Exercise – 16A

1. Find the distance between the points

(i) $A(9,3)$ and $B(15,11)$

(ii) $A(7,-4)$ and $B(-5,1)$

(iii) $A(-6,-4)$ and $B(9,-12)$

(iv) $A(1,-3)$ *and* $B(4,-6)$

(v) $P(a+b,a-b)$ *and* $Q(a-b,a+b)$

(vi) $P(a\sin\alpha, a\cos\alpha)$ *and* $Q(a\cos\alpha, -a\sin\alpha)$

Sol:

(i) $A(9,3)$ and $B(15,11)$

The given points are $A(9,3)$ and $B(15,11)$.

Then $(x_1=9, y_1=3)$ *and* $(x_2=15, y_2=11)$

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(15-9)^2+(11-3)^2}$

$=\sqrt{(15-9)^2+(11-3)^2}$

$=\sqrt{(6)^2+(8)^2}$

$=\sqrt{36+64}$

$=\sqrt{100}$

$=10\,units$

(ii) $A(7,-4)$ and $B(-5,1)$

The given points are $A(7,-4)$ *and* $B(-5,1)$.

Then, $(x_1=7, y_1=-4)$ and $(x_2=-5, y_2=1)$

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-5-7)^2+\{1-(-4)\}^2}$

$=\sqrt{(-5-7)^2+(1+4)^2}$

$=\sqrt{(-12)^2+(5)^2}$

$=\sqrt{144+25}$

$=\sqrt{169}$

$= 13 \ units$

$= \sqrt{144+25}$

$= \sqrt{169}$

$= 13 \ units$

(iii) $A(-6,-4)$ and $B(9,-12)$

The given points are $A(-6,-4) \ and \ B(9,-12)$

Then $(x_1 = -6, y_1 = -4) \ and \ (x_2 = 9, y_2 = -12)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(9-(-6))^2 + \{-12-(-4)\}^2}$

$= \sqrt{(9+6)^2 + (-12+4)^2}$

$= \sqrt{(15)^2 + (-8)^2}$

$= \sqrt{225+64}$

$= \sqrt{289}$

$= 17 \ units$

(iv) $A(1,-3) \ and \ B(4,-6)$

The given points are $A(1,-3) \ and \ B(4,-6)$

Then $(x_1 = 1, y_1 = -3) \ and \ (x_2 = 4, y_2 = -6)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-1)^2 + \{-6-(-3)\}^2}$

$= \sqrt{(4-1)^2 + (-6+3)^2}$

$= \sqrt{(3)^2 + (-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= \sqrt{9 \times 2}$

$= 3\sqrt{2} \ units$

(v) $P(a+b, a-b) \ and \ Q(a-b, a+b)$

The given points are $P(a+b, a-b)$ and $Q(a-b, a+b)$

Then $(x_1 = a+b, y_1 = a-b) \ and \ (x_2 = a-b, y_2 = a+b)$

$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{\{(a-b)-(a+b)\}^2+\{(a+b)-(a-b)\}^2}$

$=\sqrt{(a-b-a-b)^2+(a+b-a+b)^2}$

$=\sqrt{(-2b)^2+(2b)^2}$

$=\sqrt{4b^2+4b^2}$

$=\sqrt{8b^2}$

$=\sqrt{4\times 2b^2}$

$=2\sqrt{2}b\ units$

(vi) $P(a\sin\alpha, a\cos\alpha)$ *and* $Q(a\cos\alpha, -a\sin\alpha)$

The given points are $P(a\sin\alpha, a\cos\alpha)$ and $Q(a\cos a, -a\sin\alpha)$

Then $(x_1=a\sin\alpha, y_1=a\cos\alpha)$ *and* $(x_2=a\cos a, y_2=-a\sin\alpha)$

$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(a\cos\alpha-a\sin\alpha)^2+(-a\sin\alpha-a\cos\alpha)^2}$

$=\sqrt{\begin{matrix}(a^2\cos^2\alpha+a^2\sin^2\alpha-2a^2\cos\alpha\times\sin\alpha)\\+(a^2\sin^2\alpha+a^2\cos^2\alpha+2a^2\cos\alpha\times\sin\alpha)\end{matrix}}$

$=\sqrt{2a^2\cos^2\alpha+2a^2\sin^2\alpha}$

$=\sqrt{2a^2(\cos^2\alpha+\sin^2\alpha)}$

$=\sqrt{2a^2(1)}$ (From the identity $\cos^2\alpha+\sin^2\alpha=1$)

$=\sqrt{2a^2}$

$=\sqrt{2}a\ units$

2. Find the distance of each of the following points from the origin:

(i) $A(5,-12)$ (ii) $B(-5,5)$ (iii) $C(-4,-6)$

Sol:

(i) $A(5,-12)$

Let $O(0,0)$ be the origin

$OA=\sqrt{(5-0)^2+(-12-0)^2}$

$= \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13\ units$

(ii) $B(-5,5)$

Let $O(0,0)$ be the origin.

$OB = \sqrt{(-5-0)^2 + (5-0)^2}$

$= \sqrt{(-5)^2 + (5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= \sqrt{25 \times 2}$

$= 5\sqrt{2}\ units$

(iii) $C(-4,-6)$

Let $O(0,0)$ be the origin

$OC = \sqrt{(-4-0)^2 + (-6-0)^2}$

$= \sqrt{(-4)^2 + (-6)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$

$= \sqrt{4 \times 13}$

$= 2\sqrt{13}\ units$

3. Find all possible values of x for which the distance between the points $A(x,-1)$ and $B(5,3)$ is 5 units.

Sol:

Given $AB = 5\ units$

Therefore, $(AB)^2 = 25\ units$

$\Rightarrow (5-a)^2 + \{3-(-1)\}^2 = 25$

$\Rightarrow (5-a)^2 + (3+1)^2 = 25$

$\Rightarrow (5-a)^2 + (4)^2 = 25$

$\Rightarrow (5-a)^2 + 16 = 25$

$\Rightarrow (5-a)^2 = 25 - 16$

$\Rightarrow (5-a)^2 = 9$

$\Rightarrow (5-a) = \pm\sqrt{9}$

$\Rightarrow 5 - a = \pm 3$

$\Rightarrow 5 - a = 3 \ or \ 5 - a = -3$

$\Rightarrow a = 2 \ or \ 8$

Therefore, $a = 2 \ or \ 8$.

4. Find all possible values of y for which distance between the points $A(2,-3)$ and $B(10, y)$ is 10 units.

Sol:

The given points are $A(2,-3)$ and $B(10, y)$

$\therefore AB = \sqrt{(2-10)^2 + (-3-y)^2}$

$= \sqrt{(-8)^2 + (-3-y)^2}$

$= \sqrt{64 + 9 + y^2 + 6y}$

$\because AB = 10$

$\therefore \sqrt{64 + 9 + y^2 + 6y} = 10$

$\Rightarrow 73 + y^2 + 6y = 100$ (Squaring both sides)

$\Rightarrow y^2 + 6y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y+9) - 3(y+9) = 0$

$\Rightarrow (y+9)(y-3) = 0$

$\Rightarrow y + 9 = 0 \ or \ y - 3 = 0$

$\Rightarrow y = -9 \ or \ y = 3$

Hence, the possible values of y are -9 and 3.

5. Find value of x for which the distance between the points $P(x, 4)$ and $Q(9,10)$ is 10 units.

Sol:

The given points are $P(x,4) \ and \ Q(9,10)$.

$\therefore PQ = \sqrt{(x-9)^2 + (4-10)^2}$

$=\sqrt{(x-9)^2+(-6)^2}$

$=\sqrt{x^2-18x+81+36}$

$=\sqrt{x^2-18x+117}$

$\because PQ=10$

$\therefore \sqrt{x^2-18x+117}=10$

$\Rightarrow x^2-18x+117=100$ (Squaring both sides)

$\Rightarrow x^2-18x+17=$

$\Rightarrow x^2-17x-x+17=0$

$\Rightarrow x(x-17)-1(x-17)=0$

$\Rightarrow (x-17)(x-1)=0$

$\Rightarrow x-17=0\ or\ x-1=0$

$\Rightarrow x=17\ or\ x=1$

Hence, the values of x are 1 and 17.

6. If the point $A(x,2)$ is equidistant form the points $B(8,-2)$ and $C(2,-2)$, find the value of x. Also, find the value of x. Also, find the length of AB.

Sol:

As per the question

$AB=AC$

$\Rightarrow \sqrt{(x-8)^2+(2+2)^2}=\sqrt{(x-2)^2+(2+2)^2}$

Squaring both sides, we get

$(x-8)^2+4^2=(x-2)^2+4^2$

$\Rightarrow x^2-16x+64+16=x^2+4-4x+16$

$\Rightarrow 16x-4x=64-4$

$\Rightarrow x=\frac{60}{12}=5$

Now,

$AB=\sqrt{(x-8)^2+(2+2)^2}$

$=\sqrt{(5-8)^2+(2+2)^2}$ $(\because x=2)$

$=\sqrt{(-3)^2+(4)^2}$

$=\sqrt{9+16}=\sqrt{25}=5$

Hence, $x=5$ and $AB=5\ units$.

7. If the point $A(0,2)$ is equidistant form the points $B(3,p)$ and $C(p,5)$ find the value of p. Also, find the length of AB.

Sol:

As per the question

$AB = AC$

$\Rightarrow \sqrt{(0-3)^2+(2-p)^2} = \sqrt{(0-p)^2+(2-5)^2}$

$\Rightarrow \sqrt{(-3)^2+(2-p)^2} = \sqrt{(-p)^2+(-3)^2}$

Squaring both sides, we get

$(-3)^2+(2-p)^2 = (-p)^2+(-3)^2$

$\Rightarrow 9+4+p^2-4p = p^2+9$

$\Rightarrow 4p = 4$

$\Rightarrow p = 1$

Now,

$AB = \sqrt{(0-3)^2+(2-p)^2}$

$= \sqrt{(-3)^2+(2-1)^2}$ $\quad (\because p = 1)$

$= \sqrt{9+1}$

$= \sqrt{10}\ units$

Hence, $p = 1\ and\ AB = \sqrt{10}\ units$

8. Find the point on the a-axis which is equidistant from the points (2, -5) and (-2, 9).

Sol:

Let (x, 0) be the point on the x axis. Then as per the question, we have

$\sqrt{(x-2)^2+(0+5)^2} = \sqrt{(x+2)^2+(0-9)^2}$

$\Rightarrow \sqrt{(x-2)^2+(5)^2} = \sqrt{(x+2)^2+(-9)^2}$

$\Rightarrow (x-2)^2+(5)^2 = (x+2)^2+(-9)^2$ (Squaring both sides)

$\Rightarrow x^2-4x+4+25 = x^2+4x+4+81$

$\Rightarrow 8x = 25-81$

$\Rightarrow x = -\frac{56}{8} = -7$

Hence, the point on the x-axis is $(-7,0)$.

9. Find the points on the x-axis, each of which is at a distance of 10 units from the point A(11, -8).

Sol:

Let $P(x, 0)$ be the point on the x-axis. Then as per the question we have

$AP = 10$

$\Rightarrow \sqrt{(x-11)^2+(0+8)^2} = 10$

$\Rightarrow (x-11)^2 + 8^2 = 100$ (Squaring both sides)

$\Rightarrow (x-11)^2 = 100-64 = 36$

$\Rightarrow x-11 = \pm 6$

$\Rightarrow x = 11 \pm 6$

$\Rightarrow x = 11-6, 11+6$

$\Rightarrow x = 5, 17$

Hence, the points on the x-axis are $(5,0)$ and $(17,0)$.

10. Find the points on the y-axis which is equidistant form the points $A(6,5)$ and $B(-4,3)$

Sol:

Let P $(0, y)$ be a point on the y-axis. Then as per the question, we have

$AP = BP$

$\Rightarrow \sqrt{(0-6)^2+(y-5)^2} = \sqrt{(0+4)^2+(y-3)^2}$

$\Rightarrow \sqrt{(6)^2+(y-5)^2} = \sqrt{(4)^2+(y-3)^2}$

$\Rightarrow (6)^2+(y-5)^2 = (4)^2+(y-3)^2$ (Squaring both sides)

$\Rightarrow 36+y^2-10y+25 = 16+y^2-6y+9$

$\Rightarrow 4y = 36$

$\Rightarrow y = 9$

Hence, the point on the y-axis is $(0,9)$.

11. If the points $P(x,y)$ is point equidistant from the points $A(5,1)$ and $B(-1,5)$, Prove that 3x=2y. **Sol:**

As per the question, we have

$AP = BP$

$\Rightarrow \sqrt{(x-5)^2+(y-1)^2} = \sqrt{(x+1)^2+(y-5)^2}$

$\Rightarrow (x-5)^2+(y-1)^2 = (x+1)^2+(y-5)^2$ (Squaring both sides)

$\Rightarrow x^2-10x+25+y^2-2y+1 = x^2+2x+1+y^2-10y+25$

$\Rightarrow -10x-2y=2x-10y$

$\Rightarrow 8y=12x$

$\Rightarrow 3x=2y$

Hence, $3x=2y$

12. If $p(x,y)$ is point equidistant from the points $A(6,-1)$ and $B(2,3)$, show that x – y = 3

Sol:

The given points are $A(6,-1)$ and $B(2,3)$. The point $P(x, y)$ equidistant from the points A and B So, $PA = PB$

Also, $(PA)^2=(PB)^2$

$\Rightarrow (6-x)^2+(-1-y)^2=(2-x)^2+(3-y)^2$

$\Rightarrow x^2-12x+36+y^2+2y+1=x^2-4x+4+y^2-6y+9$

$\Rightarrow x^2+y^2-12x+2y+37=x^2+y^2-4x-6y+13$

$\Rightarrow x^2+y^2-12x+2y-x^2-y^2+4x+6y=13-37$

$\Rightarrow -8x+8y=-24$

$\Rightarrow -8(x-y)=-24$

$\Rightarrow x-y=\frac{-24}{-8}$

$\Rightarrow x-y=3$

Hence proved.

13. Find the co-ordinates of the point equidistant from three given points $A(5,3), B(5,-5)$ and $C(1,-5)$

Sol:

Let the required point be $P(x,y)$. Then $AP=BP=CP$

That is, $(AP)^2=(BP)^2=(CP)^2$

This means $(AP)^2=(BP)^2$

$\Rightarrow (x-5)^2+(y-3)^2=(x-5)^2+(y+5)^2$

$\Rightarrow x^2-10x+25+y^2-6y+9=x^2-10x+25+y^2+10y+25$

$\Rightarrow x^2-10x+y^2-6y+34=x^2-10x+y^2+10y+50$

$\Rightarrow x^2-10x+y^2-6y-x^2+10x-y^2-10y=50-34$

$\Rightarrow -16y=16$

$\Rightarrow y=-\frac{16}{16}=-1$

And $(BP)^2 = (CP)^2$

$\Rightarrow (x-5)^2 + (y+5)^2 = (x-1)^2 + (y+5)^2$

$\Rightarrow x^2 - 10x + 25 + y^2 + 10y + 25 = x^2 - 2x + 1 + y^2 + 10y + 25$

$\Rightarrow x^2 - 10x + y^2 + 10y + 50 = x^2 - 2x + y^2 + 10y + 26$

$\Rightarrow x^2 - 10x + y^2 + 10y - x^2 + 2x - y^2 - 10y = 26 - 50$

$\Rightarrow -8x = -24$

$\Rightarrow x = \frac{-24}{-8} = 3$

Hence, the required point is $(3, -1)$.

14. If the points $A(4,3)$ and $B(x,5)$ lies on a circle with the centre $O(2,3)$. Find the value of x.

Sol:

Given, the points $A(4,3)$ and $B(x,5)$ lie on a circle with center $O(2,3)$.

Then $OA = OB$

Also $(OA)^2 = (OB)^2$

$\Rightarrow (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$

$\Rightarrow (2)^2 + (0)^2 = (x-2)^2 + (2)^2$

$\Rightarrow 4 = (x-2)^2 + 4$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

Therefore, $x = 2$

15. If the point $C(-2,3)$ is equidistant form the points $A(3,-1)$ and $B(x,8)$, find the value of x. Also, find the distance between BC

Sol:

As per the question, we have

$AC = BC$

$\Rightarrow \sqrt{(-2-3)^2 + (3+1)^2} = \sqrt{(-2-x)^2 + (3-8)^2}$

$\Rightarrow \sqrt{(5)^2 + (4)^2} = \sqrt{(x+2)^2 + (-5)^2}$

$\Rightarrow 25 + 16 = (x+2)^2 + 25$ (Squaring both sides)

$\Rightarrow 25 + 16 = (x+2)^2 + 25$

$\Rightarrow (x+2)^2 = 16$

$\Rightarrow x+2 = \pm 4$

$\Rightarrow x = -2 \pm 4 = -2-4, -2+4 = -6, 2$

Now

$BC = \sqrt{(-2-x)^2 + (3-8)^2}$

$= \sqrt{(-2-2)^2 + (-5)}$

$= \sqrt{16+25} = \sqrt{41}\ units$

Hence, $x = 2$ or -6 and $BC = \sqrt{41}\ units$

16. If the point $P(2,2)$ is equidistant from the points $A(-2,k)$ and $B(-2k,-3)$, find k. Also, find the length of AP.

Sol:

As per the question, we have

$AP = BP$

$\Rightarrow \sqrt{(2+2)^2 + (2+k)^2} = \sqrt{(2+2k)^2 + (2+3)^2}$

$\Rightarrow \sqrt{(4)^2 + (2-k)^2} = \sqrt{(2+2k)^2 + (5)^2}$

$\Rightarrow 16 + 4 + k^2 - 4k = 4 + 4k^2 + 8k + 25$ (Squaring both sides)

$\Rightarrow k^2 + 4k + 3 = 0$

$\Rightarrow (k+1)(k+3) = 0$

$\Rightarrow k = -3, -1$

Now for $k = -1$

$AP = \sqrt{(2+2)^2 + (2-k)^2}$

$= \sqrt{(4)^2 + (2+1)^2}$

$= \sqrt{16+9} = 5\ units$

For $k = -3$

$AP = \sqrt{(2+2)^2 + (2-k)^2}$

$= \sqrt{(4)^2 + (2+3)^2}$

$= \sqrt{16+25} = \sqrt{41}\ units$

Hence, $k = -1, -3; AP = 5\ units\ \ for\ \ k = -1\ and\ \ AP = \sqrt{41}\ units\ \ for\ \ k = -3.$

17. If the point (x, y) is equidistant form the points $(a+b, b-a)$ and $(a-b, a+b)$, prove that bx = ay.

Sol:

As per the question, we have

$$\sqrt{(x-a-b)^2+(y-b+a)^2}=\sqrt{(x-a+b)^2+(y-a-b)^2}$$

$$\Rightarrow (x-a-b)^2+(y-b+a)^2=(x-a+b)^2+(y-a-b)^2 \quad \text{(Squaring both sides)}$$

$$\Rightarrow x^2+(a+b)^2-2x(a+b)+y^2+(a-b)^2-2y(a-b)=x^2+(a-b)^2-2x(a-b)+y^2$$
$$+(a+b)^2-2y(a+b)$$

$$\Rightarrow -x(a+b)-y(a-b)=-x(a-b)-y(a+b)$$

$$\Rightarrow -xa-xb-ay+by=-xa+bx-ya-by$$

$$\Rightarrow by=bx$$

Hence, $bx=ay$.

18. Using the distance formula, show that the given points are collinear:

(i) (1, -1), (5, 2) and (9, 5) (ii) (6, 9), (0, 1) and (-6, -7)

(iii) (-1, -1), (2, 3) and (8, 11) (iv) (-2, 5), (0,1) and (2, -3)

Sol:

(i) Let $A(1,-1), B(5,2)$ and $C(9,5)$ be the give points. Then

$$AB=\sqrt{(5-1)^2+(2+1)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5\ units$$

$$BC=\sqrt{(9-5)^2+(5-2)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5\ units$$

$$AC=\sqrt{(9-1)^2+(5+1)^2}=\sqrt{8^2+6^2}=\sqrt{100}=10\ units$$

$$\therefore AB+BC=(5+5)units=10\ units=AC$$

Hence, the given points are collinear

(ii) Let $A(6,9), B(0,1)$ and $C(-6,-7)$ be the give points. Then

$$AB=\sqrt{(0-6)^2+(1-9)^2}=\sqrt{(-6)^2+(-8)^2}=\sqrt{100}=10\ units$$

$$BC=\sqrt{(-6-0)^2+(-7-1)^2}=\sqrt{(-6)^2+(-8)^2}=\sqrt{100}=10\ units$$

$$AC=\sqrt{(-6-6)^2+(-7-9)^2}=\sqrt{(-12)^2+(16)^2}=\sqrt{400}=20\ units$$

$$\therefore AB+BC=(10+10)units=20\ units=AC$$

Hence, the given points are collinear

(iii) Let $A(-1,-1), B(2,3)$ and $C(8,11)$ be the give points. Then

$AB = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5\ units$

$BC = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10\ units$

$AC = \sqrt{(8+1)^2 + (11+1)^2} = \sqrt{(9)^2 + (12)^2} = \sqrt{225} = 15\ units$

$\therefore AB + BC = (5+10)units = 15\ units = AC$

Hence, the given points are collinear

(iv) Let $A(-2,5), B(0,1)$ and $C(2,-3)$ be the give points. Then

$AB = \sqrt{(0+2)^2 + (1-5)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}\ units$

$BC = \sqrt{(2-0)^2 + (-3-1)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}\ units$

$AC = \sqrt{(2+2)^2 + (-3-5)^2} = \sqrt{(4)^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5}\ units$

$\therefore AB + BC = (2\sqrt{5} + 2\sqrt{5})units = 4\sqrt{5}\ units = AC$

Hence, the given points are collinear

19. Show that the points A (7, 10), B(-2, 5) and C(3, -4) are the vertices of an isosceles right triangle.

Sol:

The given points are $A(7,10), B(-2,5)\ and\ C(3,-4)$.

$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{(-9)^2 + (-5)^2} = \sqrt{81+25} = \sqrt{106}$

$BC = \sqrt{(3-(-2))^2 + (-4-5)^2} = \sqrt{(5)^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$

$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$

Since, AB and BC are equal, they form the vertices of an isosceles triangle

Also, $(AB)^2 + (BC)^2 = (\sqrt{106})^2 + (\sqrt{106})^2 = 212$

and $(AC)^2 = (\sqrt{212})^2 = 212.$

Thus, $(AB)^2 + (BC)^2 = (AC)^2$

This show that ΔABC is right- angled at B.

Therefore, the points $A(7,10), B(-2,5)\ and\ C(3,-4)$ are the vertices of an isosceles right-angled triangle.

20. Show that the points A (3, 0), B(6, 4) and C(-1, 3) are the vertices of an isosceles right triangle.

Sol:

The given points are $A(3,0), B(6,4)$ and $C(-1,3)$. Now,

$$AB = \sqrt{(3-6)^2 + (0-4)^2} = \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

$$BC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{(7)^2 + (1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$AC = \sqrt{(3+1)^2 + (0-3)^2} = \sqrt{(4)^2 + (-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

$$\because AB = AC \ and \ AB^2 + AC^2 = BC^2$$

Therefore, $A(3,0), B(6,4)$ and $C(-1,3)$ are die vertices of an isosceles right triangle

21. If A(5,2), B(2, -2) and C(-2, t) are the vertices of a right triangle with $\angle$B=90°, then find the value of t.

Sol:

$$\because \angle B = 90°$$

$$\therefore AC^2 = AB^2 + BC^2$$

$$\Rightarrow (5+2)^2 + (2-t)^2 = (5-2)^2 + (2+2)^2 + (2+2)^2 + (-2-t)^2$$

$$\Rightarrow (7)^2 + (t-2)^2 = (3)^2 + (4)^2 + (4)^2 + (t+2)^2$$

$$\Rightarrow 49 + t^2 - 4t + 4 = 9 + 16 + 16 + t^2 + 4t + 4$$

$$\Rightarrow 8 - 4t = 4t$$

$$\Rightarrow 8t = 8$$

$$\Rightarrow t = 1$$

Hence, $t = 1$.

22. Prove that the points A(2, 4), b(2, 6) and $C(2+\sqrt{3}, 5)$ are the vertices of an equilateral triangle.

Sol:

The given points are $A(2,4), B(2,6) \ and \ C\left(2+\sqrt{3},5\right).Now$

$$AB = \sqrt{(2-2)^2 + (4-6)^2} = \sqrt{(0)^2 + (-2)^2}$$

$$= \sqrt{0+4} = 2$$

$BC = \sqrt{\left(2-2-\sqrt{3}\right)^2 + (6-5)^2} = \sqrt{\left(-\sqrt{3}\right)^2 + (1)^2}$

$= \sqrt{3+1} = 2$

$AC = \sqrt{\left(2-2-\sqrt{3}\right)^2 + (4-5)^2} = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}$

$= \sqrt{3+1} = 2$

Hence, the points $A(2,4), B(2,6)$ and $C\left(2+\sqrt{3},5\right)$ are the vertices of an equilateral triangle.

23. Show that the points (-3, -3),(3,3) and $C\left(-3\sqrt{3},3\sqrt{3}\right)$ are the vertices of an equilateral triangle.

Sol:

Let the given points be $A(-3,-3), B(3,3)$ *and* $C\left(-3\sqrt{3},3\sqrt{3}\right)$.Now

$AB = \sqrt{(-3-3)^2 + (-3-3)^2} = \sqrt{(-6)^2 + (-6)^2}$

$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$BC = \sqrt{\left(3+3\sqrt{3}\right)^2 + \left(3-3\sqrt{3}\right)^2}$

$= \sqrt{9+27+18\sqrt{3}+9+27-18\sqrt{3}} = \sqrt{72} = 6\sqrt{2}$

$AC = \sqrt{\left(-3+3\sqrt{3}\right)^2 + \left(-3-3\sqrt{3}\right)^2} = \sqrt{\left(3-3\sqrt{3}\right)^2 + \left(3+3\sqrt{3}\right)^2}$

$= \sqrt{9+27-18\sqrt{3}+9+27+18\sqrt{3}}$

$= \sqrt{72} = 6\sqrt{2}$

Hence, the given points are the vertices of an equilateral triangle.

24. Show that the points A(-5,6), B(3,0) and C(9,8) are the vertices of an isosceles right-angled triangle. Calculate its area.

Sol:

Let the given points be $A(-5,6)B(3,0)$ *and* $C(9,8)$.

$AB = \sqrt{(3-(-5))^2 + (0-6)^2} = \sqrt{(8)^2 + (-6)^2} = \sqrt{64+36} = \sqrt{100} = 10\ units$

$BC = \sqrt{(9-3)^2 + (8-0)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{36+64} = \sqrt{100} = 10\ units$

$AC = \sqrt{(9-(-5))^2 + (8-6)^2} = \sqrt{(14)^2 + (2)^2} = \sqrt{196+4} = \sqrt{200} = 10\sqrt{2}\ units$

Therefore, $AB = BC = 10\ units$

Also, $(AB)^2 + (BC)^2 = (10)^2 + (10)^2 = 200$

and $(AC)^2 = \left(10\sqrt{2}\right)^2 = 200$

Thus, $(AB)^2 + (BC)^2 = (AC)^2$

This show that ΔABC is right angled at B.

Therefore, the points $A(-5,6)B(3,0)$ *and* $C(9,8)$ are the vertices of an isosceles right-angled triangle

Also, area of a triangle $= \frac{1}{2} \times base \times height$

If AB is the height and BC is the base,

Area $= \frac{1}{2} \times 10 \times 10$

= 50 square units

25. Show that the points O(0,0), $A\left(3,\sqrt{3}\right)$ and $B\left(3,-\sqrt{3}\right)$ are the vertices of an equilateral triangle. Find the area of this triangle.

Sol:

The given points are $O(0,0)\,A\left(3,\sqrt{3}\right)$ *and* $B\left(3,-\sqrt{3}\right)$.

$$OA = \sqrt{(3-0)^2 + \left\{\left(\sqrt{3}\right)-0\right\}^2} = \sqrt{(3)^2 + \left(\sqrt{3}\right)^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

$$AB = \sqrt{(3-3)^2 + \left(-\sqrt{3}-\sqrt{3}\right)^2} = \sqrt{(0) + \left(2\sqrt{3}\right)^2} = \sqrt{4(3)} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

$$OB = \sqrt{(3-0)^2 + \left(-\sqrt{3}-0\right)^2} = \sqrt{(3)^2 + \left(\sqrt{3}\right)^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

Therefore, $OA = AB = OB = 2\sqrt{3}$ *units*

Thus, the points $O(0,0)$ $A\left(3,\sqrt{3}\right)$ and $B\left(3,-\sqrt{3}\right)$ are the vertices of an equilateral triangle

Also, the area of the triangle $OAB = \frac{\sqrt{3}}{4} \times (side)^2$

$= \frac{\sqrt{3}}{4} \times \left(2\sqrt{3}\right)^2$

$= \frac{\sqrt{3}}{4} \times 12$

$= 3\sqrt{3}$ square units.

26. Show that the following points are the vertices of a square:
(i) A (3,2), B(0,5), C(-3,2) and D(0,-1)
(ii) A (6,2), B(2,1), C(1,5) and D(5,6)
(iii) A (0,-2), B(3,1), C(0,4) and D(-3,1)

Sol:

(i) The given points are $A(3,2), B(0,5), C(-3,2)$ and $D(0,-1)$.

$$AB=\sqrt{(0-3)^2+(5-2)^2}=\sqrt{(-3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$$BC=\sqrt{(-3-0)^2+(2-5)^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$$CD=\sqrt{(0+3)^2+(-1-2)^2}=\sqrt{(3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$$DA=\sqrt{(0-3)^2+(-1-2)^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

Therefore, $AB=BC=CD=DA=3\sqrt{2}\ units$

Also, $AC=\sqrt{(-3-3)^2+(2-2)^2}=\sqrt{(-6)^2+(0)^2}=\sqrt{36}=6\ units$

$$BD=\sqrt{(0-0)^2+(-1-5)^2}=\sqrt{(0)^2+(-6)^2}=\sqrt{36}=6\ units$$

Thus, diagonal $AC=$ diagonal BD
Therefore, the given points from a square.

(ii) The given points are $A(6,2), B(2,1), C(1,5)\ and\ D(5,6)$

$$AB=\sqrt{(2-6)^2+(1-2)^2}=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}\ units$$

$$BC=\sqrt{(1-2)^2+(5-1)^2}=\sqrt{(-1)^2+(-4)^2}=\sqrt{1+16}=\sqrt{17}\ units$$

$$CD=\sqrt{(5-1)^2+(6-5)^2}=\sqrt{(4)^2+(1)^2}=\sqrt{16+1}=\sqrt{17}\ units$$

$$DA=\sqrt{(5-6)^2+(6-2)^2}=\sqrt{(1)^2+(4)^2}=\sqrt{1+16}=\sqrt{17}\ units$$

Therefore, $AB=BC=CD=DA=17\ units$

Also, $AC=\sqrt{(1-6)^2+(5-2)^2}=\sqrt{(-5)^2+(3)^2}=\sqrt{25+9}=\sqrt{34}\ units$

$$BD=\sqrt{(5-2)^2+(6-1)^2}=\sqrt{(3)^2+(5)^2}=\sqrt{9+25}=\sqrt{34}\ units$$

Thus, diagonal $AC=$ diagonal BD
Therefore, the given points from a square.

(iii) The given points are $P(0,-2), Q(3,1), R(0,4)\ and\ S(-3,1)$

$$PQ=\sqrt{(3-0)^2+(1+2)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$$QR=\sqrt{(0-3)^2+(4-1)^2}=\sqrt{(-3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$RS = \sqrt{(-3-0)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ *units*

$SP = \sqrt{(-3-0)^2 + (1+2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ *units*

Therefore, $PQ = QS = RS = SP = 3\sqrt{2}$ *units*

Also, $PR = \sqrt{(0-0)^2 + (4+2)^2} = \sqrt{(0)^2 + (6)^2} = \sqrt{36} = 6$ *units*

$QS = \sqrt{(-3-3)^2 + (1-1)^2} = \sqrt{(-6)^2 + (0)^2} = \sqrt{36} = 6$ *units*

Thus, diagonal $PR =$ diagonal QS

Therefore, the given points from a square.

27. Show that the points $A(-3,2), B(-5,-5), C(2,-3)$ *and* $D(4,4)$. are the vertices of a rhombus. Find the area of this rhombus

Sol:

The given points are $A(-3,2), B(-5,-5), C(2,-3)$ *and* $D(4,4)$.

$AB = \sqrt{(-5+3)^2 + (-5-2)^2} = \sqrt{(-2)^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$ *units*

$BC = \sqrt{(2+5)^2 + (-3+5)^2} = \sqrt{(7)^2 + (2)^2} = \sqrt{49+4} = \sqrt{53}$ *units*

$CD = \sqrt{(4-2)^2 + (4+3)^2} = \sqrt{(2)^2 + (7)^2} = \sqrt{4+49} = \sqrt{53}$ *units*

$DA = \sqrt{(4+3)^2 + (4-2)^2} = \sqrt{(7)^2 + (2)^2} = \sqrt{49+4} = \sqrt{53}$ *units*

Therefore, $AB = BC = CD = DA = \sqrt{53}$ *units*

Also, $AC = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{(5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = \sqrt{25\times 2} = 5\sqrt{2}$ *units*

$BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{(9)^2 + (9)^2} = \sqrt{81+81} = \sqrt{162} = \sqrt{81\times 2} = 9\sqrt{2}$ *units*

Thus, diagonal AC is not equal to diagonal BD.

Therefore ABCD is a quadrilateral with equal sides and unequal diagonals

Hence, ABCD a rhombus

Area of a rhombus $= \frac{1}{2} \times$ (product of diagonals)

$= \frac{1}{2} \times (5\sqrt{2}) \times (9\sqrt{2})$

$= \frac{45(2)}{2}$

$= 45$ square units.

28. Show that the points A(3,0), B(4,5), C(-1,4) and D(-2,-1) are the vertices of a rhombus. Find its area.

Sol:

The given points are $A(3,0), B(4,5), C(-1,4)$ and $D(-2,-1)$

$AB = \sqrt{(3-4)^2+(0-5)^2} = \sqrt{(-1)^2+(-5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

$BC = \sqrt{(4+1)^2+(5-4)^2} = \sqrt{(5)^2+(1)^2}$

$= \sqrt{25+1} = \sqrt{26}$

$CD = \sqrt{(-1+2)^2+(4+1)^2} = \sqrt{(1)^2+(5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

$AD = \sqrt{(3+2)^2+(0+1)^2} = \sqrt{(5)^2+(1)^2}$

$= \sqrt{25+1} = \sqrt{26}$

$AC = \sqrt{(3+1)^2+(0-4)^2} = \sqrt{(4)^2+(-4)^2}$

$= \sqrt{16+16} = 4\sqrt{2}$

$BD = \sqrt{(4+2)^2+(5+1)^2} = \sqrt{(6)^2+(6)^2}$

$= \sqrt{36+36} = 6\sqrt{2}$

$\because AB = BC = CD = AD = 6\sqrt{2}$ *and* $AC \neq BD$

Therefore, the given points are the vertices of a rhombus

Area $(\Delta ABCD) = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24\, sq.\ units$

Hence, the area of the rhombus is 24 sq. units.

29. Show that the points A(6,1), B(8,2), C(9,4) and D(7,3) are the vertices of a rhombus. Find its area.

Sol:

The given points are $A(6,1), B(8,2), C(9,4)$ and $D(7,3)$.

$AB = \sqrt{(6-8)^2+(1-2)^2} = \sqrt{(-2)^2+(-1)^2}$

$= \sqrt{4+1} = \sqrt{5}$

$BC = \sqrt{(8-9)^2+(2-4)^2} = \sqrt{(-1)^2+(-2)^2}$

$= \sqrt{1+4} = \sqrt{5}$

$CD = \sqrt{(9-7)^2 + (4-3)^2} = \sqrt{(2)^2 + (1)^2}$

$= \sqrt{4+1} = \sqrt{5}$

$AD = \sqrt{(7-6)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2}$

$= \sqrt{1+4} = \sqrt{5}$

$AC = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9} = 3\sqrt{2}$

$BD = \sqrt{(8-7)^2 + (2-3)^2} = \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{1+1} = \sqrt{2}$

$\because AB = BC = CD = AD = \sqrt{5}$ *and* $AC \neq BD$

Therefore, the given points are the vertices of a rhombus. Now

Area $(\Delta ABCD) = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 3\sqrt{2} \times \sqrt{2} = 3$ *sq. units*

Hence, the area of the rhombus is 3 sq. units.

30. Show that the points A(2,1), B(5,2), C(6,4) and D(3,3) are the angular points of a parallelogram. Is this figure a rectangle?

Sol:

The given points are $A(2,1), B(5,2), C(6,4)$ *and* $D(3,3)$

$AB = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$ *units*

$BC = \sqrt{(6-5)^2 + (4-2)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$ *units*

$CD = \sqrt{(3-6)^2 + (3-4)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$ *units*

$AD = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$ *units*

Thus, $AB = CD = \sqrt{10}$ units and $BC = AD = \sqrt{5}$ units

So, quadrilateral $ABCD$ is a parallelogram

Also, $AC = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ *units*

$BD = \sqrt{(3-5)^2 + (3-2)^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$ *units*

But diagonal AC is not equal to diagonal BD.

Hence, the given points do not form a rectangle.

31. Show hat A(1,2), B(4,3),C(6,6) and D(3,5) are the vertices of a parallelogram. Show that ABCD is not rectangle.

Sol:

The given vertices are $A(1,2), B(4,3), C(6,6)$ and $D(3,5)$.

$AB = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2}$

$= \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(4-6)^2 + (3-6)^2} = \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4+9} = \sqrt{13}$

$CD = \sqrt{(6-3)^2 + (6-5)^2} = \sqrt{(3)^2 + (1)^2}$

$= \sqrt{9+1} = \sqrt{10}$

$AD = \sqrt{(1-3)^2 + (2-5)^2} = \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4+9} = \sqrt{13}$

$\because AB = CD = \sqrt{10}$ *units and* $BC = AD = \sqrt{13}$ *units*

Therefore, *ABCD* is a parallelogram

$AC = \sqrt{(1-6)^2 + (2-6)^2} = \sqrt{(-5)^2 + (-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$

$BD = \sqrt{(4-3)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2}$

$= \sqrt{1+4} = \sqrt{5}$

Thus, the diagonal *AC* and *BD* are not equal and hence *ABCD* is not a rectangle

32. Show that the following points are the vertices of a rectangle.

(i) A (-4,-1), B(-2,-4), C(4,0) and D(2,3)

(ii) A (2, -2), B(14,10), C(11,13) and D(-1,1)

(iii) A (0,-4), B(6,2), C(3,5) and D(-3,-1)

Sol:

(i) The given points are $A(-4,-1), B(-2,-4) C(4,0)$ *and* $D(2,3)$

$AB = \sqrt{\{-2-(-4)\}^2 + \{-4-(-1)\}^2} = \sqrt{(2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$ *units*

$BC = \sqrt{\{4-(-2)\}^2 + \{0-(-4)\}^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ *units*

$CD = \sqrt{(2-4)^2 + (3-0)^2} = \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$ *units*

$AD = \sqrt{\{2-(-4)\}^2 + \{3-(-1)\}^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ *units*

Thus, $AB = CD = \sqrt{13}$ *units and* $BC = AD = 2\sqrt{13}$ *units*

Also, $AC = \sqrt{\{4-(-4)\}^2 + \{0-(-1)\}^2} = \sqrt{(8)^2 + (1)^2} = \sqrt{64+1} = \sqrt{65}$ *units*

$BD = \sqrt{\{2-(-2)\}^2 + \{3-(-4)\}^2} = \sqrt{(4)^2 + (7)^2} = \sqrt{16+49} = \sqrt{65}$ *units*

Also, diagonal AC = diagonal BD

Hence, the given points form a rectangle

(ii) The given points are $A(2,-2), B(14,10) C(11,13)$ *and* $D(-1,1)$

$AB = \sqrt{(14-2)^2 + \{10-(-2)\}^2} = \sqrt{(12)^2 + (12)^2} = \sqrt{144+144} = \sqrt{288} = 12\sqrt{2}$ *units*

$BC = \sqrt{(11-14)^2 + (13-10)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ *units*

$CD = \sqrt{(-1-11)^2 + (1-13)^2} = \sqrt{(-12)^2 + (-12)^2} = \sqrt{144+144} = \sqrt{288} = 12\sqrt{2}$ *units*

$AD = \sqrt{(-1-2)^2 + \{1-(-2)\}^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ *units*

Thus, $AB = CD = 12\sqrt{2}$ *units and* $BC = AD = 3\sqrt{2}$ *units*

Also,

$AC = \sqrt{(11-2)^2 + \{13-(-2)\}^2} = \sqrt{(9)^2 + (15)^2} = \sqrt{81+225} = \sqrt{306} = 3\sqrt{34}$ *units*

$BD = \sqrt{(-1-14)^2 + (1-10)^2} = \sqrt{(-15)^2 + (-9)^2} = \sqrt{81+225} = \sqrt{306} = 3\sqrt{34}$ *units*

Also, diagonal AC = diagonal BD

Hence, the given points from a rectangle

(iii) The given points are $A(0,-4), B(6,2) C(3,5)$ *and* $D(-3,-1)$.

$AB = \sqrt{(6-0)^2 + \{2-(-4)\}^2} = \sqrt{(6)^2 + (6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ *units*

$BC = \sqrt{(3-6)^2 + (5-2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ *units*

$CD = \sqrt{(-3-3)^2 + (-1-5)^2} = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ *units*

$AD = \sqrt{(-3-0)^2 + \{-1-(-4)\}^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ *units*

Thus, AB = CD = $\sqrt{10}$ units and BC = AD = $\sqrt{5}$ units

Also, $AC = \sqrt{(3-0)^2 + \{5-(-4)\}^2} = \sqrt{(3)^2 + (9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$ *units*

$BD = \sqrt{(-3-6)^2 + (-1-2)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$ *units*

Also, diagonal AC = diagonal BD

Hence, the given points from a rectangle

Exercise – 16B

1. Find the coordinates of the point which divides the join of $A(-1,7)$ *and* $B(4,-3)$. in the ratio $2:3$

Sol:

The end points of AB are $A(-1,7)$ *and* $B(4,-3)$.

Therefore, $(x_1=-1, y_1=7)$ *and* $(x_2=4, y_2=-3)$

Also, $m=2$ *and* $n=3$

Let the required point be $P(x,y)$.

By section formula, we get

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{\{2\times 4+3\times(-1)\}}{2+3}, y=\frac{\{2\times(-3)+3\times 7\}}{2+3}$$

$$\Rightarrow x=\frac{8-3}{5}, y=\frac{-6+21}{5}$$

$$\Rightarrow x=\frac{5}{5}, y=\frac{15}{5}$$

Therefore, $x=1$ *and* $y=3$

Hence, the coordinates of the required point are $(1,3)$.

2. Find the co-ordinates of the point which divides the join of A(-5, 11) and B(4,-7) in the ratio 7 : 2

Sol:

The end points of AB are $A(-5,11)$ *and* $B(4,-7)$.

Therefore, $(x_1=-5, y_1=11)$ *and* $(x_2=4, y_2=-7)$

Also, $m=7$ *and* $n=2$

Let the required point be $P(x,y)$.

By section formula, we get

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{\{7\times 4+2\times(-5)\}}{7+2}, y=\frac{\{7\times(-7)+2\times 11\}}{7+2}$$

$$\Rightarrow x=\frac{28-10}{9}, y=\frac{-49+22}{9}$$

$\Rightarrow x = \frac{18}{9}, y = -\frac{27}{9}$

Therefore, $x = 2$ *and* $y = -3$

Hence, the required point are $P(2, -3)$.

3. If the coordinates of points A and B are (-2, -2) and (2, -4) respectively. Find the coordinates of the point P such that AP=$\frac{3}{7}$ AB, where P lies on the segment AB.

Sol:

The coordinates of the points A and $B$are $(-2, -2)$ and $(2, -4)$ respectively, where

$AP = \frac{3}{7} AB$ *and* P lies on the line segment AB. So

$AP + BP = AB$

$\Rightarrow AP + BP = \frac{7AP}{3}$ $\quad \because AP = \frac{3}{7} AB$

$\Rightarrow BP = \frac{7AP}{3} - AP$

$\Rightarrow \frac{AP}{BP} = \frac{3}{4}$

Let (x, y) be the coordinates of P which divides AB in the ratio 3 : 4 internally Then

$x = \frac{3 \times 2 + 4 \times (-2)}{3+4} = \frac{6-8}{7} = -\frac{2}{7}$

$y = \frac{3 \times (-4) + 4 \times (-2)}{3+4} = \frac{-12-8}{7} = -\frac{20}{7}$

Hence, the coordinates of point $P$are$\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

4. Point A lies on the line segment PQ joining P(6, -6) and Q(-4, -1) in such a way that $\frac{PA}{PQ} = \frac{2}{5}$. If that point A also lies on the line 3x + k(y + 1) = 0, find the value of k.

Sol:

Let the coordinates of A be (x, y). Here $\frac{PA}{PQ} = \frac{2}{5}$. So,

$PA + AQ = PQ$

$\Rightarrow PA + AQ = \frac{5PA}{2}$ $\quad \left[\because PA = \frac{2}{5} PQ\right]$

$$\Rightarrow AQ = \frac{5PA}{2} - PA$$

$$\Rightarrow \frac{AQ}{PA} = \frac{3}{2}$$

$$\Rightarrow \frac{PA}{AQ} = \frac{2}{3}$$

Let (x, y) be the coordinates of A, which dives PQ in the ratio 2 : 3 internally Then using section formula, we get

$$x = \frac{2\times(-4)+3\times(6)}{2+3} = \frac{-8+18}{5} = \frac{10}{5} = 2$$

$$y = \frac{2\times(-1)+3\times(-6)}{2+3} = \frac{-2-18}{5} = \frac{-20}{5} = -4$$

Now, the point $(2,-4)$ lies on the line $3x+k(y+1)=0$, therefore

$$3\times 2+k(-4+1)=0$$

$$\Rightarrow 3k = 6$$

$$\Rightarrow k = \frac{6}{3} = 2$$

Hence, $k = 2$.

5. Points P, Q, R and S divide the line segment joining the points A(1,2) and B(6,7) in five equal parts. Find the coordinates of the points P,Q and R

Sol:

Since, the points P, Q, R and S divide the line segment joining the points $A(1,2)$ *and* $B(6,7)$ in five equal parts, so

$$AP = PQ = QR = R = SB$$

Here, point P divides AB in the ratio of 1 : 4 internally So using section formula, we get

$$\text{Coordinates of } P = \left(\frac{1\times(6)+4\times(1)}{1+4}, \frac{1\times(7)+4\times(2)}{1+4}\right)$$

$$= \left(\frac{6+4}{5}, \frac{7+8}{5}\right) = (2,3)$$

The point Q divides AB in the ratio of 2 : 3 internally. So using section formula, we get

$$\text{Coordinates of } Q = \left(\frac{2\times(6)+3\times(1)}{2+3}, \frac{2\times(7)+3\times(2)}{2+3}\right)$$

$$= \left(\frac{12+3}{5}, \frac{14+6}{5}\right) = (3,4)$$

The point R divides AB in the ratio of 3 : 2 internally So using section formula, we get

Coordinates of $R=\left(\frac{3\times(6)+2\times(1)}{3+2},\frac{3\times(7)+2\times(2)}{3+2}\right)$

$=\left(\frac{18+2}{5},\frac{21+4}{5}\right)=(4,5)$

Hence, the coordinates of the points *P, Q* and R are $(2,3),(3,4)$ and $(4,5)$ respectively

6. Points P, Q, and R in that order are dividing line segment joining A (1,6) and B(5, -2) in four equal parts. Find the coordinates of P, Q and R.

Sol:

The given points are $A(1,6)$ and $B(5,-2)$.

Then, $P(x,y)$ is a point that devices the line AB in the ratio $1:3$

By the section formula:

$x=\frac{(mx_2+nx_1)}{(m+n)},y=\frac{(my_2+ny_1)}{(m+n)}$

$\Rightarrow x=\frac{(1\times5+3\times1)}{1+3},y=\frac{(1\times(-2)+3\times6)}{1+3}$

$\Rightarrow x=\frac{5+3}{4},y=\frac{-2+18}{4}$

$\Rightarrow x=\frac{8}{4},y=\frac{16}{4}$

$\Rightarrow x=2\ and\ \ y=4$

Therefore, the coordinates of point P are $(2,4)$

Let Q be the mid-point of AB

Then, $Q(x,y)$

$x=\frac{x_1+x_2}{2},y=\frac{y_1+y_2}{2}$

$\Rightarrow x=\frac{1+5}{2},y=\frac{6+(-2)}{2}$

$\Rightarrow x=\frac{6}{2},y=\frac{4}{2}$

$\Rightarrow x=3,y=2$

Therefore, the coordinates of Q are $(3,2)$

Let $R(x,y)$ be a point that divides AB in the ratio $3:1$

Then, by the section formula:

$$x = \frac{(mx_2 + nx_1)}{(m+n)}, y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$\Rightarrow x = \frac{(3\times5+1\times1)}{3+1}, y = \frac{(3\times(-2)+1\times6)}{3+1}$$

$$\Rightarrow x = \frac{15+1}{4}, y = \frac{-6+6}{4}$$

$$\Rightarrow x = \frac{16}{4}, y = \frac{0}{4}$$

$$\Rightarrow x = 4 \text{ and } y = 0$$

Therefore, the coordinates of R are $(4,0)$.

Hence, the coordinates of point P, Q and R are $(2,4),(3,2)$ *and* $(4,0)$ respectively.

7. The line segment joining the points A(3,-4) and B(1,2) is trisected at the points P(p, -2) and $Q\left(\frac{5}{3},q\right)$. Find the values of p and q.

Sol:
Let P and Q be the points of trisection of AB.
Then, P divides AB in the radio 1:2
So, the coordinates of P are

$$x = \frac{(mx_2 + nx_1)}{(m+n)}, y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$\Rightarrow x = \frac{\{1\times1+2\times(3)\}}{1+2}, y = \frac{\{1\times2+2\times(-4)\}}{1+2}$$

$$\Rightarrow x = \frac{1+6}{3}, y = \frac{2-8}{3}$$

$$\Rightarrow x = \frac{7}{3}, y = -\frac{6}{3}$$

$$\Rightarrow x = \frac{7}{3}, y = -2$$

Hence, the coordinates of P are $\left(\frac{7}{3},-2\right)$

But $(p,-2)$ are the coordinates of P.

So, $p = \frac{7}{3}$

Also, Q divides the line AB in the ratio 2:1
So, the coordinates of Q are

$$x=\frac{(mx_2+mx_1)}{(m+n)}, y=\frac{(my_2+my_1)}{(m+n)}$$
$$\Rightarrow x=\frac{(2\times1+1\times3)}{2+1}, y=\frac{\{2\times2+1\times(-4)\}}{2+1}$$
$$\Rightarrow x=\frac{2+3}{3}, y=\frac{4-4}{3}$$
$$\Rightarrow x=\frac{5}{3}, y=0$$

Hence, coordinates of Q are $\left(\frac{5}{3},0\right)$.

But the given coordinates of Q are $\left(\frac{5}{3},q\right)$.

So, $q=0$

Thus, $p=\frac{7}{3}$ *and* $q=0$

8. Find the coordinates of the midpoints of the line segment joining
(i) A(3,0) and B(-5, 4) (ii) P(-11,-8) and Q(8,-2)

Sol:

(i) The given points are $A(3,0)$ *and* $B(-5,4)$.

Let (x,y) be the midpoint of AB. Then:

$$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$$
$$\Rightarrow x=\frac{3+(-5)}{2}, y=\frac{0+4}{2}$$
$$\Rightarrow x=\frac{-2}{2}, y=\frac{4}{2}$$
$$\Rightarrow x=-1, y=2$$

Therefore, $(-1,2)$.are the coordinates of midpoint of AB.

(ii) The given points are $P(-11,-8)$ *and* $Q(8,-2)$.

Let (x,y) be the midpoint of PQ. Then:

$$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$$
$$\Rightarrow x=\frac{-11+8}{2}, y=\frac{-8--2}{2}$$

$\Rightarrow x=-\frac{3}{2}, y=-\frac{10}{2}$

$\Rightarrow x=-\frac{3}{2}, y=-5$

Therefore, $\left(-\frac{3}{2},-5\right)$ are the coordinates of midpoint of PQ.

9. If (2, p) is the midpoint of the line segment joining the points A(6, -5) and B(-2,11) find the value of p.

Sol:

The given points are $A(6,-5)$ *and* $B(-2,11)$.

Let (x, y) be the midpoint of AB. Then,

$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$

$\Rightarrow x=\frac{6+(-2)}{2}, y=\frac{-5+11}{2}$

$\Rightarrow x=\frac{6-2}{2}, y=\frac{-5+11}{2}$

$\Rightarrow x=\frac{4}{2}, y=\frac{6}{2}$

$\Rightarrow x=2, y=3$

So, the midpoint of AB is $(2,3)$.

But it is given that midpoint of AB is $(2, p)$.

Therefore, the value of $p=3$.

10. The midpoint of the line segment joining A (2a, 4) and B (-2, 3b) is C (1, 2a+1). Find the values of a and b.

Sol:

The points are $A(2a,4)$ *and* $B(-2,3b)$.

Let $C(1,2a+1)$ be the mid-point of AB. Then:

$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$

$\Rightarrow 1=\frac{2a(-2)}{2}, 2a+1=\frac{4+3b}{2}$

$\Rightarrow 2=2a-2, 4a+2=4+3b$

$\Rightarrow 2a=2+2, 4a-3b=4-2$

$\Rightarrow a=\frac{4}{2}, 4a-3b=2$

$\Rightarrow a=2, 4a-3b=2$

Putting the value of a in the equation $4a+3b=2$, we get:

$4(2)-3b=2$

$\Rightarrow -3b=2-8=-6$

$\Rightarrow b=\frac{6}{3}=2$

Therefore, $a=2$ and $b=2$.

11. The line segment joining $A(-2,9)$ and $B(6,3)$ is a diameter of a circle with center C. Find the coordinates of C.

Sol:

The given points are $A(-2,9)$ *and* $B(6,3)$

Then, $C(x,y)$ is the midpoint of *AB*.

$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$

$\Rightarrow x=\frac{-2+6}{2}, y=\frac{9+3}{2}$

$\Rightarrow x=\frac{4}{2}, y=\frac{12}{2}$

$\Rightarrow x=2, y=6$

Therefore, the coordinates of point C are $(2,6)$.

12. Find the coordinates of a point *A*, where *AB* is a diameter of a circle with center $C(2,-3)$ and the other end of the diameter is $B(1,4)$.

Sol:

$C(2,-3)$ is the center of the given circle. Let $A(a,b)$ *and* $B(1,4)$ be the two end-points of the given diameter *AB*. Then, the coordinates of C are

$x=\frac{a+1}{2}, y=\frac{b+4}{2}$

It is given that $x=2$ and $y=-3$.

$\Rightarrow 2=\frac{a+1}{2}, -3=\frac{b+4}{2}$

$\Rightarrow 4=a+1, -6=b+4$

$\Rightarrow a=4-1, b=-6-4$

$\Rightarrow a = 3, b = -10$

Therefore, the coordinates of point A are $(3, -10)$.

13. In what ratio does the point P(2,5) divide the join of A (8,2) and B(-6, 9)?

Sol:

Let the point $P(2,5)$ divide AB in the ratio $k:1$.

Then, by section formula, the coordinates of P are

$$x = \frac{-6k+8}{k+1}, y = \frac{9k+2}{k+1}$$

It is given that the coordinates of P are $(2,5)$.

$$\Rightarrow 2 = \frac{-6k+8}{k+1}, 5 = \frac{9k+2}{k+1}$$

$$\Rightarrow 2k + 2 = -6k + 8, 5k + 5 = 9k + 2$$

$$\Rightarrow 2k + 6k = 8 - 2, 5 - 2 = 9k - 5k$$

$$\Rightarrow 8k = 6, 4k = 3$$

$$\Rightarrow k = \frac{6}{8}, k = \frac{3}{4}$$

$\Rightarrow k = \frac{3}{4}$ in each case..

Therefore, the point $P(2,5)$ divides AB in the ratio $3:4$

14. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, -5).

Sol:

Let $k:1$ be the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ *and* $(2, -5)$. Then

$$\left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{k(2) + \frac{1}{2}}{k+1}, \frac{k(-5) + \frac{2}{2}}{k+1}\right)$$

$$\Rightarrow \frac{k(2) + \frac{1}{2}}{k+1} = \frac{3}{4} \text{ and } \frac{k(-5) + \frac{3}{2}}{k+1} = \frac{5}{12}$$

$\Rightarrow 8k+2=3k+3 \ and \ -60k+18=5k+5$

$\Rightarrow k=\frac{1}{5} \ and \ k=\frac{1}{5}$

Hence, the required ratio is $1:5$.

15. Find the ratio in which the point P(m, 6) divides the join of A(-4, 3) and B(2, 8) Also, find the value of m.

Sol:

Let the point $P(m,6)$ divide the line AB in the ratio $k:1$.

Then, by the section formula:

$x=\frac{mx_2+nx_1}{m+n}, y=\frac{my_2+ny_1}{m+n}$

The coordinates of P are $(m,6)$.

$m=\frac{2k-4}{k+1}, 6=\frac{8k+3}{k+1}$

$\Rightarrow m(k+1)=2k-4, 6k+6=8k+3$

$\Rightarrow m(k+1)=2k-4, 6-3=8k-6k$

$\Rightarrow m(k+1)=2k-4, 2k=3$

$\Rightarrow m(k+1)=2k-4, k=\frac{3}{2}$

Therefore, the point P divides the line AB in the ratio $3:2$

Now, putting the value of k in the equation $m(k+1)=2k-4$, we get:

$m\left(\frac{3}{2}+1\right)=2\left(\frac{3}{2}\right)-4$

$\Rightarrow m\left(\frac{3+2}{2}\right)=3-4$

$\Rightarrow \frac{5m}{2}=-1 \Rightarrow 5m=-2 \Rightarrow m=-\frac{2}{5}$

Therefore, the value of $m=-\frac{2}{5}$

So, the coordinates of P are $\left(-\frac{2}{5},6\right)$.

16. Find the ratio in which the pint (-3, k) divide the join of A(-5, -4) and B(-2, 3),Also, find the value of k.

Sol:

Let the point $P(-3,k)$ divide the line AB in the ratio $s : 1$

Then, by the section formula:

$$x=\frac{mx_1+nx_1}{m+n}, y=\frac{my_2+ny_1}{m+n}$$

The coordinates of P are $(-3,k)$.

$$-3=\frac{-2s-5}{s+1}, k=\frac{3s-4}{s+1}$$

$$\Rightarrow -3s-3=-2s-5, k(s+1)=3s-4$$

$$\Rightarrow -3s+2s=-5+3, k(s+1)=3s-4$$

$$\Rightarrow -s=-2, k(s+1)=3s-4$$

$$\Rightarrow s=2, k(s+1)=3s-4$$

Therefore, the point P divides the line AB in the ratio 2 : 1.

Now, putting the value of s in the equation $k(s+1)=3s-4,$ we get:

$$k(2+1)=3(2)-4$$

$$\Rightarrow 3k=6-4$$

$$\Rightarrow 3k=2\Rightarrow k=\frac{2}{3}$$

Therefore, the value of $k=\frac{2}{3}$

That is, the coordinates of P are $\left(-3,\frac{2}{3}\right)$.

17. In what ratio is the line segment joining A(2, -3) and B(5, 6) divide by the x-axis? Also, find the coordinates of the pint of division.

Sol:

Let AB be divided by the x-axis in the ratio $k:1$ at the point P.

Then, by section formula the coordination of P are

$$P=\left(\frac{5k+2}{k+1},\frac{6k-3}{k+1}\right)$$

But P lies on the x-axis; so, its ordinate is 0.

Therefore, $\frac{6k-3}{k+1}=0$

$$\Rightarrow 6k-3=0\Rightarrow 6k=3\Rightarrow k=\frac{3}{6}\Rightarrow k=\frac{1}{2}$$

Therefore, the required ratio is $\frac{1}{2}:1,$ which is same as 1 : 2

Thus, the x-axis divides the line AB li the ratio 1 : 2 at the point P.

Applying $k=\frac{1}{2}$, we get the coordinates of point.

$$P\left(\frac{5k+1}{k+1},0\right)$$

$$=P\left(\frac{5\times\frac{1}{2}+2}{\frac{1}{2}+1},0\right)$$

$$=p\left(\frac{\frac{5+4}{2}}{\frac{5+2}{2}},0\right)$$

$$=P\left(\frac{9}{3},0\right)$$

$$=P(3,0)$$

Hence, the point of intersection of AB and the x-axis is $P(3,0)$

18. In what ratio is the line segment joining the points A(-2, -3) and B(3,7) divided by the y-axis? Also, find the coordinates of the point of division.

Sol:

Let AB be divided by the x-axis in the ratio $k:1$ at the point P.

Then, by section formula the coordination of P are

$$P=\left(\frac{3k-2}{k+1},\frac{7k-3}{k+1}\right)$$

But P lies on the y-axis; so, its abscissa is 0.

Therefore, $\frac{3k-2}{k+1}=0$

$$\Rightarrow 3k-2=0\Rightarrow 3k=2\Rightarrow k=\frac{2}{3}\Rightarrow k=\frac{2}{3}$$

Therefore, the required ratio is $\frac{2}{3}:1$, which is same as 2 : 3

Thus, the x-axis divides the line AB in the ratio 2 : 3 at the point P.

Applying $k=\frac{2}{3}$, we get the coordinates of point.

$$P\left(0,\frac{7k-3}{k+1}\right)$$

$$= P\left(0, \frac{7\times\frac{2}{3}-3}{\frac{2}{3}+1}\right)$$

$$= p\left(0, \frac{\frac{14-9}{3}}{\frac{2+3}{3}}\right)$$

$$= P\left(0, \frac{5}{5}\right)$$

$$= P(0,1)$$

Hence, the point of intersection of AB and the x-axis is $P(0,1)$.

19. In what ratio does the line $x-y-2=0$ divide the line segment joining the points $A(3,-1)$ *and* $B(8,9)$?

Sol:

Let the line $x-y-2=0$ divide the line segment joining the points $A(3,-1)$ and $B(8,9)$ in the ratio $k:1$ at P.

Then, the coordinates of P are

$$P\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$$

Since, P lies on the line $x-y-2=0$, we have:

$$\left(\frac{8k+3}{k+1}\right)-\left(\frac{9k-1}{k+1}\right)-2=0$$

$$\Rightarrow 8k+3-9k+1-2k-2=0$$

$$\Rightarrow 8k-9k-2k+3+1-2=0$$

$$\Rightarrow -3k+2=0$$

$$\Rightarrow -3k=-2$$

$$\Rightarrow k=\frac{2}{3}$$

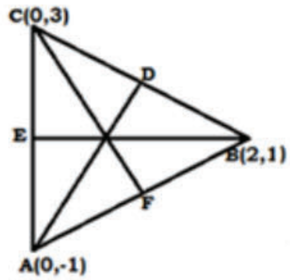
So, the required ratio is $\frac{2}{3}:1$, which is equal to $2:3$.

20. Find the lengths of the medians of a ΔABC whose vertices are $A(0,-1), B(2,1)$ and $C(0,3)$.

Sol:

The vertices of ΔABC are $A(0,-1), B(2,1)$ and $C(0,3)$.

Let AD, BE and CF be the medians of ΔABC.



Let D be the midpoint of BC. So, the coordinates of D are

$D\left(\frac{2+0}{2},\frac{1+3}{2}\right)$ i.e. $D\left(\frac{2}{2},\frac{4}{2}\right)$ i.e. $D(1,2)$

Let E be the midpoint of AC. So the coordinate of E are

$E\left(\frac{0+0}{2},\frac{-1+3}{2}\right)$ i.e. $E\left(\frac{0}{2},\frac{0}{2}\right)$ i.e. $E(0,1)$

Let F be the midpoint of AB. So, the coordinates of F are

$F\left(\frac{0+2}{2},\frac{-1+1}{2}\right)$ i.e. $F\left(\frac{2}{2},\frac{0}{2}\right)$ i.e. $F(1,0)$

$AD=\sqrt{(1-0)^2+(2-(-1))^2}=\sqrt{(1)^2+(3)^2}=\sqrt{1+9}=\sqrt{10}$ *units.*

$BE=\sqrt{(0-2)^2+(1-1)^2}=\sqrt{(-2)^2+(0)^2}=\sqrt{4+0}=\sqrt{4}=2$ *units.*

$CF=\sqrt{(1-0)^2+(0-3)^2}=\sqrt{(1)^2+(-3)^2}=\sqrt{1+9}=\sqrt{10}$ *units.*

Therefore, the lengths of the medians: $AD=\sqrt{10}$ *units*, $BE=2$ *units and* $CF=\sqrt{10}$ *units.*

21. Find the centroid of ΔABC whose vertices are A(-1, 0) B(5, -2) and C(8,2)

Sol:

Here, $(x_1=-1, y_1=0), (x_2=5, y_2=-2)$ *and* $(x_3=8, y_3=2)$

Let $G(x,y)$ be the centroid of the ΔABC. Then,

$$x=\frac{1}{3}(x_1+x_2+x_3)=\frac{1}{3}(-1+5+8)=\frac{1}{3}(12)=4$$

$$y=\frac{1}{3}(y_1+y_2+y_3)=\frac{1}{3}(0-2+2)=\frac{1}{3}(0)=0$$

Hence, the centroid of ΔABC is $G(4,0)$.

22. If G(-2, 1) is the centroid of a ΔABC and two of its vertices are A(1, -6) and B(-5, 2) , find the third vertex of the triangle.

Sol:

Two vertices of ΔABC are $A(1,-6)$ and $B(-5,2)$. Let the third vertex be C(a, b).

Then the coordinates of its centroid are

$$C\left(\frac{1-5+a}{3},\frac{-6+2+b}{3}\right)$$

$$C\left(\frac{-4+a}{3},\frac{-4+b}{3}\right)$$

But it is given that $G(-2,1)$ is the centroid. Therefore,

$$-2=\frac{-4+a}{3},1=\frac{-4+b}{3}$$

$$\Rightarrow -6=-4+a,3=-4+b$$

$$\Rightarrow -6+4=a,3+4=b$$

$$\Rightarrow a=-2,b=7$$

Therefore, the third vertex of ΔABC is $C(-2,7)$.

23. Find the third vertex of a ΔABC if two of its vertices are $B(-3,1)$ and $C(0,-2)$, and its centroid is at the origin

Sol:

Two vertices of ΔABC are $B(-3,1)$ and $C(0,-2)$. Let the third vertex be $A(a, b)$.

Then, the coordinates of its centroid are

$$\left(\frac{-3+0+a}{3},\frac{1-2+b}{3}\right)$$

i.e., $\left(\frac{-3+a}{3},\frac{-1+b}{3}\right)$

But it is given that the centroid is at the origin, that is $G(0,0)$. Therefore

$$0=\frac{-3+a}{3},0=\frac{-1+b}{3}$$

$$\Rightarrow 0=-3+a,0=-1+b$$

$$\Rightarrow 3=a,1=b$$

$$\Rightarrow a=3,b=1$$

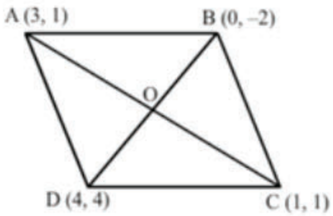
Therefore, the third vertex of ΔABC is $A(3,1)$.

24. Show that the points $A(3,1), B(0,-2), C(1,1)$ and $D(4,4)$ are the vertices of parallelogram ABCD.

Sol:

The points are $A(3,1), B(0,-2), C(1,1)$ and $D(4,4)$

Join AC and BD, intersecting at O.



We know that the diagonals of a parallelogram bisect each other.

Midpoint of $AC = \left(\frac{3+1}{2}, \frac{1+1}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$

Midpoint of $BD = \left(\frac{0+4}{2}, \frac{-2+4}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$

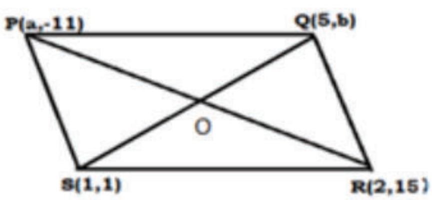
Thus, the diagonals AC and BD have the same midpoint

Therefore, $ABCD$ is a parallelogram.

25. If the points $P(a,-11), Q(5,b), R(2,15)$ *and* $S(1,1)$. are the vertices of a parallelogram PQRS, find the values of a and b.

Sol:

The points are $P(a,-11), Q(5,b), R(2,15)$ *and* $S(1,1)$.



Join PR and QS, intersecting at O.

We know that the diagonals of a parallelogram bisect each other

Therefore, O is the midpoint of PR as well as QS.

Midpoint of $PR = \left(\frac{a+2}{2}, \frac{-11+15}{2}\right) = \left(\frac{a+2}{2}, \frac{4}{2}\right) = \left(\frac{a+2}{2}, 2\right)$

Midpoint of $QS = \left(\frac{5+1}{2}, \frac{b+1}{2}\right) = \left(\frac{6}{2}, \frac{b+1}{2}\right) = \left(3, \frac{b+1}{2}\right)$

Therefore, $\frac{a+2}{2} = 3, \frac{b+1}{2} = 2$

$\Rightarrow a+2=6, b+1=4$

$\Rightarrow a=6-2, b=4-1$

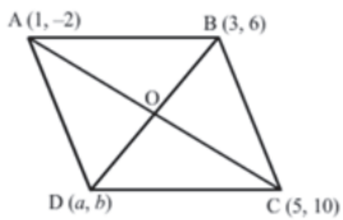
$\Rightarrow a=4 \text{ and } b=3$

26. If three consecutive vertices of a parallelogram $ABCD$ are $A(1,-2), B(3,6) \text{ and } C(5,10)$, find its fourth vertex *D*.

Sol:

Let $A(1,-2), B(3,6) \text{ and } C(5,10)$ be the three vertices of a parallelogram *ABCD* and the fourth vertex be $D(a,b)$.

Join *AC* and *BD* intersecting at *O*.



We know that the diagonals of a parallelogram bisect each other

Therefore, *O* is the midpoint of *AC* as well as *BD*.

Midpoint of $AC = \left(\frac{1+5}{2}, \frac{-2+10}{2}\right) = \left(\frac{6}{2}, \frac{8}{2}\right) = (3,4)$

Midpoint of $BD = \left(\frac{3+a}{2}, \frac{6+b}{2}\right)$

Therefore, $\frac{3+a}{2} = 3$ and $\frac{6+b}{2} = 4$

$\Rightarrow 3+a=6 \text{ and } 6+b=8$

$\Rightarrow a=6-3 \text{ and } b=8-6$

$\Rightarrow a=3 \text{ and } b=2$

Therefore, the fourth vertex is $D(3,2)$.

27. In what ratio does y-axis divide the line segment joining the points (-4, 7) and (3, -7)?

Sol:

Let *y*-axis divides the e segment pining the points $(-4,7)$ and $(3,-7)$ in the ratio $k:1$. Then

$0 = \frac{3k-4}{k+1}$

$\Rightarrow 3k = 4$

$\Rightarrow k = \frac{4}{3}$

Hence, the required ratio is $4:3$

28. If the point $P\left(\frac{1}{2}, y\right)$ lies on the line segment joining the points A(3, -5) and B(-7, 9) then find the ratio in which P divides AB. Also, find the value of y.

Sol:

Let the point $P\left(\frac{1}{2}, y\right)$ divides the line segment joining the points $A(3,-5)$ *and* $B(-7,9)$ in the ratio $k:1$. Then

$$\left(\frac{1}{2}, y\right)=\left(\frac{k(-7)+3}{k+1}, \frac{k(9)-3}{k+1}\right)$$

$$\Rightarrow \frac{-7k+3}{k+1}=\frac{1}{2} \text{ and } \frac{9k-5}{k+1}=y$$

$$\Rightarrow k+1=-14k+6 \Rightarrow k=\frac{1}{3}$$

Now, substituting $k=\frac{1}{3}$ in $\frac{9k-5}{k+1}=y$, we get

$$\frac{\frac{9}{3}-5}{\frac{1}{3}+1}=y \Rightarrow y=\frac{9-15}{1+3}=-\frac{3}{2}$$

Hence, required ratio is $1:3$ and $y=-\frac{3}{2}$.

29. Find the ratio which the line segment joining the pints A(3, -3) and B(-2,7) is divided by x-axis Also, find the point of division.

Sol:

The line segment joining the points $A(3,-3)$ *and* $B(-2,7)$ is divided by x-axis. Let the required ratio be $k:1$. So,

$$0=\frac{k(7)-3}{k+1} \Rightarrow k=\frac{3}{7}$$

Now,

$$\text{Point of division}=\left(\frac{k(-2)+3}{k+1}, \frac{k(7)-3}{k+1}\right)$$

$$=\left(\frac{\frac{3}{7}\times(-2)+3}{\frac{3}{7}+1}, \frac{\frac{3}{7}\times(7)-3}{\frac{3}{7}+1}\right) \qquad \left(\because k=\frac{3}{7}\right)$$