Exercise 7(D)

Solution:

Given, a/b = 3/5
(10a + 3b)/ (5a + 2b)

$$= \frac{10(a/b)+3}{5(a/b)+2}$$

$$= \frac{10(3/5)+3}{5(3/5)+2}$$

$$= \frac{6+3}{3+2}$$

$$= \frac{9}{5}$$

Solution:

Given, $\frac{5x+6y}{8x+5y} = \frac{8}{9}$

On cross multiplying, we get
45x + 54y = 64x + 40y
14y = 19x
Thus,
x/y = 14/19

Solution:

Given, (3x - 4y): (2x - 3y) = (5x - 6y): (4x - 5y)
This can be rewritten as,

$$\frac{3x-4y}{2x-3y} = \frac{5x-6y}{4x-5y}$$

Applying componendo and dividendo,

$$\frac{3x-4y+2x-3y}{3x-4y-2x+3y} = \frac{5x-6y+4x-5y}{5x-6y-4x+5y}$$

$$\frac{5x-7y}{x-y} = \frac{9x-11y}{x-y}$$

5x – 7y = 9x – 11y
4y = 4x

$x/y = 1/1$
Thus,
$x : y = 1 : 1$

1. Find the:
(i) duplicate ratio of $2\sqrt{2} : 3\sqrt{5}$
(ii) triplicate ratio of 2a: 3b
(iii) sub-duplicate ratio of $9x^2a^4 : 25y^6b^2$
(iv) sub-triplicate ratio of 216: 343
(v) reciprocal ratio of 3: 5
(vi) ratio compounded of the duplicate ratio of 5: 6, the reciprocal ratio of 25: 42 and the sub-duplicate ratio of 36: 49.
Solution:

(i) Duplicate ratio of $2\sqrt{2} : 3\sqrt{5} = (2\sqrt{2})^2 : (3\sqrt{5})^2 = 8 : 45$
(ii) Triplicate ratio of $2a : 3b = (2a)^3 : (3b)^3 = 8a^3 : 27b^3$
(iii) Sub-duplicate ratio of $9x^2a^4 : 25y^6b^2 = \sqrt{(9x^2a^4)} : \sqrt{(25y^6b^2)} = 3xa^2 : 5y^3b$
(iv) Sub-triplicate ratio of $216 : 343 = (216)^{1/3} : (343)^{1/3} = 6 : 7$
(v) Reciprocal ratio of 3: 5 = 5: 3
(vi) Duplicate ratio of 5: 6 = 25: 36
Reciprocal ratio of 25: 42 = 42: 25
Sub-duplicate ratio of 36: 49 = 6: 7

Required compound ratio = $\dfrac{25 \times 42 \times 6}{36 \times 25 \times 7} = 1 : 1$

2. Find the value of x, if:
(i) $(2x + 3) : (5x - 38)$ is the duplicate ratio of $\sqrt{5} : \sqrt{6}$.
(ii) $(2x + 1) : (3x + 13)$ is the sub-duplicate ratio of 9: 25.
(iii) $(3x - 7) : (4x + 3)$ is the sub-triplicate ratio of 8: 27.
Solution:

(i) $(2x + 3) : (5x - 38)$ is the duplicate ratio of $\sqrt{5} : \sqrt{6}$
And, the duplicate ratio of $\sqrt{5} : \sqrt{6} = 5 : 6$
So,
$(2x + 3)/(5x - 38) = 5/6$
$12x + 18 = 25x - 190$
$25x - 12x = 190 + 18$
$13x = 208$
$x = 208/13 = 16$

(ii) $(2x + 1) : (3x + 13)$ is the sub-duplicate ratio of 9: 25
Then the sub-duplicate ratio of 9: 25 = 3: 5
$(2x + 1)/(3x + 13) = 3/5$
$10x + 5 = 9x + 39$
$x = 34$

(iii) (3x - 7): (4x + 3) is the sub-triplicate ratio of 8: 27
And the sub-triplicate ratio of 8: 27 = 2: 3
(3x – 7)/ (4x + 3) = 2/3
9x – 8x = 6 + 21
x = 27

3. What quantity must be added to each term of the ratio x: y so that it may become equal to c: d?
Solution:

Let's assume the required quantity which has to be added be p.
So, we have

$$\frac{x+p}{y+p} = \frac{c}{d}$$

dx + pd = cy + cp
pd – cp = cy – dx
p(d - c) = cy – dx
p = cy –dx/ (d - c)

4. A woman reduces her weight in the ratio 7: 5. What does her weight become if originally it was 84 kg?
Solution:

Let's consider the woman's reduced weight as x.
Given, the original weight = 84 kg
So, we have
84: x = 7: 5
84/x = 7/5
84 x 5 = 7x
x = (84 x 5)/ 7
x = 60
Therefore, the reduced weight of the woman is 60 kg.

5. If $15(2x^2 - y^2) = 7xy$, find x: y; if x and y both are positive.
Solution:

$15(2x^2 - y^2) = 7xy$

$$\frac{2x^2 - y^2}{xy} = \frac{7}{15}$$

$$\frac{2x}{y} - \frac{y}{x} = \frac{7}{15}$$

Let's take the substitution as x/y = a
2a – 1/a = 7/15
$(2a^2 - 1)/ a = 7/15$
$30a^2 – 15 = 7a$
$30a^2 – 7a – 15 = 0$

$30a^2 - 25a + 18a - 15 = 0$
$5a(6a - 5) + 3(6a - 5) = 0$
$(6a - 5)(5a + 3) = 0$
So, $6a - 5 = 0$ or $5a + 3 = 0$
$a = 5/6$ or $a = -3/5$
As, a cannot be taken negative (ratio)
Thus, $a = 5/6$
$x/y = 5/6$
Hence, $x : y = 5 : 6$

6. Find the:
(i) fourth proportional to $2xy$, x^2 and y^2.
(ii) third proportional to $a^2 - b^2$ and $a + b$.
(iii) mean proportional to $(x - y)$ and $(x^3 - x^2y)$.
Solution:

(i) Let the fourth proportional to $2xy$, x^2 and y^2 be n.
$2xy : x^2 = y^2 : n$
$2xy \times n = x^2 \times y^2$
$$n = \frac{x^2y^2}{2xy} = \frac{xy}{2}$$

(ii) Let the third proportional to $a^2 - b^2$ and $a + b$ be n.
$a^2 - b^2$, $a + b$ and n are in continued proportion.
$a^2 - b^2 : a + b = a + b : n$
$$n = \frac{(a+b)^2}{a^2 - b^2} = \frac{(a+b)^2}{(a+b)(a-b)} = \frac{a+b}{a-b}$$

(iii) Let the mean proportional to $(x - y)$ and $(x^3 - x^2y)$ be n.
$(x - y)$, n, $(x^3 - x^2y)$ are in continued proportion
$(x - y) : n = n : (x^3 - x^2y)$
$n^2 = (x - y)(x^3 - x^2y)$
$n^2 = (x - y)x^2(x - y)$
$n^2 = x^2(x - y)^2$
$n = x(x - y)$

7. Find two numbers such that the mean proportional between them is 14 and third proportional to them is 112.
Solution:

Let's assume the required numbers be a and b.
Given, 14 is the mean proportional between a and b.
$a : 14 = 14 : b$
$ab = 196$
$a = 196/b$ …. (1)

Also, given, third proportional to a and b is 112.
a: b = b: 112
$b^2 = 112a$ …. (2)

Using (1), we have:
$b^2 = 112 \times (196/b)$
$b^3 = 14^3 \times 2^3$
b = 28
From (1),
a = 196/ 28 = 7
Therefore, the two numbers are 7 and 28.

8. If x and y be unequal and x: y is the duplicate ratio of x + z and y + z, prove that z is mean proportional between x and y.
Solution:
Given, $\frac{x}{y} = \frac{(x+z)^2}{(y+z)^2}$
$x(y^2 + z^2 + 2yz) = y(x^2 + z^2 + 2xz)$
$xy^2 + xz^2 + 2yzx = x^2y + z^2y + 2xzy$
$xy^2 + xz^2 = x^2y + z^2y$
$xy(y - x) = z^2(y - x)$
$xy = z^2$
Therefore, z is mean proportional between x and y.

9. If $x = \frac{2ab}{a+b}$, find the value of $\frac{x+a}{x-a} + \frac{x+b}{x-b}$.
Solution:

x = 2ab/ (a + b)
x/a = 2b/(a + b)
Applying componendo and dividendo,
$$\frac{x+a}{x-a} = \frac{2b+a+b}{2b-a-b}$$
$$\frac{x+a}{x-a} = \frac{3b+a}{b-a} \quad \ldots (1)$$
Also, x = 2ab/ (a + b)
x/b = 2a/ (a + b)
Applying componendo and dividendo, we have
$$\frac{x+b}{x-b} = \frac{2a+a+b}{2a-a-b}$$
$$\frac{x+b}{x-b} = \frac{3a+b}{a-b} \quad \ldots (2)$$
Now, comparing (1) and (2) we have

$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$
$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{-3b-a+3a+b}{a-b}$$
$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{2a-2b}{a-b}=2$$

10. If (4a + 9b) (4c - 9d) = (4a - 9b) (4c + 9d), prove that:
a: b = c: d.
Solution:

Given, $\frac{4a+9b}{4a-9b}=\frac{4c+9d}{4c-9d}$

Applying componendo and dividendo, we get

$$\frac{4a+9b+4a-9b}{4a+9b-4a+9b}=\frac{4c+9d+4c-9d}{4c+9d-4c+9d}$$

8a/18b = 8c/18d

a/b = c/d

- Hence Proved