EXERCISE 21.2

Sum the following series to n terms:

1. 3 + 5 + 9 + 15 + 23 + …………

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.

We have,

$S_n = 3 + 5 + 9 + 15 + 23 + ... + T_{n-1} + T_n$ … (1)

Equation (1) can be rewritten as:

$S_n = 3 + 5 + 9 + 15 + 23 + ………… + T_{n-1} + T_n$................(2)

By subtracting (2) from (1) we get

$S_n = 3 + 5 + 9 + 15 + 23 + ... + T_{n-1} + T_n$

$S_n = \quad 3 + 5 + 9 + 15 + 23 + ... + T_{n-1} + T_n$

$0 = 3 + [2 + 4 + 6 + 8 + … + (T_n – T_{n-1})] - T_n$

The difference between the successive terms are 5-3 = 2, 9-5 = 4, 15-9 = 6,

So these differences are in A.P

Now,

$$3 + \left[\frac{(n-1)}{2}\{4 + (n-2)2\}\right] - T_n = 0$$

$$3 + \left[\frac{(n-1)}{2}(2n)\right] = T_n$$

$$3 + n(n-1) = T_n$$

Now,

$$S_n = \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} \{3 + k(k-1)\}$$

$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 3 - \sum_{k=1}^{n} k$$

$$= \frac{n(n+1)(2n+1)}{6} + 3n - \frac{n(n+1)}{2}$$

$$= \frac{n}{3}\left[\frac{(n+1)(2n+1)}{2} + 9 - \frac{3}{2}(n+1)\right]$$

$$= \frac{n[n^2+8]}{3}$$

$$= \frac{n}{3}(n^2 + 8)$$

∴ The sum of the series is $n/3\ (n^2 + 8)$

2. 2 + 5 + 10 + 17 + 26 + ………..

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.

We have,

$S_n = 2 + 5 + 10 + 17 + 26 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n \ldots (1)$

Equation (1) can be rewritten as:

$S_n = 2 + 5 + 10 + 17 + 26 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n \ldots\ldots (2)$

By subtracting (2) from (1) we get

$S_n = 2 + 5 + 10 + 17 + 26 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$

$S_n = \quad 2 + 5 + 10 + 17 + 26 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$

$0 = 2 + [3 + 5 + 7 + 9 + \ldots + (T_n - T_{n-1})] - T_n$

The difference between the successive terms are 3, 5, 7, 9

So these differences are in A.P

Now,

$$2+\left[\frac{(n-1)}{2}\{6+(n-2)2\}\right]-T_n=0$$

$$2+\left[n^2-1\right]=T_n$$

$$\left[n^2+1\right]=T_n$$

Now,

$$S_n=\sum_{k=1}^{n} T_k$$

$$=\sum_{k=1}^{n}\left(k^2+1\right)$$

$$=\sum_{k=1}^{n} k^2+\sum_{k=1}^{n} 1$$

$$=\frac{n(n+1)(2n+1)}{6}+n$$

$$=\frac{n(n+1)(2n+1)+6n}{6}$$

$$=\frac{n\left(2n^2+3n+7\right)}{6}$$

$$=\frac{n}{6}(2n^2+3n+7)$$

∴ The sum of the series is $n/6\ (2n^2 + 3n + 7)$

3. 1 + 3 + 7 + 13 + 21 + …

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.

We have,

$S_n = 1 + 3 + 7 + 13 + 21 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n \ldots (1)$

Equation (1) can be rewritten as:

$S_n = 1 + 3 + 7 + 13 + 21 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ .... (2)
By subtracting (2) from (1) we get
$S_n = 1 + 3 + 7 + 13 + 21 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$
$S_n = \quad 1 + 3 + 7 + 13 + 21 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$
$0 = 1 + [2 + 4 + 6 + 8 + \ldots + (T_n - T_{n-1})] - T_n$

The difference between the successive terms are 2, 4, 6, 8
So these differences are in A.P
Now,

$$1 + \left[\frac{(n-1)}{2}\{4 + (n-2)2\}\right] - T_n = 0$$
$$1 + [n^2 - n] = T_n$$
$$[n^2 - n + 1] = T_n$$

Now,

$$S_n = \sum_{k=1}^{n} T_k$$
$$= \sum_{k=1}^{n} (k^2 - k + 1)$$
$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1 - \sum_{k=1}^{n} k$$
$$= \frac{n(n+1)(2n+1)}{6} + n - \frac{n(n+1)}{2}$$
$$= \frac{n(n+1)}{2}\left(\frac{2n-2}{3}\right) + n$$
$$= n\left(\frac{n^2-1+3}{3}\right)$$
$$= \frac{n}{3}(n^2 + 2)$$

∴ The sum of the series is $n/3\ (n^2 + 2)$

4. 3 + 7 + 14 + 24 + 37 + …

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.
We have,
$S_n = 3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ … (1)
Equation (1) can be rewritten as:
$S_n = 3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ .... (2)
By subtracting (2) from (1) we get
$S_n = 3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$
$S_n = \quad 3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$
$0 = 3 + [4 + 7 + 10 + 13 + \ldots + (T_n - T_{n-1})] - T_n$

The difference between the successive terms are 4, 7, 10, 13
So these differences are in A.P
Now,

$$3+\left[\frac{(n-1)}{2}\{8+(n-2)3\}\right]-T_n=0$$

$$3+\left[\frac{(n-1)}{2}(3n+2)\right]-T_n=0$$

$$\left[\frac{3n^2-n+4}{2}\right]=T_n$$

$$\left[\frac{3}{2}n^2-\frac{n}{2}+2\right]=T_n$$

Now,

$$S_n=\sum_{k=1}^{n}T_k$$

$$=\sum_{k=1}^{n}\left(\frac{3}{2}k^2-\frac{k}{2}+2\right)$$

$$=\frac{3}{2}\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}2-\frac{1}{2}\sum_{k=1}^{n}k$$

$$=\frac{n(n+1)(2n+1)}{4}+2n-\frac{n(n+1)}{4}$$

$$=\frac{n(n+1)(2n)+8n}{4}$$

$$=\frac{(n+1)(2n^2)+8n}{4}$$

$$=\frac{n}{2}[n(n+1)+4]$$

$$=\frac{n}{2}[n^2+n+4]$$

∴ The sum of the series is $n/2\,[n^2 + n + 4]$

5. 1 + 3 + 6 + 10 + 15 + …

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.
We have,
$S_n = 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ … (1)
Equation (1) can be rewritten as:
$S_n = 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$……………. (2)
By subtracting (2) from (1) we get
$S_n = 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$
$S_n = \quad 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$
$0 = 1 + [2 + 3 + 4 + 5 + \ldots + (T_n - T_{n-1})] - T_n$

The difference between the successive terms are 2, 3, 4, 5

So these differences are in A.P

Now,

$$1+\left[\frac{(n-1)}{2}(4+(n-2)1)\right]-T_n=0$$

$$1+\left[\frac{(n-1)}{2}(n+2)\right]-T_n=0$$

$$\left[\frac{n^2+n}{2}\right]=T_n$$

Now,

$$S_n=\sum_{k=1}^{n}T_k$$

$$=\sum_{k=1}^{n}\left(\frac{k^2+k}{2}\right)$$

$$=\frac{1}{2}\sum_{k=1}^{n}k^2+\frac{1}{2}\sum_{k=1}^{n}k$$

$$=\frac{n(n+1)(2n+1)}{12}+\frac{n(n+1)}{4}$$

$$=\frac{n(n+1)}{4}\left(\frac{2n+1}{3}+1\right)$$

$$=\frac{n(n+1)}{4}\left(\frac{2n+4}{3}\right)$$

$$=\frac{n(n+1)}{2}\left(\frac{n+2}{3}\right)$$

$$=\frac{n(n+1)(n+2)}{6}$$

$$=\frac{n}{6}(n+1)(n+2)$$

∴ The sum of the series is n/6 (n+1) (n+2)