Now consider the y values for some random x values between 0 and 2π for the function $y = \cos x$.

X	Y
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



From the table we can clearly draw the graph for $y = \cos x$



From the given curve, we can say that,

For $0 < x < \frac{\pi}{2}$, $y = \cos x$

For $\frac{\pi}{2} < x < \frac{3\pi}{2}$, $y = -\cos x$

For $\frac{3\pi}{2} < x < 2\pi$, $y = \cos x$

The required area under the curve is given by:

Area required = Area under of OA + Area of ABC + Area under AC

$$\begin{aligned}
 \text{Area required} &= \int_0^{\frac{\pi}{2}} \cos x \, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos x) \, dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x \, dx \\
 &= \int_0^{\frac{\pi}{2}} \cos x \, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x \, dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x \, dx \\
 &= (\sin x)_0^{\frac{\pi}{2}} - (\sin x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + (\sin x)_{\frac{3\pi}{2}}^{2\pi}
 \end{aligned}$$

[using the formula, $\int \cos x \, dx = \sin x$]

$$\begin{aligned}
 &= \left[\sin \frac{\pi}{2} - \sin 0 \right] - \left[\sin \frac{3\pi}{2} - \sin \frac{\pi}{2} \right] + \left[\sin 2\pi - \sin \frac{3\pi}{2} \right] \\
 &= [1 - 0] - [-1 - 1] + [0 - (-1)] = 1 + 2 + 1 = 4
 \end{aligned}$$

[as $\sin \frac{\pi}{2} = 1$, $\sin 2\pi = 0$, $\sin \frac{3\pi}{2} = -1$, $\sin 0 = 0$]

Hence the required area of the curve $y = \cos x$ from $x = 0$ to $x = 2\pi$ is 4 sq. units.

25. Question

Compare the areas under the curves $y = \cos^2 x$ and $y = \sin^2 x$ between $x = 0$ and $x = \pi$.

Answer

Given

- First curve $y = \cos^2 x$
- Second curve $y = \sin^2 x$
- $x = 0$
- $x = \pi$



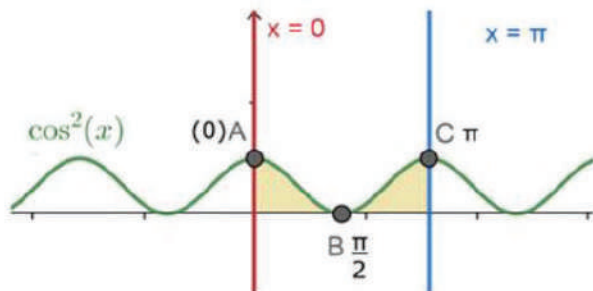
Consider the curve $y = \cos^2 x$

Now consider the y values for some random x values between 0 and π for the function $y = \cos^2 x$.

X	Y
0	1
$\frac{\pi}{6}$	$\frac{3}{4}$
$\frac{\pi}{4}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{4}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$\frac{1}{4}$
π	1



From the table we can clearly draw the graph for $y = \cos^2 x$



The required area under the curve is given by:

$$\begin{aligned} \text{Area required} &= \int_0^{\pi} \cos^2 x \, dx \\ &= \int_0^{\pi} \cos^2 x \, dx = \int_0^{\pi} \frac{1 + \cos 2x}{2} \, dx \end{aligned}$$

[using the property $\cos 2x = 2 \cos^2 x - 1$]

$$= \frac{1}{2} \left[x + \frac{\sin 2x}{2} \right]_0^\pi$$

[using the formula, $\int \cos x \, dx = \sin x$]

$$= \frac{1}{2} \left\{ [\pi - 0] + \frac{1}{2} [\sin 2(\pi) - \sin 2(0)] \right\}$$

$$= \left\{ \frac{\pi}{2} + \frac{1}{4} [0 - 0] \right\} = \frac{\pi}{2}$$

[as $\sin 2\pi = 0$, $\sin 0 = 0$]

Hence the required area of the curve $y = \cos^2 x$ from $x = 0$ to $x = \pi$ is $= \frac{\pi}{2}$ sq. units. ----- (1)

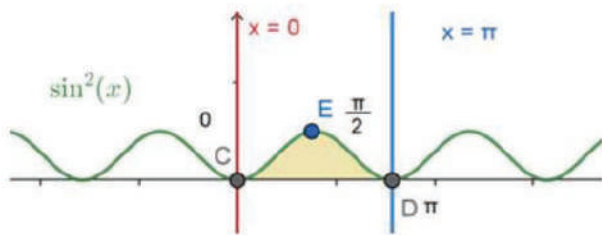
Consider the curve $y = \sin^2 x$

Now consider the y values for some random x values between 0 and π for the function $y = \sin^2 x$.

X	Y
0	0
$\frac{\pi}{6}$	$\frac{1}{4}$
$\frac{\pi}{4}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{3}{4}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{3}{4}$
π	0



From the table we can clearly draw the graph for $y = \sin^2 x$



The required area under the curve is given by:

$$\begin{aligned} \text{Area required} &= \int_0^{\pi} \sin^2 x \, dx \\ &= \int_0^{\pi} \sin^2 x \, dx = \int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx \end{aligned}$$

[using the property $\cos 2x = 1 - 2 \sin^2 x$]

$$= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi}$$

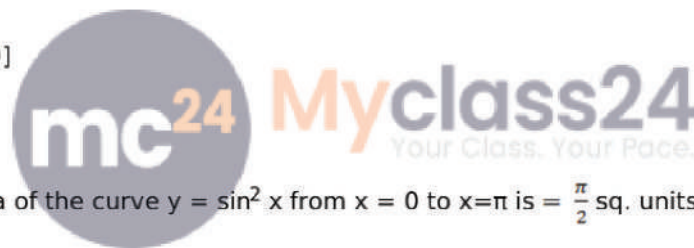
[using the formula, $\int \sin x \, dx = \cos x$]

$$= \frac{1}{2} \left\{ [\pi - 0] - \frac{1}{2} [\sin 2(\pi) - \sin 2(0)] \right\}$$

$$= \left\{ \frac{\pi}{2} - \frac{1}{4} [0 - 0] \right\}$$

[as $\sin 2\pi = 0$, $\sin 0 = 0$]

$$= \frac{\pi}{2}$$



Hence the required area of the curve $y = \sin^2 x$ from $x = 0$ to $x = \pi$ is $= \frac{\pi}{2}$ sq. units. ----- (2)

From (1) and (2), we can clearly state that, the areas under

$y = \cos^2 x$ and $y = \sin^2 x$ are similar which is $= \frac{\pi}{2}$ sq. units.

26. Question

Using integration, find the area of the triangle, the equations of whose sides are $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

Answer

Given,

- ABC is a triangle
- Equation of side AB of $y = 2x + 1$
- Equation of side BC of $y = 3x + 1$
- Equation of side CA of $x = 4$

By solving AB & BC we get the point B,

$$\text{AB : } y = 2x + 1, \text{ BC: } y = 3x + 1$$

$$2x + 1 = 3x + 1$$

$$x = 0$$

by substituting $x = 0$ in AB we get $y = 1$

The point B = (0,1)

By solving BC & CA we get the point C,

$$AC : x = 4, BC: y = 3x + 1$$

$$y = 12 + 1$$

$$y = 13$$

The point C = (4,13)

By solving AB & AC we get the point A,

$$AB : y = 2x+1, AC : x = 4$$

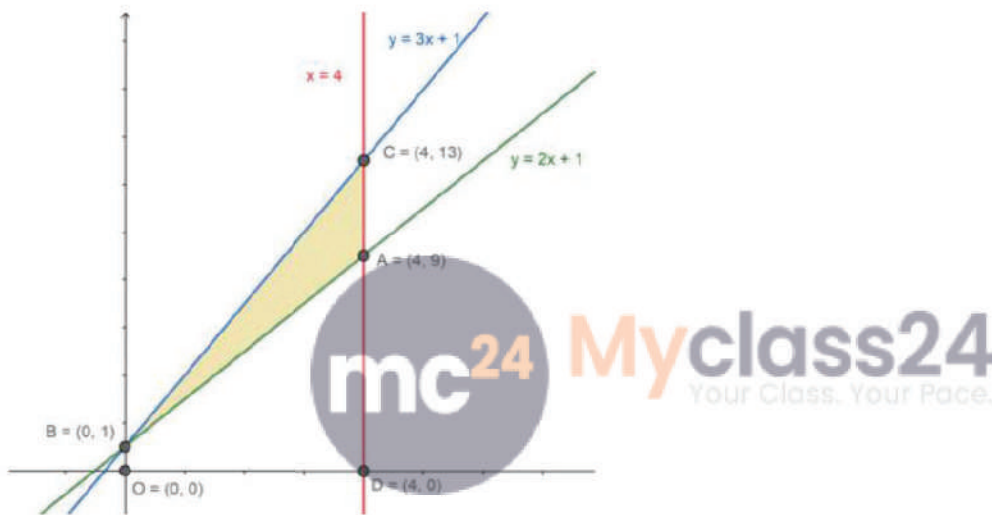
$$y = 8 + 1$$

$$y = 9$$

The point A = (4,9)

These points are used for obtaining the upper and lower bounds of the integral.

From the given information, the area under the triangle (colored) can be given by the below figure.



From above figure we can clearly say that, the area between ABC is the area to be found.

The required area is

$$\text{Area of } ABC = \text{Area under } OBCD - \text{Area under } OBAD$$

Now, the combined area under the rABC is given by

Area under rABC

$$= \text{Area under } AB + \text{Area under } BC - \text{Area under } AC$$

Area of the rABC=

$$\begin{aligned} \text{Area of } ABC &= \int_0^4 (3x + 1)dx - \int_0^4 (2x + 1)dx \\ &= \left[\frac{3x^2}{2} + x \right]_0^4 - \left[\frac{2x^2}{2} + x \right]_0^4 \\ &= \left\{ \left[\frac{3(4)^2}{2} - \frac{3(0)^2}{2} \right] + [4 - 0] \right\} - \left\{ [4^2 - 0] + [4 - 0] \right\} \\ &= \left\{ \left[\frac{3(16)}{2} \right] + 4 \right\} - \left\{ [16] + [4] \right\} \\ &= 24 + 4 - 20 = 8 \end{aligned}$$

Therefore, area of the rABC is 8 sq.units.

27. Question

Find area of region $\{(x, y) : x^2 \leq y \leq x\}$

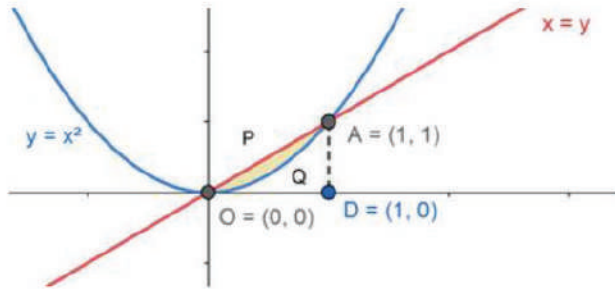
Answer

Given,

$$\bullet R = \{(x, y) : x^2 \leq y \leq x\}$$

From the set we have the curve, $y = x^2$ ----- (1)

Also the line equation $y = x$ ----- (2)



As per the given boundaries,

- The curve $y = x^2$, has only the positive numbers as x has even power, so it is about the y-axis equally distributed on both sides.

- $y = x$ is a line passing through the origin.

The boundaries of the region to be found are,

- Point A, where the curve $y = x^2$ and $y = x$ meet, i.e. A (1,1)

- Point O, which is the origin

Drop a perpendicular D on the x-axis from A, where D = (1,0)

Now,

Area of the required region = Area of OPAQO.

Area of OPAQO = Area of OPAD - Area of OQADO

$$\text{Area of OPAQO} = \int_0^1 y \, dx - \int_0^1 y \, dx$$

$$= \int_0^1 x \, dx - \int_0^1 x^2 \, dx$$

$$\left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^3}{3} \right]_0^1$$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$$= \frac{1}{2}(1^2 - 0^2) - \frac{1}{3}(1^3 - 0^3) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

The Area of the required region = $\frac{1}{6}$ sq. units

28. Question

Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y-axis.

Answer

Given the boundaries of the area O befound are,

- Curve is $y^2 = 2y - x$
- Y-axis.

Consider the curve, $y^2 = 2y - x$

$$y^2 - 2y = -x$$

by adding 1 on both sides

$$y^2 - 2y + 1 = -(x-1)$$

$$(y-1)^2 = -(x-1)$$

From the above equation, we can say that, the given equation is that of a parabola with vertex at A(1,1)

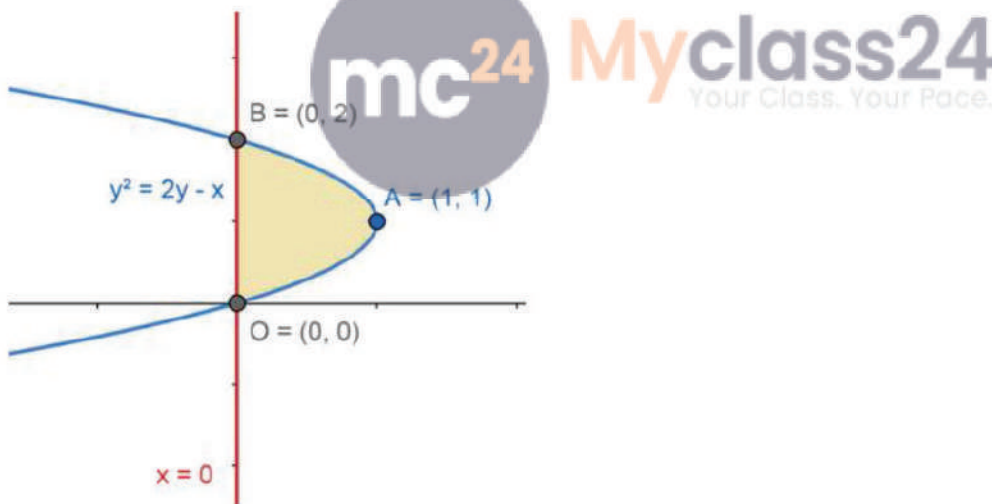
Consider the line $x = 0$ which is the y-axis, now substituting $x = 0$ in the curve equation we get

$$y^2 - 2y = 0$$

$$y(y-2) = 0$$

$$y = 0 \text{ (or) } y = 2$$

So, the parabola meets the y-axis at 2 points, B (0,2) and O(0,0)



As per the given boundaries,

- The parabola $y^2 = 2y - x$, with vertex at A(1,1).
- $X = 0$ which is the y-axis.

The boundaries of the region to be found are,

- Point A, where the curve $y^2 = 2y - x$ has the extreme end the vertex i.e. A (1,1)
- Point O, which is the origin
- Point B, where the curve $y^2 = 2y - x$ and the y - axis meet i.e. B (0,2)

Consider the curve,

$$y^2 = 2y - x$$

$$x = 2y - y^2$$

Area of the required region = Area of OBAO.

$$\text{Area of OBAO} = \int_0^1 x \, dy$$

$$= \int_0^1 (2y - y^2) \, dy$$

$$= \left[\frac{2y^2}{2} \right]_0^1 - \left[\frac{y^3}{3} \right]_0^1$$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$$= [1^2 - 0^2] - \frac{1}{3} [1^3 - 0^3] = 1 - \frac{1}{3} = \frac{2}{3}$$

The Area of the required region = $\frac{2}{3}$ sq. units

29. Question

Draw a rough sketch of the curves $y = \sin x$ and $y = \cos x$, as x varies from 0 to $\frac{\pi}{2}$, and find the area of the region enclosed between them and the x -axis.

Answer

Given

- First curve $y = \cos x$
- Second curve $y = \sin x$
- $x = 0$
- $x = \frac{\pi}{2}$

Consider the curves $y = \cos x$ and $y = \sin x$

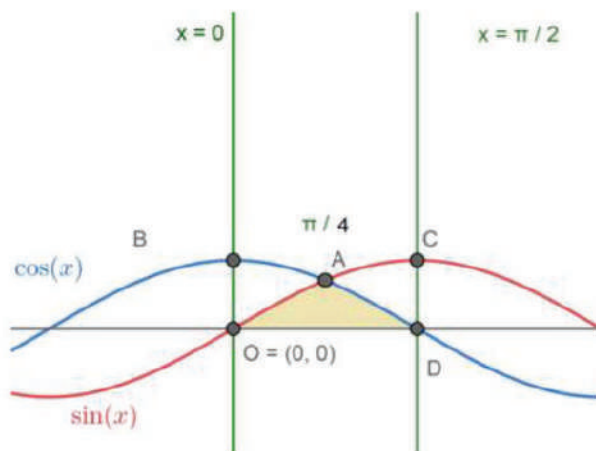
Now consider the y values for some random x values between 0 and $\frac{\pi}{2}$ for the functions $y = \cos x$ and $y = \sin x$.



$y = \cos x$		$Y = \sin x$	
X	Y	X	Y
0	1	0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	$\frac{\pi}{2}$	1



From the above table we can clearly draw the below graphs.



The required area under the curve is given by:

Area of OAD = Area under the curve OA + Area under the curve AD

$$\text{Area required} = \int_0^{\pi/4} y_{OA} dx + \int_{\pi/4}^{\pi/2} y_{AD} dx$$

$$= \int_0^{\frac{\pi}{4}} \sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$$

$$= [-\cos x]_0^{\frac{\pi}{4}} + [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

[using the formula, $\int \cos x \, dx = \sin x$ and $\int \sin x \, dx = -\cos x$]

$$= -\left[\cos\left(\frac{\pi}{4}\right) - \cos 0\right] + \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right]$$

$$= -\left(\frac{1}{\sqrt{2}} - 1\right) + \left(1 - \frac{1}{\sqrt{2}}\right) = 2 - \frac{2}{\sqrt{2}} = 2 - \sqrt{2}$$

Thus the area under the curves $y = \cos x$ and $y = \sin x$ is $2 - \sqrt{2}$ sq. units.

30. Question

Find the area of the region bounded by the parabola $y^2 = 2x + 1$ and the lines $x - y = 1$.

Answer

Given the boundaries of the area O befound are,

- Curve is $y^2 = 2x + 1$
- Line $x - y = 1$

Consider the curve

$$y^2 = 2x + 1$$

$$(y - 0)^2 = 2\left(x + \frac{1}{2}\right)$$



This clearly shows, the curve is a parabola with vertex $A\left(-\frac{1}{2}, 0\right)$

Consider the curve, $y^2 = 2x + 1$ and substitute the line $x = y + 1$ in the curve

$$y^2 = 2(y+1) + 1$$

$$y^2 = 2y + 2 + 1$$

$$y^2 = 2y + 3$$

$$y^2 - 2y - 3 = 0$$

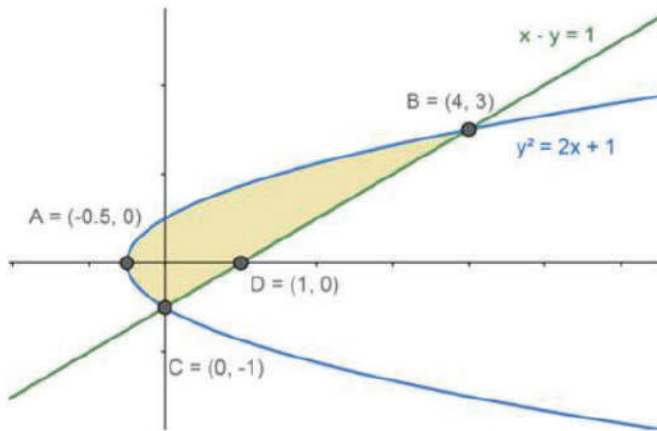
$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)} = \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$$

$$y = 3 \text{ (or) } y = -1$$

substituting y in $x - y = 1$

$$x = 4 \text{ (or) } x = 0$$

So, the parabola meets the line $x - y = 1$ at 2 points, B (4,3) and C (0,-1)



As per the given boundaries,

- The parabola $y^2 = 2x + 1$, with vertex at $A(-0.5, 0)$ and symmetric about the x -axis as y has even powers.
- Line $x - y = 1$

The boundaries of the region to be found are,

- Point A, where the curve $y^2 = 2x + 1$ has the extreme end the vertex i.e. $A(-0.5, 0)$
- Point B, where the curve $y^2 = 2x + 1$ and the line $x - y = 1$ meet i.e. $B(4, 3)$
- Point C, where the curve $y^2 = 2x + 1$ and the line $x - y = 1$ meet i.e. $C(0, -1)$ on the negative y
- Point D, where the line $x - y = 1$ meets the x -axis i.e. $D(1, 0)$

Consider the curve,

$$y^2 = 2x + 1$$

$$2x = y^2 - 1$$

$$x = \frac{y^2 - 1}{2}$$



Consider the line $x - y = 1$

$$x = y + 1$$

Area of the required region = Area of ABDC

Area of ABDC = Area above CDB - Area above CAB

$$\text{Area of ABDC} = \int_{-1}^3 x_{\text{CDB}} dy - \int_{-1}^3 x_{\text{ABDC}} dy$$

$$= \int_{-1}^3 (y + 1) dy - \frac{1}{2} \int_{-1}^3 (y^2 - 1) dy$$

$$= \left[\frac{y^2}{2} + y \right]_{-1}^3 - \frac{1}{2} \left[\frac{y^3}{3} - y \right]_{-1}^3$$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$$= \left\{ \frac{1}{2} [3^2 - (-1)^2] + (3 - (-1)) \right\} - \frac{1}{2} \left\{ \frac{1}{3} [3^3 - (-1)^3] - [3 - (-1)] \right\}$$

$$= \frac{1}{2} [9 - 1] + 4 - \frac{1}{2} \left\{ \frac{1}{3} [27 + 1] - [4] \right\} = \frac{1}{2} [8] - \frac{1}{2} \left\{ \frac{28}{3} - [4] \right\}$$

$$= 4 + 4 - \frac{14}{3} + 2 = \frac{30 - 14}{3} = \frac{16}{3}$$

The Area of the required region = $\frac{16}{3}$ sq.units

31. Question

Find the area of the region bounded by the curve $y=2x-x^2$ and the straight line $y=-x$.

Answer

Given the boundaries of the area O befound are,

- Curve is $y = 2x - x^2$
- Line $y = -x$

Consider the curve

$$y = 2x - x^2$$

$$x^2 - 2x = -y$$

adding 1 on both sides

$$x^2 - 2x + 1 = -(y-1)$$

$$(x-1)^2 = -(y-1)$$

This clearly shows, the curve is a parabola with vertex B (1,1)

Consider the curve, $y = 2x - x^2$ and substitute the line $-x = y$ in the curve

$$-x = 2x - x^2$$

$$x^2 - 2x - x = 0$$

$$x^2 - 3x = 0$$

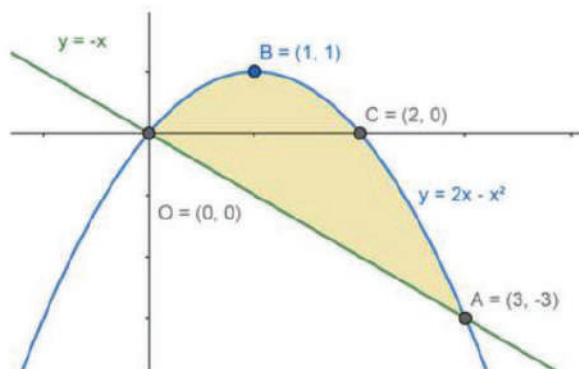
$$x(x-3) = 0$$

$$x = 3 \text{ (or) } x = 0$$

substituting x in $y = -x$

$$y = -3 \text{ (or) } y = 0$$

So , the parabola meets the line $y = -x$ at 2 points, A (3,-3) and •(0,0)



As per the given boundaries,

- The parabola $y = 2x - x^2$, with vertex at B(1,1).
- Line $y = -x$

The boundaries of the region to be found are,

- Point A, where the curve $y = 2x - x^2$ and the line $y = -x$ meet i.e. A (3,-3)
- Point B, where the curve $y = 2x - x^2$ has the extreme end the vertex i.e. B (1,1)
- Point C, where the curve $y = 2x - x^2$ and the line $y = -x$ meet i.e. C (2,0)
- Point O, the origin

Area of the required region = Area of OACBO

Area of OACBO = Area under OBCA - Area under line OA

$$\text{Area of OBCA} = \int_0^3 y_{\text{OBCA}} dx - \int_0^3 y_{\text{OA}} dx$$

$$= \int_0^3 2x - x^2 dx - \int_0^3 (-x) dx$$

$$= \int_0^3 (2x - x^2 + x) dx = \int_0^3 3x - x^2 dx$$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3$$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$$= \left\{ \frac{3}{2} [3^2 - (0)^2] \right\} - \frac{1}{3} \{ [3^3 - (0)^3] \}$$

$$= \frac{3}{2} (9) - \frac{1}{3} (27) = \frac{81 - 54}{6} = \frac{27 - 9}{2}$$

The Area of the required region = $\frac{9}{2}$ sq. units

32. Question

Find the area of the region bounded by the curve $(y-1)^2 = 4(x+1)$ and the line $y = x-1$.

Answer

Given the boundaries of the area O befound are,

- Curve is $(y-1)^2 = 4(x+1)$
- Line $y = x-1$

Consider the curve

$$(y-1)^2 = 4(x+1)$$

Substitute $y = x-1$

$$(x-1-1)^2 = 4(x+1)$$

$$(x-2)^2 = 4x + 4$$

$$x^2 - 4x + 4 = 4x + 4$$

$$x^2 - 8x = 0$$

$$x(x-8) = 0$$

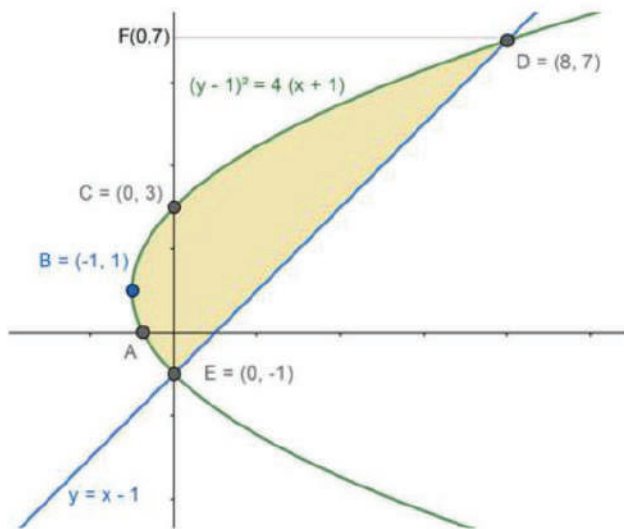
$$x = 8 \text{ (or) } x = 0$$



substituting x in $y = x-1$

$$y = 7 \text{ (or) } y = -1$$

So, the parabola meets the line $y = x-1$ at 2 points, D (8,7) and E (0,-1)



As per the given boundaries,

- The parabola $(y-1)^2 = 4(x+1)$, with vertex at B(-1,1).
- Line $y = x-1$

The boundaries of the region to be found are,

- Point B, where the curve $(y-1)^2 = 4(x+1)$ has the extreme end the vertex i.e. B (-1,1)
- Point D, where the curve $(y-1)^2 = 4(x+1)$ and the line $y = x + 1$ meet i.e. D (8,7)
- Point E, where the curve $(y-1)^2 = 4(x+1)$ and the line $y = x-1$ meet i.e. E (0,-1)
- Point O, the origin

Consider the parabola,

$$(y-1)^2 = 4(x+1)$$

$$x = \frac{(y-1)^2 - 4}{4}$$

Area of the required region = Area of EABCD

Area of EABCD = Area above line ED - Area above EABCD

$$\begin{aligned} \text{Area of EABCD} &= \int_{-1}^7 x_{ED} dy - \int_{-1}^7 x_{EABCD} dy \\ &= \int_{-1}^7 (y+1) dy - \int_{-1}^7 \left(\frac{(y-1)^2 - 4}{4} \right) dy \\ &= \int_{-1}^7 (y+1) dy - \int_{-1}^7 \left(\frac{y^2 - 2y + 1 - 4}{4} \right) dy \\ &= \int_{-1}^7 (y+1) dy - \frac{1}{4} \int_{-1}^7 (y^2 - 2y - 3) dy \end{aligned}$$

$$= \left[\frac{y^2}{2} + y \right]_{-1}^7 - \frac{1}{4} \left\{ \left[\frac{y^3}{3} \right] - 2 \left[\frac{y^2}{2} \right] - 3y \right\}_{-1}^7$$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$$= \left\{ \frac{1}{2} [7^2 - (-1)^2] + [7 - (-1)] \right\} - \frac{1}{4} \left\{ \frac{1}{3} [7^3 - (-1)^3] - [7^2 - (-1)^2] - 3[7 - (-1)] \right\}$$

$$= \left\{ \frac{1}{2} [48] + [8] \right\} - \frac{1}{4} \left\{ \frac{1}{3} [344] - [48] - 3[8] \right\}$$

$$= \{24 + 8\} - \left\{ \frac{1}{3} [86] - [12] - 3[2] \right\} = 32 - \frac{86}{3} + 18 = 50 - \frac{86}{3} = \frac{150 - 86}{3} = \frac{64}{3}$$

The Area of the required region = $\frac{64}{3}$ sq. units

33. Question

Find the area of the region bounded by the curve $y = \sqrt{x}$ and the line $y = x$.

Answer

Given the boundaries of the area O befound are,

- Curve is $y = \sqrt{x}$
- Line $y = x$

Consider the curve

$$y^2 = x$$

Substitute $y = x$

$$(x)^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

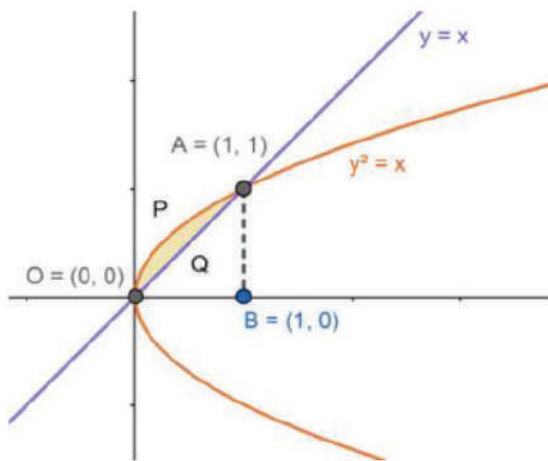
$$x = 1 \text{ (or) } x = 0$$

substituting x in $y = x$

$$y = 1 \text{ (or) } y = 0$$

So , the parabola meets the line $y = \sqrt{x}$ at 2 points, A (1,1) and •(0,0)





As per the given boundaries,

- The parabola $(y)^2 = x$, with vertex at $O(0,0)$.
- Line $y = x$

The boundaries of the region to be found are,

- Point A, where the curve $(y)^2 = x$ and the line $y = x$ meet i.e. $A(1,1)$
- Point O, the origin

Now, drop a perpendicular B on the x-axis from A, the point will be $B(1,0)$

Area of the required region = Area of OPAQO

Area of OPAQO = Area under OPAB - Area under OQAB

$$\text{Area of OPAQO} = \int_0^1 y_{OPAB} dx - \int_0^1 y_{OQAB} dx$$

$$= \int_0^1 \sqrt{x} dx - \int_0^1 x dx$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 - \left[\frac{x^2}{2} \right]_0^1 = \frac{2}{3} \left[\frac{3}{2} - 0 \right] - \frac{1}{2} [1^2 - 0]$$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$$= \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$$

The Area of the required region = $\frac{1}{6}$ sq. units

34. Question

Find the area of the region included between the parabola $y^2 = 3x$ and the circle $x^2 + y^2 - 6x = 0$, lying in the first quadrant.

Answer

Given the boundaries of the area to be found are,

- the circle, $x^2 + y^2 - 6x = 0$ ---(1)
- the parabola, $y^2 = 3x$ ---- (2)

- Area under 1st quadrant.

From the equation, of the first circle, $x^2 + y^2 - 6x = 0$

$$x^2 - 6x + 9 + y^2 - 9 = 0$$

$$(x-3)^2 + y^2 = 9$$

- the vertex at (3,0)
- the radius is 3 units.

From the equation, of the parabola, $y^2 = 3x$

- the vertex at (0,0) i.e. the origin
- Symmetric about the x-axis, as it has the even power of y.

Now to find the point of intersection of (1) and (2), substitute $y^2 = 3x$ in (1)

$$x^2 + 3x - 6x = 0$$

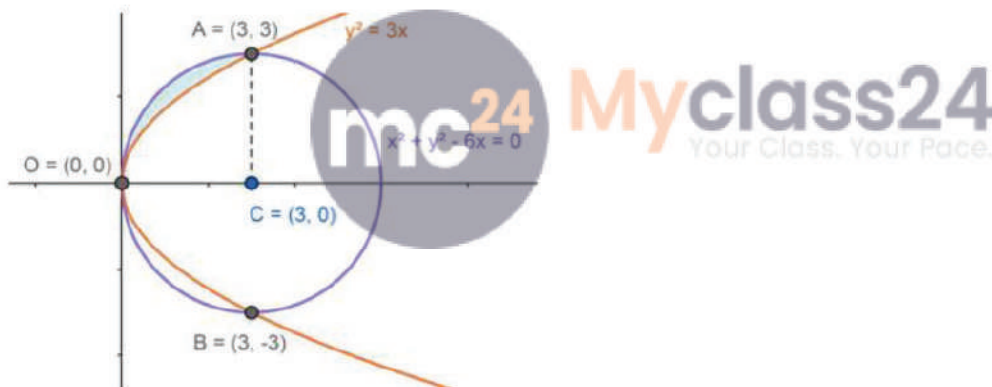
$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 3 \text{ (or) } x = 0$$

Substituting x in (2), we get $y = \pm 3$ or $y = 0$

So the three points, A, B and •where (1) and (2) meet are $A = (3,3)$, $B = (3,-3)$ and $O=(0,0)$



Consider the circle, $x^2 + y^2 - 6x$, can be re-written as

$$y^2 = 6x - x^2$$

$$y = \sqrt{9 - (x - 3)^2} \text{ ----- (3)}$$

Consider the parabola, $y^2 = 3x$, can be re-written as

$$y = \sqrt{3x} \text{ ----- (4)}$$

Let us drop a perpendicular from A on to x-axis. The base of the perpendicular is $C = (3, 0)$

Now, the area to be found will be the area is

Area of the required region = Area between the circle and the parabola at OA.

Area of OA = Area under circle OAC - Area under parabola OAC

Area of OA is

$$\text{Area of OA} = \int_0^3 y_{\text{circle}} dx - \int_0^3 y_{\text{parabola}} dx$$

$$= \int_0^3 \sqrt{9 - (x-3)^2} dx - \int_0^3 \sqrt{3x} dx$$

[Using the formula, $\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right)$ and $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$$= \left[\frac{(x-3)\sqrt{9 - (x-3)^2}}{2} + \frac{9}{2} \sin^{-1}\left(\frac{x-3}{3}\right) \right]_0^3 - \sqrt{3} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3$$

$$= \left\{ \left[\frac{(3-3)\sqrt{9 - (3-3)^2}}{2} + \frac{9}{2} \sin^{-1}\left(\frac{3-3}{3}\right) \right] - \left[\frac{(0-3)\sqrt{9 - (0-3)^2}}{2} + \frac{9}{2} \sin^{-1}\left(\frac{0-3}{3}\right) \right] \right\} - \frac{2\sqrt{3}}{3} [3^{\frac{3}{2}} - 0^{\frac{3}{2}}]$$

$$= \left\{ \left[\frac{0\sqrt{9 - (3-3)^2}}{2} + \frac{9}{2} \sin^{-1}\left(\frac{0}{3}\right) \right] - \left[\frac{(-3)\sqrt{9 - 9}}{2} + \frac{9}{2} \sin^{-1}\left(\frac{-3}{3}\right) \right] \right\} - \frac{2\sqrt{3}}{3} [3\sqrt{3}]$$

$$= \left\{ \left[\frac{9}{2} (0) \right] - \left[0 + \frac{9}{2} \sin^{-1}(-1) \right] \right\} - 6 = -\frac{9}{2} \left(-\frac{\pi}{2} \right) - 6 = \frac{9\pi}{4} - 6$$

$$= \frac{3}{4} (3\pi - 8)$$

[$\sin^{-1}(-1) = -90^\circ$]

Area of the required region is $\frac{3}{4} (3\pi - 8)$ sq. units.



35. Question

Find the area bounded by the curve $y = \cos x$ between $x=0$ to $x=2\pi$.

Answer

Given

- Curve is $y = \cos x$
- $x = 0$ and
- $x = 2\pi$

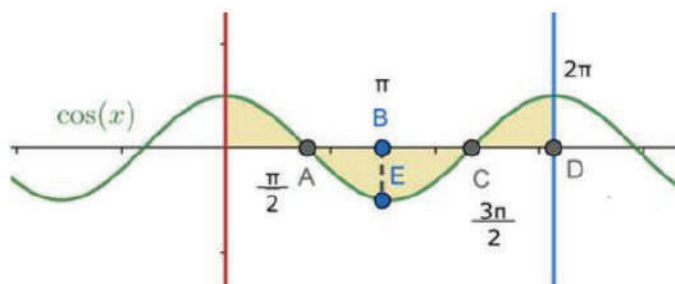
The given curve is $y = \cos x$.

Now consider the y values for some random x values between 0 and 2π for the function $y = \cos x$.

X	Y
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-1
$\frac{3\pi}{2}$	0
2π	1



From the table we can clearly draw the graph for $y = \cos x$



From the given curve, we can say that,

For $0 < x < \frac{\pi}{2}$, $y = \cos x$

For $\frac{\pi}{2} < x < \frac{3\pi}{2}$, $y = -\cos x$

For $\frac{3\pi}{2} < x < 2\pi$, $y = \cos x$

The required area under the curve is given by:

Area required = Area under of OA + Area of ABC + Area under AC

$$\text{Area required} = \int_0^{\frac{\pi}{2}} \cos x \, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos x) \, dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \cos x \, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x \, dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x \, dx$$

$$= (\sin x)_0^{\frac{\pi}{2}} - (\sin x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + (\sin x)_{\frac{3\pi}{2}}^{2\pi}$$

[Using the formula, $\int \cos x \, dx = \sin x$]

$$= \left[\sin \frac{\pi}{2} - \sin 0 \right] - \left[\sin \frac{3\pi}{2} - \sin \frac{\pi}{2} \right] + \left[\sin 2\pi - \sin \frac{3\pi}{2} \right]$$

$$= [1 - 0] - [-1 - 1] + [0 - (-1)] = 1 + 2 + 1 = 4$$

[as $\sin \frac{\pi}{2} = 1$, $\sin 2\pi = 0$, $\sin \frac{3\pi}{2} = -1$, $\sin 0 = 0$]

Hence the required area of the curve $y = \cos x$ from $x = 0$ to $x = 2\pi$ is 4 sq. units.

36. Question

Using integration, find the area of the region in the first quadrant, enclosed by the x-axis, the line $y=x$ and the circle $x^2+y^2=32$

Answer

Given the boundaries of the area to be found are,

- the circle, $x^2 + y^2 = 32$ ---(1)
- the line, $y = x$ --- (2)
- Area should be in first quadrant.

From the equation, of the first circle, $x^2 + y^2 = 32$

- the vertex at (0,0) i.e. the origin
- the radius is $4\sqrt{2}$ unit.

Now to find the point of intersection of (1) and (2), substitute $y = x$ in (1)

$$x^2 + x^2 = 32$$

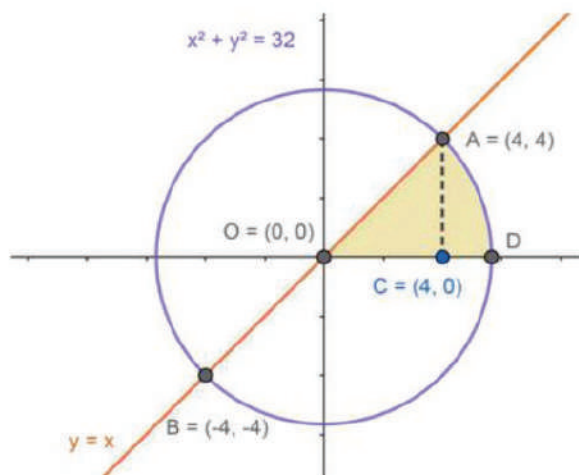
$$2x^2 = 32$$

$$x^2 = 16$$

$$x = \pm 4$$

Substituting x in (2), we get $y = \pm 4$

So the two points, A and B where (1) and (2) meet are $A = (4,4)$ and $B = (-4,-4)$



As x and y have even powers for both the circles, they will be symmetrical about the x -axis and y -axis.

Consider the circle, $x^2 + y^2 = 32$, can be re-written as

$$y^2 = 32 - x^2$$

$$y = \sqrt{(32 - x^2)} \text{----- (3)}$$

Let us drop a perpendicular from A on to x -axis. The base of the perpendicular is $C = (4, 0)$

Now, the area to be found will be the area is

Area of the required region = Area of OADO.

Area of OADO = Area of OAC + Area of CAD

Area of OADO is

$$\text{Area of OADO} = \int_{4\sqrt{2}}^4 \sqrt{32 - x^2} dx + \int_0^4 x dx$$

$$= \left[\frac{x\sqrt{32 - x^2}}{2} + \frac{32}{2} \sin^{-1}\left(\frac{x}{4\sqrt{2}}\right) \right]_{4\sqrt{2}}^4 + \left[\frac{x^2}{2} \right]_0^4$$

[Using the formula, $\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right)$ and $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$$= \left\{ \left[\frac{(4)\sqrt{32 - (4)^2}}{2} + \frac{32}{2} \sin^{-1}\left(\frac{4}{4\sqrt{2}}\right) \right] - \frac{(4\sqrt{2})\sqrt{32 - (4\sqrt{2})^2}}{2} - \frac{32}{2} \sin^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) \right\} + \left\{ \left[\frac{(4)^2}{2} \right] - \left[\frac{0^2}{2} \right] \right\}$$

$$= \left\{ \left[\frac{(4)\sqrt{32 - 16}}{2} + \frac{32}{2} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \right] - \frac{4\sqrt{2}\sqrt{32 - 32}}{2} - \frac{32}{2} \sin^{-1}(1) \right\} + \left\{ \left[\frac{16}{2} \right] \right\}$$

[$\sin^{-1}(1) = 90^\circ$ and $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$]

$$= \left[8 + 16 \left(\frac{\pi}{4}\right) \right] - \left[\frac{32}{2} (0) \right] - \{16\} = 8 + 4\pi - 8 = 4\pi$$

$$= 4\pi$$

Area of the required region is 4π sq. units.

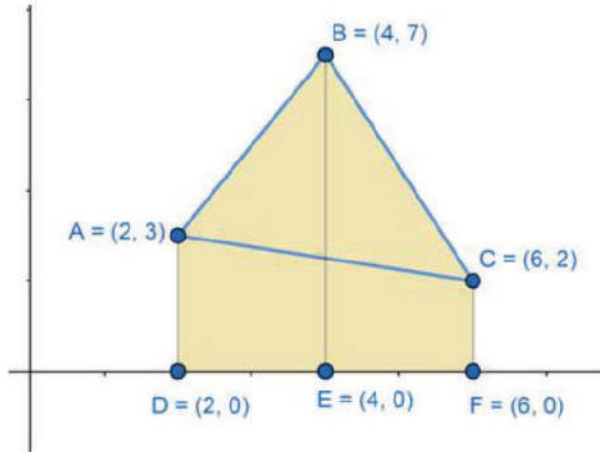
37. Question

Using integration, find the area of the triangle whose vertices are $A(2,3)$, $B(4,7)$ and $C(6,2)$.

Answer

Given,

- $A(2,3)$, $B(4,7)$ and $C(6,2)$ are the 3 vertices of a triangle.



From above figure we can clearly say that, the area between ABC and DEF is the area to be found.

For finding this area, we can consider the lines AB, BC and CA which are the sides of the given triangle. By calculating the area under these lines we can find the area of the complete region.

Consider the line AB,

If (x_1, y_1) and (x_2, y_2) are two points, the equation of a line passing through these points can be given by

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Using this formula, equation of the line $A(2,3)$ $B(4,7)$

$$\frac{y - (3)}{7 - 3} = \frac{x - (2)}{4 - (2)}$$

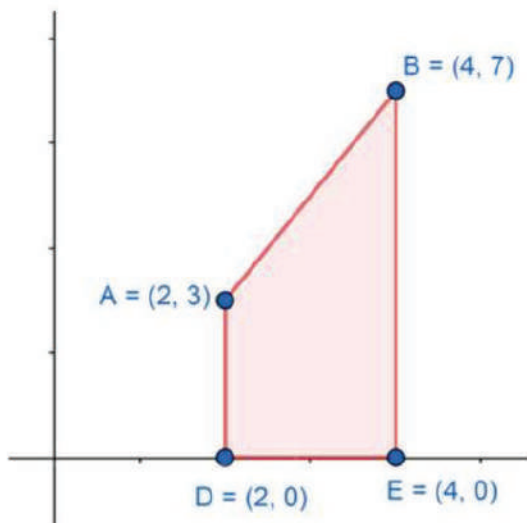
$$\frac{y - (3)}{4} = \frac{x - 2}{2}$$

$$y = \frac{4}{2}(x - 2) + 3$$

$$y = 2x - 4 + 3$$

$$y = 2x - 1$$

Consider the area under AB:



From the above figure, the area under the line AB will be given by,

$$\begin{aligned} \text{Area of } ABED &= \int_2^4 y \, dx = \int_2^4 (2x - 1) \, dx \\ &= \int_2^4 (2x - 1) \, dx = 2 \left[\frac{x^2}{2} \right]_2^4 - [x]_2^4 = [x^2 - x]_2^4 \end{aligned}$$

[using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int c \, dx = cx$]

$$= [4^2 - 4] - [2^2 - 2] = 12 - 2 = 10$$

Area of ABDE = 10 sq. units. ---- (1)

Consider the line BC,



Using this 2-point formula for line, equation of the line B(4,7) and C(6,2)

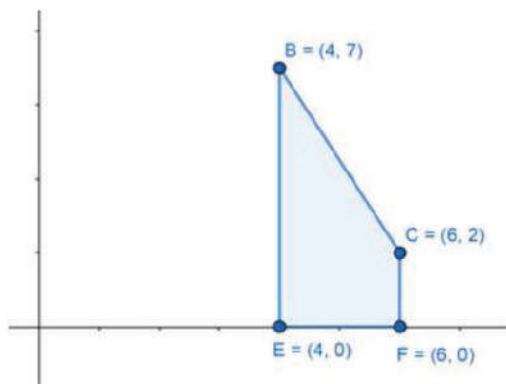
$$\frac{y - (7)}{2 - 7} = \frac{x - (4)}{6 - (4)}$$

$$\frac{y - (7)}{-5} = \frac{x - 4}{2}$$

$$y = \frac{5}{2} (4 - x) + 7 = \frac{20 - 5x + 14}{2} = \frac{34 - 5x}{2}$$

$$y = \frac{34 - 5x}{2}$$

Consider the area under BC:



From the above figure, the area under the line BC will be given by,

$$\text{Area of } BCDF = \int_4^6 y \, dx = \int_4^6 \left(\frac{34 - 5x}{2} \right) dx$$

$$= \int_4^6 \frac{1}{2} (34 - 5x) \, dx = \frac{1}{2} \left[34x - \frac{5x^2}{2} \right]_4^6$$

[using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int c \, dx = cx$]

$$= \frac{1}{2} \left\{ \left[34(6) - \frac{5(6)^2}{2} \right] - \left[34(4) - \frac{5(4)^2}{2} \right] \right\}$$

$$= \frac{1}{2} \left\{ \left[204 - \frac{180}{2} \right] - \left[136 - \frac{80}{2} \right] \right\} = \frac{1}{2} [204 - 90] - \frac{1}{2} [136 - 40]$$

$$= \frac{114 - 96}{2} = 9$$

Area of BCFE = 9 sq. units. (2)

Consider the line CA,

Using this 2-point formula for line, equation of the line C(6,2) and A(2,3)

$$\frac{y - (2)}{3 - 2} = \frac{x - (6)}{2 - (6)}$$

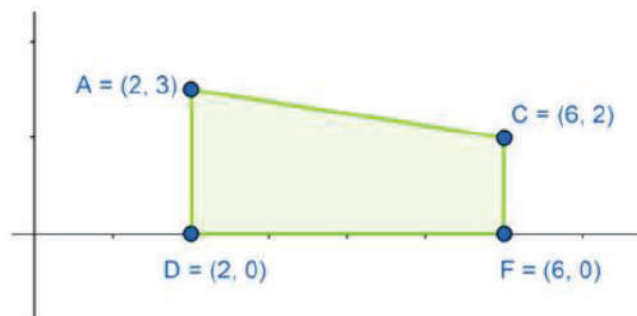
$$\frac{y - (2)}{1} = \frac{x - 6}{-4}$$

$$y = \frac{1}{4} (6 - x) + 2 = \frac{6 - x + 8}{4} = \frac{14 - x}{4}$$

$$y = \frac{14 - x}{4}$$



Consider the area under CA:



From the above figure, the area under the line CA will be given by,

$$\text{Area of } ACFE = \int_2^6 y \, dx = \int_2^6 \left(\frac{14 - x}{4} \right) dx$$

$$= \int_2^6 \frac{1}{4} (14 - x) \, dx = \frac{1}{4} \left[14x - \frac{x^2}{2} \right]_2^6$$

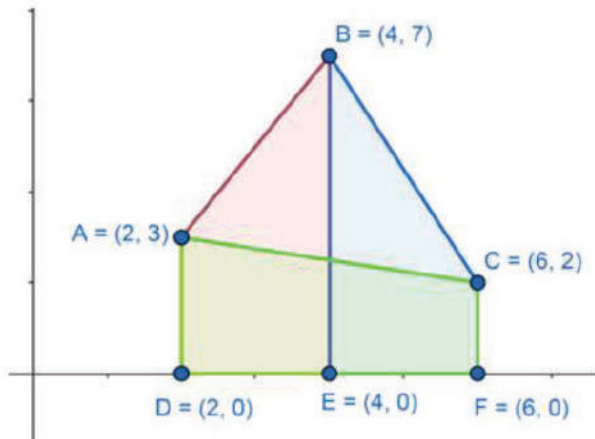
[using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int c \, dx = cx$]

$$= \frac{1}{4} \left\{ \left[14(6) - \frac{6^2}{2} \right] - \left[14(2) - \frac{2^2}{2} \right] \right\}$$

$$= \frac{1}{4} \left\{ \left[84 - \frac{36}{2} \right] - [28 - 2] \right\} = \frac{1}{4} [66 - 26] = \frac{40}{4} = 10$$

Area of ACFD = 10 sq.units ---- (3)

If we combined, the areas under AB, BC and AC in the below graph, we can clearly say that the area under AC (3) is overlapping the previous two areas under AB & BC.



Now, the combined area under the rABC is given by

Area under rABC

$$= \text{Area under AB} + \text{Area under BC} - \text{Area under AC}$$

From (1), (2) and (3), we get

$$\text{Area under rABC} = 10 + 9 - 10 = 9$$

Therefore, area under rABC = 9 sq.units.



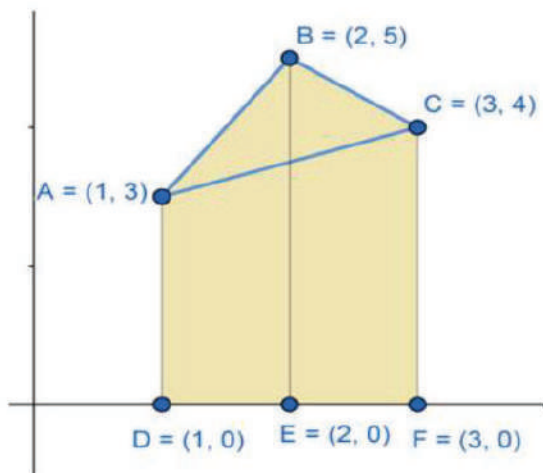
38. Question

Using integration, find the area of the triangle whose vertices are A(1,3), B(2,5) and C(3,4).

Answer

Given,

- A (1,3), B (2,5) and C (3,4) are the 3 vertices of a triangle.



From above figure we can clearly say that, the area between ABC and DEF is the area to be found.

For finding this area, we can consider the lines AB, BC and CA which are the sides of the given triangle. By calculating the area under these lines we can find the area of the complete region.

Consider the line AB,

If (x_1, y_1) and (x_2, y_2) are two points, the equation of a line passing through these points can be given by

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Using this formula, equation of the line A(1,3) B=(2,5)

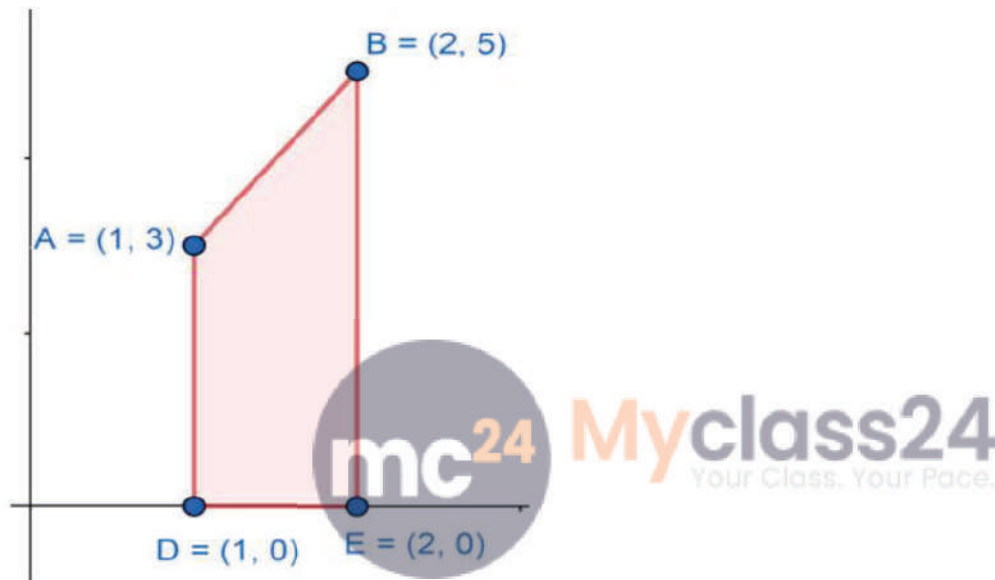
$$\frac{y - (3)}{5 - 3} = \frac{x - (1)}{2 - (1)}$$

$$\frac{y - (3)}{2} = \frac{x - 1}{1}$$

$$y = 2x - 2 + 3$$

$$y = 2x + 1$$

Consider the area under AB:



From the above figure, the area under the line AB will be given by,

$$\text{Area of } ABED = \int_1^2 y \, dx = \int_1^2 (2x + 1) \, dx$$

$$= \int_1^2 (2x + 1) \, dx = 2 \left[\frac{x^2}{2} \right]_1^2 + [x]_1^2 = [x^2 + x]_1^2$$

$$[\text{using the formula, } \int x^n dx = \frac{x^{n+1}}{n+1} \text{ and } \int c \, dx = cx]$$

$$= [2^2 + 2] - [1^2 + 1] = 6 - 2 = 4$$

$$\text{Area of } ABDE = 4 \text{ sq. units. ---- (1)}$$

Consider the line BC,

Using this 2-point formula for line, equation of the line B(2,5) and C(3,4)

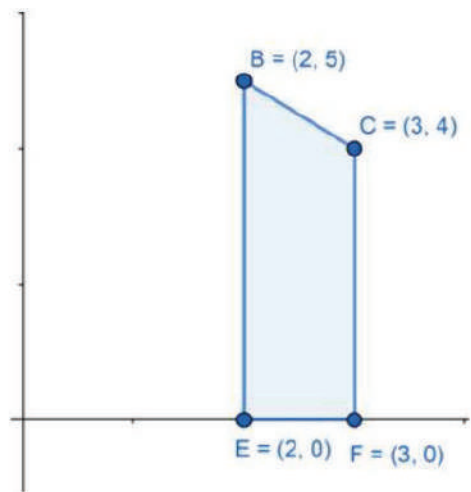
$$\frac{y - (5)}{4 - 5} = \frac{x - (2)}{3 - (2)}$$

$$\frac{y - (5)}{-1} = \frac{x - 2}{1}$$

$$y - 5 = 2 - x$$

$$y = 7 - x$$

Consider the area under BC:



From the above figure, the area under the line BC will be given by,

$$\begin{aligned} \text{Area of BCDF} &= \int_2^3 y \, dx = \int_2^3 (7 - x) \, dx \\ &= \int_2^3 (7 - x) \, dx = \left[7x - \frac{x^2}{2} \right]_2^3 \end{aligned}$$

[using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int c \, dx = cx$]

$$\begin{aligned} &= \left\{ \left[7(3) - \frac{(3)^2}{2} \right] - \left[7(2) - \frac{(2)^2}{2} \right] \right\} \\ &= \left\{ \left[21 - \frac{9}{2} \right] - [14 - 2] \right\} = \frac{42 - 9}{2} - 12 = \frac{33 - 24}{2} = \frac{9}{2} \end{aligned}$$

$$\text{Area of BCFE} = \frac{9}{2} \text{ sq. units. (2)}$$

Consider the line CA,

Using this 2-point formula for line, equation of the line C(3,4) and A(1,3)

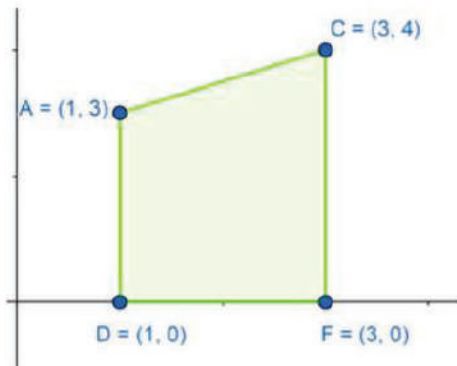
$$\frac{y - (4)}{3 - 4} = \frac{x - (3)}{1 - (3)}$$

$$\frac{y - (4)}{-1} = \frac{x - 3}{-2}$$

$$y = \frac{1}{2}(x - 3) + 4 = \frac{x - 3 + 8}{2} = \frac{x + 5}{2}$$

$$y = \frac{x + 5}{2}$$

Consider the area under CA:



From the above figure, the area under the line CA will be given by,

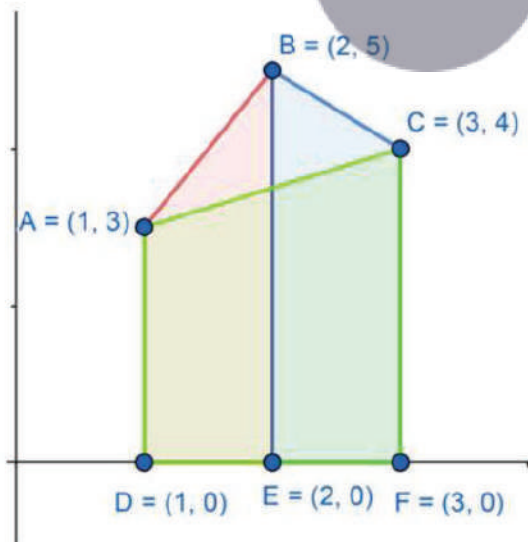
$$\begin{aligned} \text{Area of ACFE} &= \int_1^3 y \, dx = \int_1^3 \left(\frac{x+5}{2} \right) dx \\ &= \int_1^3 \frac{1}{2}(x+5) \, dx = \frac{1}{2} \left[\frac{x^2}{2} + 5x \right]_1^3 \end{aligned}$$

[using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int c \, dx = cx$]

$$\begin{aligned} &= \frac{1}{2} \left\{ \left[\frac{3^2}{2} + 5(3) \right] - \left[\frac{1^2}{2} + 5(1) \right] \right\} \\ &= \frac{1}{2} \left\{ \left[\frac{9}{2} + 15 \right] - \left[\frac{1}{2} + 5 \right] \right\} = \frac{1}{2} \left[\frac{8}{2} + 10 \right] = \frac{1}{2} (14) = 7 \end{aligned}$$

Area of ACFD = 7 sq.units ---- (3)

If we combined, the areas under AB, BC and AC in the below graph, we can clearly say that the area under AC (3) is overlapping the previous two areas under AB & BC.



Now, the combined area under the rABC is given by

Area under rABC

= Area under AB + Area under BC - Area under AC

From (1), (2) and (3), we get

$$\text{Area under rABC} = 4 + \frac{9}{2} - 7 = \frac{9}{2} - 3 = \frac{3}{2}$$

Therefore, area under rABC = $\frac{3}{2}$ sq.units.

39. Question

Using integration, find the area of the triangular region bounded by the lines $y=2x+1$, $y=3x+1$ and $x=4$.

Answer

Given,

- ABC is a triangle
- Equation of side AB of $y = 2x + 1$
- Equation of side BC of $y = 3x+1$
- Equation of side CA of $x=4$

By solving AB & BC we get the point B,

$$AB : y = 2x+1, BC: y = 3x+1$$

$$2x+1 = 3x+1$$

$$x = 0$$

by substituting $x = 0$ in AB we get $y = 1$

The point B = (0,1)

By solving BC & CA we get the point C,

$$AC : x = 4, BC: y = 3x+1$$

$$y = 12+1 = 13$$

$$y = 13$$

The point C = (4,13)

By solving AB & AC we get the point A,

$$AB : y = 2x+1, AC : x=4$$

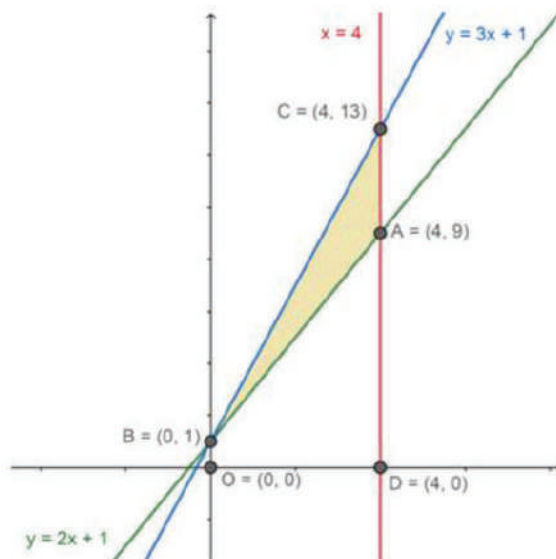
$$y = 8+1 = 9$$

$$y = 9$$

The point A = (4,9)

These points are used for obtaining the upper and lower bounds of the integral.

From the given information, the area under the triangle (colored) can be given by the below figure.

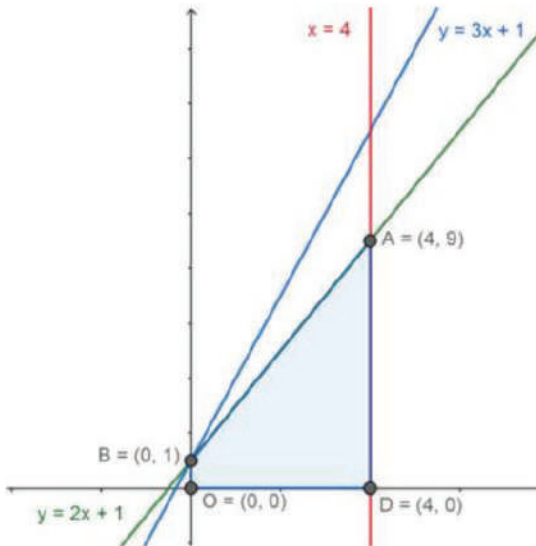


From above figure we can clearly say that, the area between ABC is the area to be found.

For finding this area, the line equations of the sides of the given triangle are considered. By calculating the area under these lines we can find the area of the complete region.

Consider the line AB, $y = 4x + 5$

The area under line AB:



From the above figure, the area under the line AB will be given by,

$$\text{Area of } AB = \int_0^4 y_{AB} dx = \int_0^4 (2x + 1) dx$$

$$= \int_0^4 (2x + 1) dx = \left[\frac{2x^2}{2} + x \right]_0^4$$



[using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int c dx = cx$]

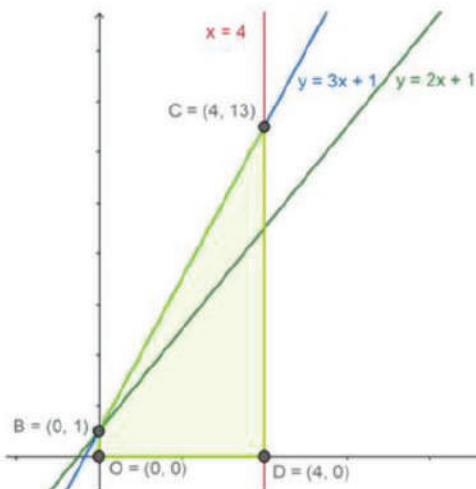
$$= \{ [(4^2) + (4)] - [(0)^2 + 0] \}$$

$$= 20$$

Area under AB = 20 sq. units. ---- (1)

Consider the line BC, $y = 3x + 1$

Consider the area under BC:



From the above figure, the area under the line BC will be given by,

$$\text{Area of } BC = \int_0^4 y_{BC} dx = \int_0^4 (3x + 1) dx$$

$$= \int_0^4 (3x + 1) dx = \left[\frac{3x^2}{2} + x \right]_0^4$$

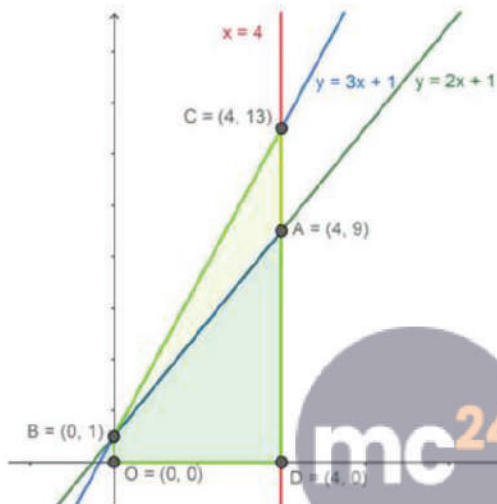
[using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int c dx = cx$]

$$= \left\{ \left[\frac{3(4)^2}{2} + 4 \right] - \left[\frac{(0)^2}{2} + (0) \right] \right\}$$

$$= 24 + 4 - 0 = 28$$

Area under BC = 28 sq. units. ---- (2)

If we area under AB is removed from BC from the graph, we can obtain the area required.



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Now, the combined area under the rABC is given by

$$\text{Area under rABC} = \text{Area under BC} - \text{Area under AB}$$

From (1), (2), we get

$$\text{Area under rABC} = 28 - 20 = 8$$

Therefore, area under rABC = 8 sq.units.