Chapter 6

Chapter 6: Triangles

EXERCISE 6.5:

1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.
   (i) 7 cm, 24 cm, 25 cm
   (ii) 3 cm, 8 cm, 6 cm
   (iii) 50 cm, 80 cm, 100 cm
   (iv) 13 cm, 12 cm, 5 cm

Solution: (i) Geven: sides of the triangle are 7 cm, 24 cm, and 25 cm
Squaring the lengths of these sides, we get: 49, 576, and 625.
$49 + 576 = 625$
Or, $7^2 + 24^2 = 25^2$
The sides of the given triangle satisfy Pythagoras theorem
Hence, it is a right triangle
We know that the longest side of a right triangle is the hypotenuse
Therefore, the length of the hypotenuse of this triangle is 25 cm

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm
Squaring the lengths of these sides, we will obtain 9, 64, and 36
However, $9 + 36 \neq 64$
Or, $32 + 62 \neq 82$
Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side
Therefore, the given triangle is not satisfying Pythagoras theorem
Hence, it is not a right triangle

(iii)Given that sides are 50 cm, 80 cm, and 100 cm.
Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.
And, $2500 + 6400 \neq 10000$
Or, $502 + 802 \neq 1002$
Now, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side
Therefore, the given triangle is not satisfying Pythagoras theorem
Hence, it is not a right triangle

(iv)Given: Sides are 13 cm, 12 cm, and 5 cm
Squaring the lengths of these sides, we get 169, 144, and 25.

Clearly, 144 +25 = 169

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem

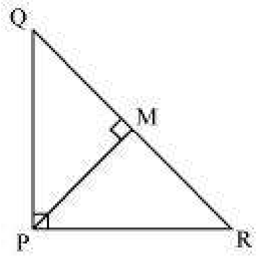
Therefore, it is a right triangle

We know that the longest side of a right triangle is the hypotenuse

Therefore, the length of the hypotenuse of this triangle is 13 cm

2. PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show thatPM2 = QM, MR.

Solution: Let, ∠MPR = x



In triangle MPR,

∠MRP = $180^o - 90^o - x$

∠MRP = $90^o - x$

Similarly,

In triangle MPQ,

∠MPQ = 90^o - ∠MPR

= $90^o - x$

In triangle QMP and PMR,

∠MPQ = ∠MRP

∠PMQ = ∠RMP

∠MQP = ∠MPR

Therefore,

ΔQMP ~ΔPMR (By SSS similarity)

$$\frac{QM}{PM} = \frac{MP}{MR}$$

PM^2 = QM * MR

3. In Fig. 6.53, ABD is a triangle right angled at Aand AC ⊥ BD. Show that:

(i) AB^2 = BC . BD

(ii) AC^2 = BC . DC

(iii) AD^2 = BD . CD

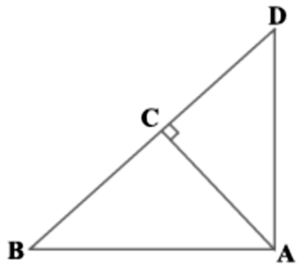


Fig. 6.53

Solution: (i) In ΔADB and ΔCAB, we have

∠DAB = ∠ACB (Each of 90°)

∠ABD = ∠CBA (Common angle)

Therefore,

ΔADB ~ΔCAB (AA similarity)

$$\frac{AB}{CB} = \frac{BD}{AB}$$

$AB^2 = CB * BD$

(ii) Let ∠CAB = x

In ΔCBA,

∠CBA = 180° – 90° – x

∠CBA = 90° – x

Similarly, in ΔCAD

∠CAD = 90° - ∠CBA

= 90° – x

∠CDA = 180° – 90° – (90° – x)

∠CDA = x

In triangle CBA and CAD, we have

∠CBA = ∠CAD

∠CAB = ∠CDA

∠ACB = ∠DCA (Each 90°)

Therefore,

ΔCBA ~ΔCAD (By AAA similarity)

$$\frac{AC}{DC} = \frac{BC}{AC}$$

$AC^2 = DC * BC$

(iii) In ΔDCA and ΔDAB, we have

∠DCA = ∠DAB (Each 90°)

∠CDA = ∠ADB (Common angle)

Therefore,

ΔDCA ~ΔDAB (By AA similarity)

$$\frac{DC}{DA} = \frac{DA}{DA}$$

$AD^2 = BD * CD$

4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Solution: Given: ΔABC is an isosceles triangle

AC = CB

Using Pythagoras theorem in ΔABC (i.e., right-angled at point C), we get

$AC^2 + CB^2 = AB^2$

$AC^2 + AC^2 = AB^2$ (AC = CB)

$2AC^2 = AB^2$

5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle

Solution: Given that,

$AB^2 = 2AC^2$

$AB^2 = AC^2 + AC^2$

$AB^2 = AC^2 + BC^2$ (As, AC = BC)

The triangle is satisfying the Pythagoras theorem

Therefore, the given triangle is a right-angled triangle

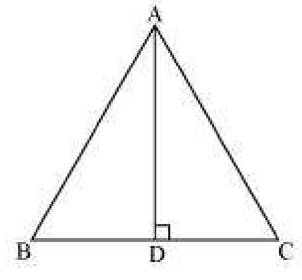
###Topic###Maths||Triangles||Pythagoras Theorem### (Easy)

6. ABC is an equilateral triangle of side 2a. Find each of its altitudes

Solution: Let AD be the altitude of the given equilateral triangle, ΔABC

We know that altitude bisects the opposite side

BD = DC = *a*



In triangle ADB,

$\angle ADB = 90^O$

Using Pythagoras theorem, we get

$AD^2 + DB^2 = AB^2$

$AD^2 + a^2 = (2a)^2$

$AD^2 + a^2 = 4a^2$

$AD^2 = 3a^2$

$AD = a\sqrt{3}$

In an equilateral triangle, all the altitudes are equal in length.

Hence, the length of each altitude will be $\sqrt{3}a$

7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals

Solution:In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$AB^2 = AO^2 + OB^2$ (i)

$BC^2 = BO^2 + OC^2$ (ii)

$CD^2 = CO^2 + OD^2$ (iii)

$AD^2 = AO^2 + OD^2$ (iv)

Now after adding all equations, we get,

$AB^2 + BC^2 + CD^2 + AD^2 = 2 (AO^2 + OB^2 + OC^2 + OD^2)$

$= 2[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2]$

As we know that the Diagonals bisect each other,

$= 2 [\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2]$

$= (AC)^2 + (BD)^2$

8. In Fig. 6.54, O is a point in the interior of a triangleABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

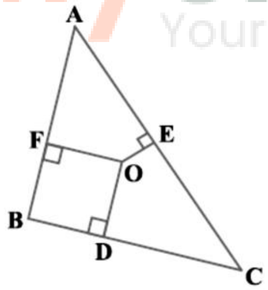


Fig. 6.54

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$,

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

Solution: Join OA, OB, and OC

(i) Applying Pythagoras theorem in ΔAOF, we obtain

$OA^2 = OF^2 + AF^2$

Similarly, in ΔBOD,

$OB^2 = OD^2 + BD^2$

Similarly, in ΔCOE,

$OC^2 = OE^2 + EC^2$

Adding these equations, we get

$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$

$OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2 = AF^2 + BD^2 + EC^2$

(ii) From the above given result,

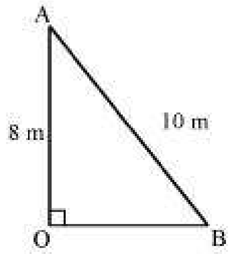
$AF^2 + BD^2 + EC^2 = (AO^2 – OE^2) + (OC^2 – OD^2) + (OB^2 – OF^2)$

Therefore,

$AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$

9. A ladder 10 m long reaches a window 8 m above theground. Find the distance of the foot of the ladder from base of the wall

Solution: Let OA be the wall and AB be the ladder



By Pythagoras theorem,

$AB^2 = OA^2 + BO^2$

$(10)^2 = (8)^2 + OB^2$

$100 = 64 + OB^2$

$OB^2 = 36$

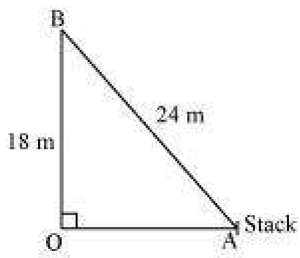
OB = 6 cm

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m

10. A guy wire attached to a vertical pole of height 18 mis 24 m long and has a stake attached to the otherend. How far from the base of the pole should thestake be driven so that the wire will be taut?

Solution: Let OB be the pole and AB be the wire

By Pythagoras theorem,



$AB^2 = OB^2 + OA^2$

$(24)^2 = (18)^2 + OA^2$

$OA^2 = (576 – 324)$

$OA^2 = 252$

$OA = 6\sqrt{7}$ m

11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At thesame time, another aeroplane leaves the same airport and flies due west at a speed of1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Solution: Distance travelled by the plane flying towards north in $1\frac{1}{2}hrs$ = 1,000 * $1\frac{1}{2}$

= 1,500 km

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}hrs$ = 1,200 * $1\frac{1}{2}$

= 1,800 km

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

Distance between these planes after $1\frac{1}{2}hrs$, AB = $\sqrt{OA*OA+OB*OB}$

= ($\sqrt{1500*1500+1800*1800}$ km

= $\sqrt{2250000+3240000}$ km

= $\sqrt{5490000}$ km

= $\sqrt{9*610000}$ km

= $300\sqrt{61}$ km

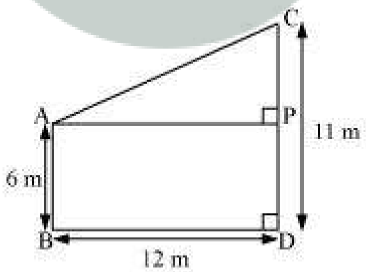
12. Two poles of heights 6 m and 11 m stand on aplane ground. If the distance between the feetof the poles is 12 m, find the distance between their tops

Solution: Let CD and AB be the poles of height 11 m and 6 m

Therefore, CP = 11 - 6 = 5 m

From the figure, it can be observed that AP = 12m

Applying Pythagoras theorem for ΔAPC, we obtain



$AP^2 + PC^2 = AC^2$

$(12)^2 + (5)^2 = AC^2$

$AC^2 = (144 + 25)$

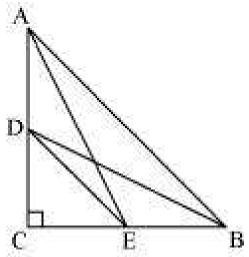
$AC^2 = 169$

AC = 13 m

Therefore, the distance between their tops is 13 m

13. D and E are points on the sides CA and CBrespectively of a triangle ABC right angled at C.Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Solution: Applying Pythagoras theorem in ΔACE, we obtain



$AC^2 + CE^2 = AE^2$ (i)

Applying Pythagoras theorem in triangle BCD, we get

$BC^2 + CD^2 = BD^2$ (ii)

Using (i) and (ii), we get

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2$ (iii)

Applying Pythagoras theorem in triangle CDE, we get

$DE^2 = CD^2 + CE^2$

Applying Pythagoras in triangle ABC, we get

$AB^2 = AC^2 + CB^2$

Putting these values in (iii), we get

$DE^2 + AB^2 = AE^2 + BD^2$

14. The perpendicular from A on side BC of aΔABC intersects BC at D such that DB = 3 CD(see Fig. 6.55). Prove that $2AB^2 = 2AC^2 + BC^2$.

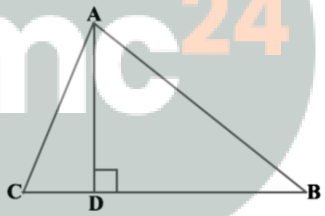


Fig. 6.55

Solution: Applying Pythagoras theorem for ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

$AD^2 = AC^2 - DC^2$ (i)

Applying Pythagoras theorem in ΔABD, we obtain

$AB^2 = AD^2 + DB^2$

$AD^2 = AB^2 - DB^2$ (ii)

From (i) and (ii), we get

$AC^2 - DC^2 = AB^2 - DB^2$ (iii)

It is given that 3DC = DB

Therefore,

DC = $\frac{BC}{4}$ and DB = $\frac{3BC}{4}$

Putting these values in (iii), we get

$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$

$AC^2 - BC * \frac{BC}{16} = AB^2 - \frac{9*BC*BC}{16}$

$16AC^2 - BC^2 = 16AB^2 - 9BC^2$

$16AB^2 - 16AC^2 = 8BC^2$

$2AB^2 = 2AC^2 + BC^2$

15. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Solution: Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC

BE = EC =BC/2 = a/2

And,

AE = $a\sqrt{3}/2$

Given that, BD = 1/3 BC

BD = a/3

DE = BE – BD

= a/2 – a/3

= a/6

Applying Pythagoras theorem in ΔADE, we obtain

$AD^2 = AE^2 + DE^2$

$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$

$= \frac{3*a*a}{4} + a * \frac{a}{36}$

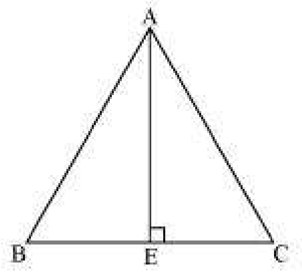
$= \frac{28*a*a}{36}$

$= \frac{7}{9} AB^2$

$9 AD^2 = 7 AB^2$

16. In an equilateral triangle, prove that three times the square of one side is equal to fourtimes the square of one of its altitudes

Solution: Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC



$BE = EC = \frac{BC}{2} = \frac{a}{2}$

Applying Pythagoras theorem in ΔABE, we obtain

$AB^2 = AE^2 + BE^2$

$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$

$AE^2 = a^2 - a * \frac{a}{4}$

$AE^2 = \frac{3*a*a}{4}$

$4AE^2 = 3a^2$

4 × (Square of altitude) = 3 × (Square of one side)

17. Tick the correct answer and justify:

In $\triangle ABC$, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm. the angle B is:

A. 120° B. 60°

C. 90° D. 45°

Solution: Given that, $AB = 6\sqrt{3}$ cm,

AC = 12 cm,

And BC = 6 cm

It can be observed that

$AB^2 = 108$

$AC^2 = 144$

And, $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

ΔABC, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B

∠B = 90°

Hence, the correct answer is (C)