Exercise 8(B)

Solution 1:

(v)

$$\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}$$

$$=\log\frac{75}{16}-\log\left(\frac{5}{9}\right)^2+\log\frac{32}{243} \quad [n\log_a m=\log_a m^n]$$

$$=\log\frac{75}{16}-\log\frac{5}{9}\times\frac{5}{9}+\log\frac{32}{243}$$

$$=\log\frac{75}{16}-\log\frac{25}{81}+\log\frac{32}{243}$$

$$=\log\left(\frac{\frac{75}{16}}{\frac{25}{81}}\right) \qquad [\log_a m-\log_a n=\log_a\frac{m}{n}]$$

$$=\log\frac{75}{16}\times\frac{81}{25}+\log\frac{32}{243}$$

$$=\log\frac{3\times25}{16}\times\frac{81}{25}+\log\frac{32}{243}$$

$$=\log\frac{3\times81}{16}+\log\frac{32}{243}$$

$$=\log\frac{243}{16}+\log\frac{32}{243}$$

$$=\log\frac{243}{16}\times\frac{32}{243} \qquad [\log_a m+\log_a n=\log_a mn]$$

$$=\log\frac{32}{16}$$

$$=\log 2$$

Solution 2:

(i)

Consider the given equation

$2\log x - \log y = 1$

$\Rightarrow \log x^2 - \log y = 1$

$\Rightarrow \log \frac{x^2}{y} = \log 10$

$\Rightarrow \frac{x^2}{y} = 10$

$\Rightarrow x^2 = 10y$

(ii)

Consider the given equation

$2\log x + 3\log y = \log a$

$\Rightarrow \log x^2 + \log y^3 = \log a$

$\Rightarrow \log x^2 y^3 = \log a$

$\Rightarrow x^2 y^3 = a$

(iii)

Consider the given equation

$a\log x - b\log y = 2\log 3$

$\Rightarrow \log x^a - \log y^b = \log 3^2$

$\Rightarrow \log \frac{x^a}{y^b} = \log 9$

$\Rightarrow \frac{x^a}{y^b} = 9$

$\Rightarrow x^a = 9y^b$

Solution 3:

(i) Consider the given expression

$\log 5+\log 8-2\log 2=\log 5+\log 8\times 8-\log 2^2$ $[n\log_a m=\log_a m^n]$

$=\log 5\times 8-\log 2^2$ $[\log_a m+\log_a n=\log_a mn]$

$=\log 40-\log 4$

$=\log\frac{40}{4}$ $[\log_a m-\log_a n=\log_a\frac{m}{n}]$

$=\log 10$

$=1$

(ii) Consider the given expression

$\log_{10}8+\log_{10}25+2\log_{10}3-\log_{10}18$

$=\log_{10}8+\log_{10}25+\log_{10}3^2-\log_{10}18$

$[n\log_a m=\log_a m^n]$

$=\log_{10}8+\log_{10}25+\log_{10}9-\log_{10}18$

$=\log_{10}8\times 25\times 9-\log_{10}18$

$[\log_a \ell+\log_a m+\log_a n=\log_a \ell mn]$

$=\log_{10}1800-\log_{10}18$

$=\log_{10}\frac{1800}{18}$ $[\log_a m-\log_a n=\log_a\frac{m}{n}]$

$=\log_{10}100$

$=2$ $[\because \log_{10}100=2]$

(iii) Consider the given expression

$\log 4+\frac{1}{3}\log 125-\frac{1}{5}\log 32$

$=\log 4+\log(125)^{\frac{1}{3}}-\log(32)^{\frac{1}{5}}$ $[n\log_a m=\log_a m^n]$

$=\log 4+\log(5^3)^{\frac{1}{3}}-\log(2^5)^{\frac{1}{5}}$

$=\log 4+\log 5-\log 2$

$=\log 4\times 5-\log 2$ $[\log_a m+\log_a n=\log_a mn]$

$=\log\frac{20}{2}$ $[\log_a m-\log_a n=\log_a\frac{m}{n}]$

$=\log 10$

$=1$

Solution 4:

We need to prove that

$$2\log\frac{15}{18}-\log\frac{25}{162}+\log\frac{4}{9}=\log 2$$

$$L.H.S=2\log\frac{15}{18}-\log\frac{25}{162}+\log\frac{4}{9}$$

$$=\log\left(\frac{15}{18}\right)^2-\log\frac{25}{162}+\log\frac{4}{9} \qquad [n\log_a m=\log_a m^n]$$

$$=\log\left[\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\right]-\log\frac{25}{162}+\log\frac{4}{9}$$

$$=\log\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9}-\log\frac{25}{162} \qquad [\log_a m+\log_a n=\log_a mn]$$

$$=\log\frac{\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9}}{\frac{25}{162}} \qquad [\log_a m-\log_a n=\log_a\frac{m}{n}]$$

$$=\log\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9}\times\frac{162}{25}$$

$$=\log\frac{72}{36}$$

$$=\log 2$$

$$=R.H.S$$

Solution 5:

Consider the given equation

$$x-\log 48+3\log 2=\frac{1}{3}\log 125-\log 3$$

$$\Rightarrow x=\frac{1}{3}\log 125-\log 3+\log 48-3\log 2$$

$$\Rightarrow x=\log(125)^{\frac{1}{3}}-\log 3+\log 48-\log 2^3 \qquad [n\log_a m=\log_a m^n]$$

$$\Rightarrow x=\log(5\times 5\times 5)^{\frac{1}{3}}-\log 3+\log 48-\log 8$$

$$\Rightarrow x=\log(5^3)^{\frac{1}{3}}-\log 3+\log 48-\log 8$$

$$\Rightarrow x=\log 5-\log 3+\log 48-\log 8$$

$$\Rightarrow x=\log 5+\log 48-\log 3-\log 8$$

$$\Rightarrow x=(\log 5+\log 48)-(\log 3+\log 8)$$

$$\Rightarrow x=(\log 5\times 48)-(\log 3\times 8) \qquad [\log_a m+\log_a n=\log_a mn]$$

$$\Rightarrow x=\log\frac{5\times 48}{3\times 8} \qquad [\log_a m-\log_a n=\log_a\frac{m}{n}]$$

$$\Rightarrow x=\log\frac{5\times 6\times 8}{3\times 8}$$

$$\Rightarrow x=\log 10$$

$$\Rightarrow x=1$$

Solution 6:

$$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10 \qquad [\because \log_{10} 10 = 1]$$
$$= \log_{10} 2 \times 10 \qquad [\log_a m + \log_a n = \log_a mn]$$
$$= \log_{10} 20$$

Solution 7:

(i)

$$\log_{10}(x - 10) = 1$$
$$\Rightarrow \log_{10}(x - 10) = \log_{10} 10$$
$$\Rightarrow x - 10 = 10$$
$$\Rightarrow x = 10 + 10$$
$$\Rightarrow x = 20$$

(ii)

$$\log(x^2 - 21) = 2$$
$$\Rightarrow \log(x^2 - 21) = \log 100$$
$$\Rightarrow x^2 - 21 = 100$$
$$\Rightarrow x^2 - 21 - 100 = 0$$
$$\Rightarrow x^2 - 121 = 0$$
$$\Rightarrow x^2 = 121$$
$$\Rightarrow x = \pm\sqrt{121}$$
$$\Rightarrow x = \pm 11$$

(iii)

$$\log(x - 2) + \log(x + 2) = \log 5$$
$$\Rightarrow \log(x - 2)(x + 2) = \log 5 \quad [\log_a m + \log_a n = \log_a mn]$$
$$\Rightarrow \log(x^2 - 4) = \log 5$$
$$\Rightarrow x^2 - 4 = 5$$
$$\Rightarrow x^2 = 9$$
$$\Rightarrow x = \pm\sqrt{9}$$
$$\Rightarrow x = \pm\sqrt{3^2}$$
$$\Rightarrow x = \pm 3$$

(iv)

$log(x+5)+\log(x-5)=4\log 2+2\log 3$

$\Rightarrow \log(x+5)(x-5)=4\log 2+2\log 3 \quad [\log_a m+\log_a n=\log_a mn]$

$\Rightarrow log(x^2-25)=\log 2^4+\log 3^2 \quad [n\log_a m=\log_a m^n]$

$\Rightarrow log(x^2-25)=\log 16+\log 9$

$\Rightarrow log(x^2-25)=\log 16\times 9 \quad [\log_a m+\log_a n=\log_a mn]$

$\Rightarrow log(x^2-25)=\log 144$

$\Rightarrow x^2-25=144$

$\Rightarrow x^2=144+25$

$\Rightarrow x^2=169$

$\Rightarrow x=\pm\sqrt{169}$

$\Rightarrow x=\pm\sqrt{13^2}$

$\Rightarrow x=\pm 13$

Solution 8:

(i)

$$\frac{\log 81}{\log 27} = x$$

$$\Rightarrow x = \frac{\log 81}{\log 27}$$

$$\Rightarrow x = \frac{\log 3\times 3\times 3\times 3}{\log 3\times 3\times 3}$$

$$\Rightarrow x = \frac{\log 3^4}{\log 3^3}$$

$$\Rightarrow x = \frac{4\log 3}{3\log 3} \quad [n\log_a m = \log_a m^n]$$

$$\Rightarrow x = \frac{4}{3}$$

$$\Rightarrow x = 1\frac{1}{3}$$

(ii)

$$\frac{\log 128}{\log 32} = x$$

$$\Rightarrow x = \frac{\log 128}{\log 32}$$

$$\Rightarrow x = \frac{\log 2\times 2\times 2\times 2\times 2\times 2\times 2}{\log 2\times 2\times 2\times 2\times 2}$$

$$\Rightarrow x = \frac{\log 2^7}{\log 2^5}$$

$$\Rightarrow x = \frac{7\log 2}{5\log 2} \quad [n\log_a m = \log_a m^n]$$

$$\Rightarrow x = \frac{7}{5}$$

$$\Rightarrow x = 1.4$$

(iii)

$$\frac{\log 64}{\log 8} = \log x$$

$$\Rightarrow \log x = \frac{\log 64}{\log 8}$$

$$\Rightarrow \log x = \frac{\log 2\times 2\times 2\times 2\times 2\times 2}{\log 2\times 2\times 2}$$

$$\Rightarrow \log x = \frac{\log 2^6}{\log 2^3}$$

$$\Rightarrow \log x = \frac{6\log 2}{3\log 2} \quad [n\log_a m = \log_a m^n]$$

$$\Rightarrow \log x = \frac{6}{3}$$

$$\Rightarrow \log x = 2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 10\times 10$$

$$\Rightarrow x = 100$$

(iv)

$$\frac{\log 225}{\log 15} = \log x$$

$$\Rightarrow \log x = \frac{\log 225}{\log 15}$$

$$\Rightarrow \log x = \frac{\log 15\times 15}{\log 15}$$

$$\Rightarrow \log x = \frac{\log 15^2}{\log 15}$$

$$\Rightarrow \log x = \frac{2\log 15}{\log 15} \quad [n\log_a m = \log_a m^n]$$

$$\Rightarrow \log x = 2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 10\times 10$$

$$\Rightarrow x = 100$$

Solution 9:

Given that

$\log x = m + n;$

$\log y = m - n;$

Consider the expression $\log\frac{10x}{y^2}$:

$$\log\frac{10x}{y^2} = \log 10x - \log y^2$$

$$= \log 10x - 2\log y \qquad [n\log_a m = \log_a m^n]$$

$$= \log 10 + \log x - 2\log y \quad [\log_a m + \log_a n = \log_a mn]$$

$$= 1 + \log x - 2\log y$$

$$= 1 + m + n - 2(m - n)$$

$$= 1 + m + n - 2m + 2n$$

$$\Rightarrow \log\frac{10x}{y^2} = 1 - m + 3n$$

Solution 10:

(i)

We have,

$\log 1 = 0$ and $\log 1000 = 3$

$\therefore \log 1 \times \log 1000 = 0 \times 3 = 0$

Thus the statement, $\log 1 \times \log 1000 = 0$ is true

(ii)

We know that

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\therefore \frac{\log x}{\log y} \neq \log x - \log y$$

Thus the statement, $\frac{\log x}{\log y} = \log x - \log y$ is false

(iii)

Given that

$$\frac{\log 25}{\log 5} = \log x$$

$$\Rightarrow \frac{\log 5 \times 5}{\log 5} = \log x$$

$$\Rightarrow \frac{\log 5^2}{\log 5} = \log x$$

$$\Rightarrow \frac{2\log 5}{\log 5} = \log x \qquad [\log_a m^n = n\log_a m]$$

$$\Rightarrow 2 = \log_{10} x$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 100$$

Thus the statement, $x = 2$ is false

(iv)

We know that

$$\log x + \log y = \log xy$$

$$\therefore \log x + \log y \neq \log x \times \log y$$

Thus the statement $\log x + \log y = \log x \times \log y$ is false

Solution 11:

Given that $\log_{10} 2 = a$ and $\log_{10} 3 = b$

(i)

$$\log 12 = \log 2 \times 2 \times 3$$
$$= \log 2 \times 2 + \log 3 \quad [\log_a mn = \log_a m + \log_a n]$$
$$= \log 2^2 + \log 3$$
$$= 2\log 2 + \log 3 \quad [n \log_a m = \log_a m^n]$$
$$= 2a + b \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$

(ii)

$$\log 2.25 = \log \frac{225}{100}$$
$$= \log \frac{25 \times 9}{25 \times 4}$$
$$= \log \frac{9}{4}$$
$$= \log \left(\frac{3}{2}\right)^2$$
$$= 2\log \left(\frac{3}{2}\right) \quad [n \log_a m = \log_a m^n]$$
$$= 2(\log 3 - \log 2) \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= 2(b - a) \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$
$$= 2b - 2a$$

(iii)

$$\log 2\frac{1}{4} = \log \frac{9}{4}$$
$$= \log \left(\frac{3}{2}\right)^2$$
$$= 2\log \left(\frac{3}{2}\right) \quad [n \log_a m = \log_a m^n]$$
$$= 2(\log 3 - \log 2) \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= 2(b - a) \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$
$$= 2b - 2a$$

(iv)

$$\log 5.4 = \log \frac{54}{10}$$

$$= \log\left(\frac{2 \times 3 \times 3 \times 3}{10}\right)$$

$$= \log(2 \times 3 \times 3 \times 3) - \log_{10} 10 \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$

$$= \log_{10} 2 + \log_{10} 3^3 - \log_{10} 10 \quad [\log_a mn = \log_a m + \log_a n]$$

$$= \log_{10} 2 + 3\log_{10} 3 - \log_{10} 10 \quad [n \log_a m = \log_a m^n]$$

$$= \log_{10} 2 + 3\log_{10} 3 - 1 \quad [\because \log_{10} 10 = 1]$$

$$= a + 3b - 1 \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$

(v)

$$\log 60 = \log_{10} 10 \times 2 \times 3$$

$$= \log_{10} 10 + \log_{10} 2 + \log_{10} 3 \quad [\log_a mn = \log_a m + \log_a n]$$

$$= 1 + \log_{10} 2 + \log_{10} 3 \quad [\because \log_{10} 10 = 1]$$

$$= 1 + a + b \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$

(vi)

$$\log 3\frac{1}{8} = \log_{10}\left(\frac{25}{8} \times \frac{4}{4}\right)$$

$$= \log_{10}\left(\frac{100}{32}\right)$$

$$= \log_{10} 100 - \log_{10} 32 \quad [\log_a \frac{m}{n} = \log_a m - \log_a n]$$

$$= \log_{10} 100 - \log_{10} 2^5$$

$$= 2 - \log_{10} 2^5 \quad [\because \log_{10} 100 = 2]$$

$$= 2 - 5\log_{10} 2 \quad [\log_a m^n = n \log_a m]$$

$$= 2 - 5a \quad [\because \log_{10} 2 = a]$$

Solution 12:

We know that $\log 2 = 0.3010$ and $\log 3 = 0.4771$

(i)

$$\log 12 = \log 2 \times 2 \times 3$$

$$= \log 2 \times 2 + \log 3 \qquad [\log_a mn = \log_a m + \log_a n]$$

$$= \log 2^2 + \log 3$$

$$= 2\log 2 + \log 3 \qquad [n\log_a m = \log_a m^n]$$

$$= 2(0.3010) + 0.4771 \qquad [\because \log 2 = 0.3010 \text{ and } \log 3 = 0.4771]$$

$$= 1.0791$$

(ii)

$$\log 1.2 = \log \frac{12}{10}$$

$$= \log 12 - \log 10 \qquad \left[\log_a \frac{m}{n} = \log_a m - \log_a n\right]$$

$$= \log 2 \times 2 \times 3 - 1 \qquad [\because \log 10 = 1]$$

$$= \log 2 \times 2 + \log 3 - 1 \qquad [\log_a mn = \log_a m + \log_a n]$$

$$= \log 2^2 + \log 3 - 1$$

$$= 2\log 2 + \log 3 - 1 \qquad [n\log_a m = \log_a m^n]$$

$$= 2(0.3010) + 0.4771 - 1 \qquad [\because \log 2 = 0.3010 \text{ and } \log 3 = 0.4771]$$

$$= 1.0791 - 1$$

$$= 0.0791$$

(iii)

$$\log 3.6 = \log \frac{36}{10}$$

$$= \log 36 - \log 10 \qquad \left[\log_a \frac{m}{n} = \log_a m - \log_a n\right]$$

$$= \log 2 \times 2 \times 3 \times 3 - 1 \qquad [\because \log 10 = 1]$$

$$= \log 2 \times 2 + \log 3 \times 3 - 1 \qquad [\log_a mn = \log_a m + \log_a n]$$

$$= \log 2^2 + \log 3^2 - 1$$

$$= 2\log 2 + 2\log 3 - 1 \qquad [n \log_a m = \log_a m^n]$$

$$= 2(0.3010) + 2(0.4771) - 1 \quad \begin{bmatrix} \because \log 2 = 0.3010 \\ \text{and } \log 3 = 0.4771 \end{bmatrix}$$

$$= 1.5562 - 1$$

$$= 0.5562$$

(iv)

$$\log 15 = \log\left(\frac{15}{10} \times 10\right)$$

$$= \log\left(\frac{15}{10}\right) + \log 10$$

$$= \log\left(\frac{3}{2}\right) + 1 \qquad [\because \log 10 = 1]$$

$$= \log 3 - \log 2 + 1 \qquad \left[\because \log m - \log n = \log\left(\frac{m}{n}\right)\right]$$

$$= 0.4771 - 0.3010 + 1$$

$$= 1.1761$$

(v)

$$\log 25 = \log\left(\frac{25}{4} \times 4\right)$$

$$= \log\left(\frac{100}{4}\right) \qquad [\log_a mn = \log_a m + \log_a n]$$

$$= \log 100 - \log(2 \times 2) \quad \left[\log_a \frac{m}{n} = \log_a m - \log_a n\right]$$

$$= 2 - \log(2^2) \qquad [\log 100 = 2]$$

$$= 2 - 2\log 2 \qquad [\log_a m^n = n\log_a m]$$

$$= 2 - 2(0.3010) \qquad [\because \log 2 = 0.3010]$$

$$= 1.398$$

(vi)

$$\frac{2}{3}\log 8 = \frac{2}{3}\log 2 \times 2 \times 2$$

$$= \frac{2}{3}\log 2^3$$

$$= 3 \times \frac{2}{3}\log 2 \quad [\log_a m^n = n\log_a m]$$

$$= 2\log 2$$

$$= 2 \times 0.3010 \quad [\because \log 2 = 0.3010]$$

$$= 0.602$$

Solution 13:

(i)

Consider the given equation:

$$2\log_{10} x + 1 = \log_{10} 250$$

$$\Rightarrow \log_{10} x^2 + 1 = \log_{10} 250 \qquad [\log_a m^n = n\log_a m]$$

$$\Rightarrow \log_{10} x^2 + \log_{10} 10 = \log_{10} 250 \quad [\because \log_{10} 10 = 1]$$

$$\Rightarrow \log_{10}(x^2 \times 10) = \log_{10} 250 \qquad [\log_a m + \log_a n = \log_a mn]$$

$$\Rightarrow x^2 \times 10 = 250$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \sqrt{25}$$

$$\Rightarrow x = 5$$

(ii)

$x = 5$ (proved above in (i))

$$\log_{10} 2x = \log_{10} 2(5)$$

$$= \log_{10} 10$$

$$= 1 \qquad [\because \log_{10} 10 = 1]$$

Solution 14:

$$3\log x + \frac{1}{2}\log y = 2$$

$$\Rightarrow \log x^3 + \log \sqrt{y} = 2$$

$$\Rightarrow \log x^3\sqrt{y} = 2$$

$$\Rightarrow x^3\sqrt{y} = 10^2$$

$$\Rightarrow \sqrt{y} = \frac{10^2}{x^3}$$

Squaring both sides, we get

$$y = \frac{10000}{x^6}$$

$$\Rightarrow y = 10000x^{-6}$$

Solution 15:

$x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$

$\Rightarrow \log x = a\log 100$, $\log y = b\log 10000$ and $\log z = c\log 10$

$$\log \frac{10\sqrt{y}}{x^2z^3} = \log 10\sqrt{y} - \log\left(x^2z^3\right)$$

$$= \log\left(10y^{1/2}\right) - \log x^2 - \log z^3$$

$$= \log 10 + \log y^{1/2} - \log x^2 - \log z^3$$

$$= \log 10 + \frac{1}{2}\log y - 2\log x - 3\log z$$

$$= 1 + \frac{1}{2}\log(10000)^b - 2\log(100)^a - 3\log(10)^c \ldots\ldots (\text{Since } \log 10 = 1)$$

$$= 1 + \frac{b}{2}\log(10)^4 - a\log(10)^2 - 3c\log 10$$

$$= 1 + \frac{b}{2} \times 4\log 10 - 2 \times 2a\log 10 - 3c\log 10$$

$$= 1 + 2b - 4a - 3c$$

Solution 16:

$3(\log 5 - \log 3) - (\log 5 - 2\log 6) = 2 - \log x$

$\Rightarrow 3\log 5 - 3\log 3 - \log 5 + 2\log(2 \times 3) = 2 - \log x$

$\Rightarrow 3\log 5 - 3\log 3 - \log 5 + 2\log 2 + 2\log 3 = 2 - \log x$

$\Rightarrow 2\log 5 - \log 3 + 2\log 2 = 2 - \log x$

$\Rightarrow 2\log 5 - \log 3 + 2\log 2 + \log x = 2$

$\Rightarrow \log 5^2 - \log 3 + \log 2^2 + \log x = 2$

$\Rightarrow \log\left(\frac{25 \times 4 \times x}{3}\right) = 2$

$\Rightarrow \log\left(\frac{100x}{3}\right) = 2$

$\Rightarrow \frac{100x}{3} = 10^2$

$\Rightarrow \frac{x}{3} = 1$

$\Rightarrow x = 3$