Chapter 3

IN TWO VARIABLES 3 Exercise 3.2

Q.1 Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens togethercost Rs 46. Find the cost of one pencil and that of one pen.

Solutions :

(i) Let number of boys = x

Let number of girls = y

According to the question,

= $x + y = 10$

= $x = 10 - y$

Putting y = 0 , 5, 10 . we get,

X = 10 – 0 = 10

X = 10 – 5 = 5

X = 10 – 10 = 0

x	10	5	0
y	0	5	10

Number of girls is 4 more than number of boys ……Given

So,

Y = x + 4

Putting x = -4 ,0 , 4 we get,

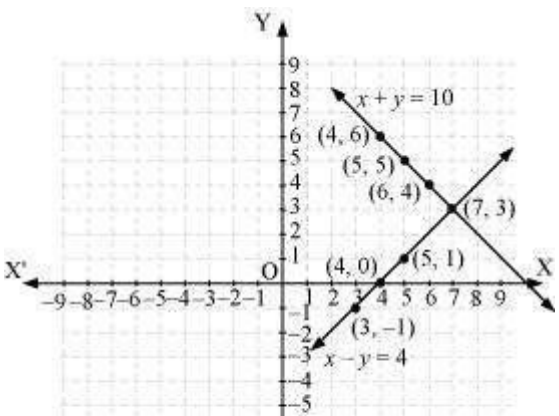
Y = -4+4 = 0

Y= 0 + 4 = 4

Y = 4 + 4 = 8

x	-4	0	4
Y	0	4	8

Graphical representation :



(ii) Let cost of one pencil = Rs. X

Let cost of one pen = Rs. Y

According to the question,

= $5x + 7y = 50$

= $5x = 50 - 7y$

= $x = 10 - \frac{7}{5}y$

Putting value of y = 5 , 10 , 15 we get,

X = $10 - \frac{7}{5} \times 5 = 10 - 7 = 3$

X = $10 - \frac{7}{5} \times 10 = 10 - 14 = -4$

X = $10 - \frac{7}{5} \times 15 = 10 - 21 = -11$

x	3	-4	-11
Y	5	10	15

Now,

7 pencils and 5 pens together cost Rs. 46

7x + 5y = 46

= 5y = 46 – 7x

= y = 9.2 – 1.4x

Putting x = 0 , 2 , 4 we get,

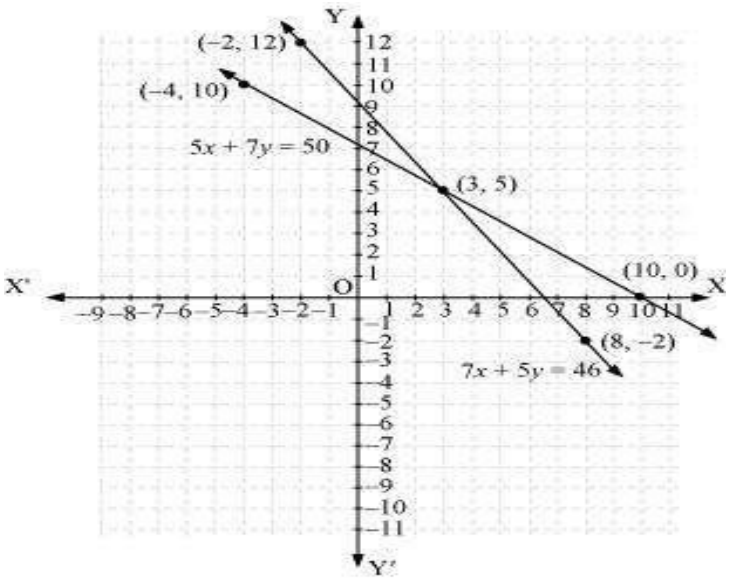
Y= 9.2 – 1.4×0 = 9.2

Y = 9.2 – 1.4 × 2 = 9.2 – 2.8 = 6.4

Y = 9.2 – 1.4 × 4 = 9.2 – 5.6 = 3.6

x	0	2	4
y	9.2	6.4	3.6

Graphical Representation:



Q.2

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing thefollowing pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$ $\quad$ $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$ $\quad$ $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$ $\quad$ $2x - y + 9 = 0$

Solutions:

i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing these equation with

$= a_1x + b_1y + c_1 = 0$

$= a_2x + b_2y + c_2 = 0$

We get,

$= a_1 = 5, b_1 = -4, c_1 = 8$

$= a_2 = 7, b_2 = 6, c_2 = -9$

Hence,

$= \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = -\frac{4}{6}$ and $\frac{c_1}{c_2} = \frac{8}{-9}$

We find that, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, both lines intersect at one point.

ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing these equations with ,

$= a_1x + b_1y + c_1 = 0$

$= a_2x + b_2y + c_2 = 0$

We get,

$= a_1 = 9, b_1 = 3, c_1 = 12$

$= a_2 = 18, b_2 = 6, c_2 = 24$

Hence,

$= \frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

We find that,

$= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore , both lines are coincident.

iii) $6x - 3y + 10 = 0$

2x – y + 9 = 0

Comparing these equations with,

= $a_1x + b_1y + c_1 = 0$

= $a_2x + b_2y + c_2 = 0$

We get,

= $a_1 = 6\,, b_1 = -3, c_1 = 10$

= $a_2 = 2, b_2 = -1\,, c_2 = 9$

Hence,

= $\frac{a_1}{a_2} = \frac{6}{2} = 3\,, \frac{b_1}{b_2} = -\frac{3}{-1} = 3$ *and* $\frac{c_1}{c_2} = \frac{10}{9}$

We find that,

= $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, both lines are parallel.

Q.3 On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linearequations are consistent, or inconsistent.

(i) $3x + 2y = 5;\quad 2x - 3y = 7$

(ii) $2x - 3y = 8;\quad 4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7;\quad 9x - 10y = 14$

(iv) $5x - 3y = 11;\quad -10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8;\quad 2x + 3y = 12$

Solutions:

i) 3x + 2y = 5

2x – 3y = 7

We get,

= $\frac{a_1}{a_2} = \frac{3}{2}$

= $\frac{b_1}{b_2} = -\frac{2}{3}$

= $\frac{c_1}{c_2} = \frac{8}{9}$

Hence,

= $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore , these linear equations will intersect at one point only and have only one possible solution.

And, $pair\ of\ linear\ equations\ is\ consistent.$

ii) 2x – 3y = 8

4x – 6y = 9

We get,

$= \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$= \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$

$= \frac{c_1}{c_2} = \frac{8}{9}$

Hence,

$= \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, these linear equations are parallel to each other and have no possible solution.

And, $pair\ of\ linear\ equations\ is\ inconsistent$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

9x – 10y = 14

We get,

$= \frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{3}{18} = \frac{1}{6}$

$= \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{5}{-30} = -\frac{1}{6}$

$= \frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$

Hence,

$= \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, these linear equations will intersect each other at one point and have only one possible solution.

And, $pair\ of\ linear\ equations\ is\ consistent.$

iv) 5x - 3y = 11

- 10x + 6y = - 22

We get,

$= \frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}$

$= \frac{b_1}{b_2} = -\frac{3}{6} = -\frac{1}{2}$

$= \frac{c_1}{c_2} = \frac{11}{-22} = -\frac{1}{2}$

Hence,

$= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore these pair of lines have infinite number of solutions. and
$pair\ of\ linear\ equation\ is\ consistent,$

v) $\frac{4}{3x} + 2y = 8$

2x + 3y = 12

We get,

$= \frac{a_1}{a_2} = \frac{4}{6} = \frac{2}{3}$

$= \frac{b_1}{b_2} = \frac{2}{3}$

$= \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Hence,

$= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore these pair of lines have infinite number of solutions and
$pair\ of\ linear\ equation\ is\ consistent.$

Q.4

Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5;$ $2x + 2y = 10$

(ii) $x - y = 8,$ $3x - 3y = 16$

(iii) $2x + y - 6 = 0,$ $4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0,$ $4x - 4y - 5 = 0$

Solution:

i) x + y = 5

2x + 2y = 10

We get,

$= \frac{a_1}{a_2} = \frac{1}{2}$

$= \frac{b_1}{b_2} = \frac{1}{2}$

$= \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$

Hence,

$= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore these pair of lines have infinite number of solutions and
pair of linear equation is consistent,

x + y = 5

x = 5 - y

putting y = 1,2,3 we get,

x = 5 -1 = 4

x = 5 - 2 = 3

x = 5 - 3 = 2

X	4	3	2
Y	1	2	3

And, 2x + 2y = 10

$x = \frac{10-2y}{2}$

X	4	3	2
Y	1	2	3

ii) x - y = 8

3x - 3y = 16

We get,

$= \frac{a_1}{a_2} = \frac{1}{3}$

$= \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$

$= \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$

Hence,

$= \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, these linear equations are parallel to each other and have no possible solution,

Hence, $the\ pair\ of\ linear\ equations\ is\ inconsistent.$

iii) 2x + y - 6 = 0

4x - 2y - 4 = 0

We get,

$= \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$= \frac{b_1}{b_2} = \frac{1}{-2}$

$= \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$

Hence,

$= \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution.

Hence, $pair\ of\ linear\ equations\ is\ consistent.$

$= 2x + y - 6 = 0$

$= y = 6 - 2x$

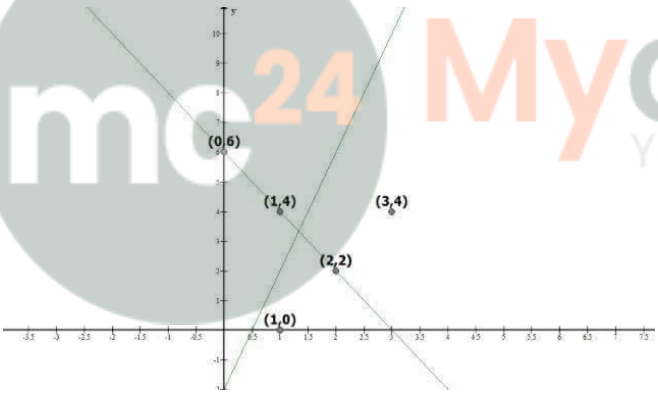
X	0	1	2
Y	6	4	2

And, $4x - 2y - 4 = 0$

= $y = \frac{4x-4}{2}$

X	1	2	3
Y	0	2	4

Graphical representation



iv) 2x - 2y -2 = 0

4x - 4y - 5 = 0

We get,

= $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$= \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$

$= \frac{c_1}{c_2} = \frac{2}{5}$S

Hence,

$= \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, these linear equations are parallel to each other and have no possible solution,

Hence, $the\ pair\ of\ linear\ equations\ is\ inconsistent$

Q.5 Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is36 m. Find the dimensions of the garden.

Solution:

Let's suppose,

width of the garden = *x*

length = *y*

so,

y - *x* = 4..... (1)

y + *x* = 36..... (2)

y - *x* = 4

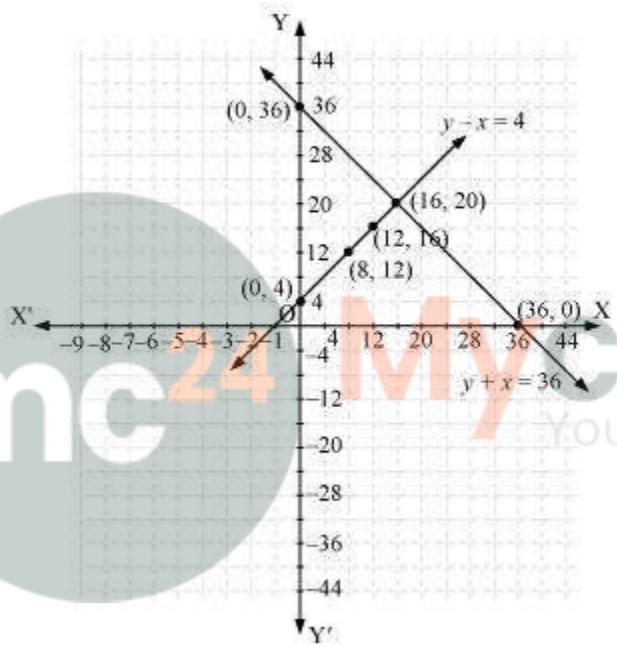
y = *x* + 4

x	0	8	12
y	4	12	16

y + *x* = 36

x	0	36	16
y	36	0	20

Hence, the graphic representation is as follows.



Q.6

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variablessuch that the geometrical representation of the pair so formed is:

(i) intersecting lines

(ii) parallel lines

(iii) coincident lines

Solution:

(i) Intersecting lines: For this condition,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The second line such that it is intersecting the given line is

2x+4y-6 = 0

As

$\frac{a_1}{a_2} = \frac{2}{2} = 1,$

$\frac{b_1}{b_2} = \frac{3}{4}\ and\ \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) Parallel lines:

For this condition,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the second line can be 4*x* + 6y - 8 = 0

As $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},$

$\frac{c_1}{c_2} = \frac{-8}{-8} = 1$

So,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) Coincident lines: For coincident lines,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence,

the second line can be 6*x* + 9*y* - 24 = 0

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$,

$\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$,

$\frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$

So,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Q.7

Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Solution:

$x - y + 1 = 0$

$x = y - 1$

X	0	1	2
Y	1	2	3

$3x + 2y - 12 = 0$

$x = \frac{12 - 2y}{3}$

X	4	2	0
Y	0	3	6

Hence, the graphic representation is as follows.