

## 6. Determinants

### Exercise 6A

#### 1. Question

If A is a  $2 \times 2$  matrix such that  $|A| \neq 0$  and  $|A| = 5$ , write the value of  $|4A|$ .

#### Answer

Theorem: If A be  $k \times k$  matrix then  $|pA| = p^k |A|$ .

Given,  $p=4, k=2$  and  $|A|=5$ .

$$|4A| = 4^2 \times 5$$

$$= 16 \times 5$$

$$= 80$$

#### 2. Question

If A is a  $3 \times 3$  matrix such that  $|A| \neq 0$  and  $|3A| = k|A|$  then write the value of k.

#### Answer

Theorem: If Let A be  $k \times k$  matrix then  $|pA| = p^k |A|$ .

Given:  $k=3$  and  $p=3$ .

$$|3A| = 3^3 \times |A|$$

$$= 27|A|.$$

Comparing above with  $k|A|$  gives  $k=27$ .

#### 3. Question

Let A be a square matrix of order 3, write the value of  $|2A|$ , where  $|A| = 4$ .

#### Answer

Theorem: If A be  $k \times k$  matrix then  $|pA| = p^k |A|$ .

Given:  $p=2, k=3$  and  $|A|=4$

$$|2A| = 2^3 \times |A|$$

$$= 8 \times 4$$

$$= 32$$

#### 4. Question

If  $A_{ij}$  is the cofactor of the element  $a_{ij}$  of  $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$  then write the value of  $(a_{32}A_{32})$ .

#### Answer

Theorem:  $A_{ij}$  is found by deleting  $i^{\text{th}}$  row and  $j^{\text{th}}$  column, the determinant of left matrix is called cofactor with multiplied by  $(-1)^{(i+j)}$ .

Given:  $i=3$  and  $j=2$ .

$$A_{32} = (-1)^{(3+2)}(2 \times 4 - 6 \times 5)$$

$$= -1 \times (-22)$$

$$=22$$

$$a_{32}=5$$

$$a_{32}A_{32}=5 \times 22$$

$$=110$$

### 5. Question

Evaluate  $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$ .

### Answer

Theorem: This evaluation can be done in two different ways either by taking out the common things and then calculating the determinants or simply take determinant.

I will prefer first method because with that chances of silly mistakes reduces.

Take out  $x+1$  from second row.

$$(x+1) \times \begin{vmatrix} x^2 - x + 1 & x - 1 \\ 1 & 1 \end{vmatrix}$$

$$\Rightarrow (x+1) \times (x^2 - x + 1 - (x - 1))$$

$$\Rightarrow (x+1) \times (x^2 - 2x + 2)$$

$$\Rightarrow x^3 - 2x^2 + 2x + x^2 - 2x + 2$$

$$\Rightarrow x^3 - x^2 + 2.$$

### 6. Question

Evaluate  $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$ .



### Answer

This we can very simply go through directly.

$$((a+ib)(a-ib)) - ((-c+id)(c+id)).$$

$$\Rightarrow (a^2 + b^2) - (-c^2 - d^2).$$

$$\Rightarrow a^2 + b^2 + c^2 + d^2$$

$$\because i \times i = -1$$

### 7. Question

If  $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$ , write the value of  $x$ .

### Answer

Here the determinant is compared so we need to take determinant both sides then find  $x$ .

$$12x + 14 = 32 - 42$$

$$\Rightarrow 12x = -10 - 14$$

$$\Rightarrow 12x = -24$$

$$\Rightarrow x = -2$$

### 8. Question

If  $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ , write the value of x.

**Answer**

this question is having the same logic as above.

$$2x^2 - 40 = 18 + 14$$

$$\Rightarrow 2x^2 = 72$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow x = \pm 6.$$

**9. Question**

If  $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$ , write the value of x.

**Answer**

Simply by equating both sides we can get the value of x.

$$2x^2 + 2x - 2(x^2 + 4x + 3) = -12$$

$$\Rightarrow -6x - 6 = -12$$

$$\Rightarrow -6x = -6$$

$$\Rightarrow x = 1$$

**10. Question**

If  $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ , find the value of  $3|A|$ .

**Answer**

Find the determinant of A and then multiply it by 3

$$|A| = 2$$

$$3|A| = 3 \times 2$$

$$= 6$$

**11. Question**

Evaluate  $2 \begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$ .

**Answer**

It is determinant multiplied by a scalar number 2, just find determinant of matrix and multiply it by 2.

$$2 \times (35 - 20)$$

$$2 \times 15 = 30$$

**12. Question**

Evaluate  $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix}$ .



**Answer**

Find determinant

$$\sqrt{6} \times \sqrt{24} - \sqrt{20} \times \sqrt{5}$$

$$\sqrt{144} - \sqrt{100}$$

$$= 12 - 10$$

$$= 2.$$

**13. Question**

Evaluate  $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ .

**Answer**

After finding determinant we will get a trigonometric identity.

$$2\cos^2\theta + 2\sin^2\theta$$

$$= 2$$

$$\therefore \sin^2\theta + \cos^2\theta = 1$$

**14. Question**

Evaluate  $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$ .

**Answer**

After finding determinant we will get a trigonometric identity. Class, Your Page.

$$\cos^2\alpha + \sin^2\alpha$$

$$= 1$$

$$\therefore \sin^2\theta + \cos^2\theta = 1$$

**15. Question**

Evaluate  $\begin{vmatrix} \sin 60^\circ & \cos 60^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{vmatrix}$ .

**Answer**

After finding determinant we will get,

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = \cos 30^\circ$$

$$\cos 60^\circ = \frac{1}{2} = \sin 30^\circ$$

$$\sin 60^\circ \times \cos 30^\circ + \sin 30^\circ \times \cos 60^\circ$$

$$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1.$$

**16. Question**

Evaluate  $\begin{vmatrix} \cos 65^\circ & \sin 65^\circ \\ \sin 25^\circ & \cos 25^\circ \end{vmatrix}$ .

**Answer**

By directly opening this determinant

$$\begin{aligned} & \cos 65^\circ \times \cos 25^\circ - \sin 25^\circ \times \sin 65^\circ \\ &= \cos(65^\circ + 25^\circ) \because \cos A \cos B - \sin A \sin B = \cos(A+B) \\ &= \cos 90^\circ \\ &= 0 \end{aligned}$$

$\therefore \cos A \cos B - \sin A \sin B = \cos(A+B)$

**17. Question**

Evaluate  $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$ .

**Answer**

$$\begin{aligned} & \cos 15^\circ \cos 75^\circ - \sin 75^\circ \sin 15^\circ \\ &= \cos(15^\circ + 75^\circ) \because \cos A \cos B - \sin A \sin B = \cos(A+B) \\ &= \cos 90^\circ \\ &= 0 \end{aligned}$$

$\therefore \cos A \cos B - \sin A \sin B = \cos(A+B)$



**18. Question**

Evaluate  $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$ .

**Answer**

We know that expansion of determinant with respect to first row is  $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ .

$$\begin{aligned} & 0(3 \times 6 - 5 \times 4) - 2(2 \times 6 - 4 \times 4) + 0(2 \times 5 - 4 \times 3) \\ &= 8. \end{aligned}$$

**19. Question**

Without expanding the determinant, prove that  $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = 0$ .

**SINGULAR MATRIX** A square matrix A is said to be singular if  $|A| = 0$ .

Also, A is called non singular if  $|A| \neq 0$ .

**Answer**

We know that  $C_1 \Rightarrow C_1 - C_2$ , would not change anything for the determinant.

Applying the same in above determinant, we get

$$\begin{bmatrix} 40 & 1 & 5 \\ 72 & 7 & 9 \\ 24 & 5 & 3 \end{bmatrix} \text{ Now it can clearly be seen that } C_1 = 8 \times C_3$$

Applying above equation we get,

$$\begin{bmatrix} 0 & 1 & 5 \\ 0 & 7 & 9 \\ 0 & 3 & 3 \end{bmatrix}$$

We know that if a row or column of a determinant is 0. Then it is singular determinant.

### 20. Question

For what value of x, the given matrix  $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$  is a singular matrix?

### Answer

For A to be singular matrix its determinant should be equal to 0.

$$0 = (3-2x) \times 4 - (x+1) \times 2$$

$$0 = 12 - 8x - 2x - 2$$

$$0 = 10 - 10x$$

$$x = 1.$$

### 21. Question

Evaluate  $\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix}$ .



### Answer

$$\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix} = 14 \times (-7) - 9 \times (-8)$$

$$= -26$$

### 22. Question

Evaluate  $\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix}$ .

### Answer

$$\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix} = 3\sqrt{3} \times \sqrt{3} - (-\sqrt{5} \times \sqrt{5})$$

$$= 14.$$

## Exercise 6B

### 1. Question

Evaluate :

$$\begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

### Answer

$$\begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} 67 & 19 & 21 \\ 78 & 26 & 28 \\ 81 & 24 & 26 \end{vmatrix} [R_2' = (1/2)R_2]$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} 67 & 19 & 21 \\ -3 & 2 & 2 \\ 81 & 24 & 26 \end{vmatrix} [R_2' = R_2 - R_3]$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} -14 & -5 & -5 \\ -3 & 2 & 2 \\ 81 & 24 & 26 \end{vmatrix} [R_1' = R_1 - R_3]$$

$$= \begin{vmatrix} -14 & -5 & -5 \\ -3 & 2 & 2 \\ 81/2 & 12 & 13 \end{vmatrix} [R_3' = 2R_3]$$

$$= (-14)\{(2 \times 13) - (2 \times 12)\} - 5\{(2 \times 81/2) - (-3) \times 13\} - 5\{(-3) \times 12 - 2 \times 81/2\}$$

[expanding by the first row]

$$= -14 \times (26 - 24) - 5(81 + 39) - 5(-36 - 81)$$

$$= -14 \times 2 - 5 \times 120 - 5 \times (-117) = -28 - 600 + 585 = -43$$

### 2. Question

Evaluate :

$$\begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$$



### Answer

$$\begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & -5 & -5 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix} [R_1' = R_1 - R_2]$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} 4 & -5 & -5 \\ 50 & 62 & 54 \\ 63 & 54 & 46 \end{vmatrix} [R_2' = 2R_2]$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} 4 & -5 & -5 \\ -13 & 8 & 8 \\ 63 & 54 & 46 \end{vmatrix} [R_2' = R_2 - R_3]$$

$$= \begin{vmatrix} 4 & -5 & -5 \\ -13 & 8 & 8 \\ 63/2 & 27 & 23 \end{vmatrix} [R_3' = 2R_3]$$

$$= 4(8 \times 23 - 8 \times 27) - 5\{8 \times 63/2 - (-13) \times 23\} - 5\{(-13) \times 27 - 8 \times 63/2\}$$

[expansion by first row]

$$= 132$$

### 3. Question

Evaluate :

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

**Answer**

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \times \begin{vmatrix} 17 & 18 & 6 \\ 1 & 6 & 4 \\ 17 & 3 & 6 \end{vmatrix} [R_1' = R_1/6]$$

Now, for any determinant, if at least two rows are identical, then the value of the determinant becomes zero.

Here, the first and third rows are identical.

So, the value of the above determinant evaluated = **0**

#### 4. Question

Evaluate :

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 2^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$$

**Answer**

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Expanding by first row, we get,

$$1(9 \times 25 - 16 \times 16) + 4(16 \times 9 - 4 \times 25) + 9(4 \times 16 - 9 \times 9) = -31 + 176 - 153 = -8$$

#### 5. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a).$$

**Answer**

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ bc-ca & ca-ab & ab \end{vmatrix} [C_1' = C_1 - C_2 \text{ \& } C_2' = C_2 - C_3]$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ -c(a-b) & -a(b-c) & ab \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -c & -a & ab \end{vmatrix} [C_1' = C_1/(a-b) \text{ \& } C_2' = C_2/(b-c)]$$

$$= (a-b)(b-c)[0 + 0 + 1\{-a - (-c)\}] \text{ [expansion by first row]}$$

$$= \mathbf{(a-b)(b-c)(c-a)}$$

## 6. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a).$$

### Answer

$$\begin{aligned} & \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} \\ &= \begin{vmatrix} 0 & b-a & b^2-a^2 \\ 0 & c-b & c^2-b^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} [R_1' = R_1 - R_2 \text{ \& } R_2' = R_2 - R_3] \\ &= \begin{vmatrix} 0 & b-a & (b-a)(b+a) \\ 0 & c-b & (c-b)(c+b) \\ 1 & a+b & a^2+b^2 \end{vmatrix} \\ &= (b-a)(c-b) \begin{vmatrix} 0 & 1 & b+a \\ 0 & 1 & c+b \\ 1 & a+b & a^2+b^2 \end{vmatrix} [R_1' = R_1/(b-a) \text{ \& } R_2' = R_2/(c-b)] \\ &= (b-a)(c-b)[0+0+1\{(c+b)-(b+a)\}] [\text{expansion by first column}] \\ &= (a-b)(b-c)(c-a) \end{aligned}$$

## 7. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1.$$

### Answer

$$\begin{aligned} & \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} \\ &= \begin{vmatrix} -1 & -2-p & -2p-q \\ -1 & -3-p & -3p-q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} [R_1' = R_1 - R_2 \text{ \& } R_2' = R_2 - R_3] \\ &= \begin{vmatrix} 0 & 1 & p \\ -1 & -3-p & -3p-q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} [R_1' = R_1 - R_2] \\ &= \left(\frac{1}{2}\right) \begin{vmatrix} 0 & 1 & p \\ -2 & -6-2p & -6p-2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} [R_2' = R_2 * 2] \\ &= \left(\frac{1}{2}\right) \begin{vmatrix} 0 & 1 & p \\ 1 & p & 1+q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} [R_2' = R_2 + R_3] \\ &= (1/2)[0+3(1+q)-(1+6p+3q)+p(6+3p-3p)] [\text{expansion by first row}] \\ &= (1/2)(3+3q-1-6p-3q+6p) = 1 \end{aligned}$$

## 8. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z).$$

**Answer**

$$\begin{aligned} & \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} \\ &= \begin{vmatrix} a & -a & 0 \\ 0 & a & -a \\ x & y & a+z \end{vmatrix} \quad [R_1' = R_1 - R_2 \text{ \& } R_2' = R_2 - R_3] \\ &= a^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ x & y & a+z \end{vmatrix} \quad [R_1' = R_1/a \text{ \& } R_2' = R_2/a] \\ &= a^2[a+z - (-y) - (-x)] \quad [\text{expansion by first row}] \\ &= a^2(a+x+y+z) \end{aligned}$$

### 9. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x+2a)(x-a)^2.$$

**Answer**

$$\begin{aligned} & \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} \\ &= \begin{vmatrix} x+2a & x+2a & x+2a \\ a & x & a \\ a & a & x \end{vmatrix} \quad [R_1' = R_1 + R_2 + R_3] \\ &= (x+2a) \begin{vmatrix} 1 & 1 & 1 \\ a & x & a \\ a & a & x \end{vmatrix} \quad [R_1' = R_1/(x+2a)] \\ &= (x+2a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-a & a-x \\ a & a & x \end{vmatrix} \quad [R_2' = R_2 - R_3] \\ &= (x+2a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-a & -(x-a) \\ a & a & x \end{vmatrix} \\ &= (x+2a)(x-a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ a & a & x \end{vmatrix} \quad [R_2' = R_2/(x-a)] \\ &= (x+2a)(x-a)[x - (-a) + (-a - 0) + (-a)] \quad [\text{expansion by first row}] \\ &= (x+2a)(x-a)(x+a-a-a) = (x+2a)(x-a)^2 \end{aligned}$$

### 10. Question

Using properties of determinants prove that:



$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(x-4)^2.$$

**Answer**

$$\begin{aligned} & \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} \\ &= \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} [R_1' = R_1 + R_2 + R_3] \\ &= (5x+4) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} [R_1' = R_1/(5x+4)] \\ &= (5x+4) \begin{vmatrix} 1 & 1 & 1 \\ 0 & -x+4 & x-4 \\ 2x & 2x & x+4 \end{vmatrix} [R_2' = R_2 - R_3] \\ &= (5x+4) \begin{vmatrix} 1 & 1 & 1 \\ 0 & -(x-4) & x-4 \\ 2x & 2x & x+4 \end{vmatrix} \\ &= (5x+4)(x-4) \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2x & 2x & x+4 \end{vmatrix} [R_2' = R_2/(x-4)] \\ &= (5x+4)(x-4)[- (x+4) - 2x + 2x - 0 + 0 - (-2x)] \text{ [expansion by first row]} \\ &= (5x+4)(x-4)(-x-4+2x) = \mathbf{(5x+4)(x-4)^2} \end{aligned}$$

### 11. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2.$$

**Answer**

$$\begin{aligned} & \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} \\ &= \begin{vmatrix} 5x+\lambda & 5x+\lambda & 5x+\lambda \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [R_1' = R_1 + R_2 + R_3] \\ &= (5x+\lambda) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [R_1' = R_1/(5x+\lambda)] \\ &= (5x+\lambda) \begin{vmatrix} 1 & 1 & 1 \\ 0 & -x+\lambda & x-\lambda \\ 2x & 2x & x+\lambda \end{vmatrix} [R_2' = R_2 - R_3] \\ &= (5x+\lambda) \begin{vmatrix} 1 & 1 & 1 \\ 0 & -(x-\lambda) & x-\lambda \\ 2x & 2x & x+\lambda \end{vmatrix} \\ &= (5x+\lambda)(x-\lambda) \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2x & 2x & x+\lambda \end{vmatrix} [R_2' = R_2/(x-\lambda)] \end{aligned}$$

$$= (5x + \lambda)(x - \lambda)[-(x + \lambda) - 2x + 2x - 0 + 0 - (-2x)] \text{ [expansion by first row]}$$

$$= (5x + \lambda)(x - \lambda)(-x - \lambda + 2x) = \mathbf{(5x + \lambda)(x - \lambda)^2}$$

### 12. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3.$$

### Answer

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2a - 2 & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \text{ [R}_1' = R_1 - R_2 \text{ \& } R_2' = R_2 - R_3]$$

$$= \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2(a - 1) & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= (a - 1)^2 \begin{vmatrix} a + 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \text{ [R}_1' = R_1/(a - 1) \text{ \& } R_2' = R_2/(a - 1)]$$

$$= (a - 1)^2 [a + 1 - 0 - 2] \text{ [expansion by first row]}$$

$$= \mathbf{(a - 1)^3}$$

### 13. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x & x + y & x + 2y \\ x + 2y & x & x + y \\ x + y & x + 2y & x \end{vmatrix} = 9y^2(x + y).$$

### Answer

$$\begin{vmatrix} x & x + y & x + 2y \\ x + 2y & x & x + y \\ x + y & x + 2y & x \end{vmatrix}$$

$$= \begin{vmatrix} 3(x + y) & 3(x + y) & 3(x + y) \\ x + 2y & x & x + y \\ x + y & x + 2y & x \end{vmatrix} \text{ [R}_1' = R_1 + R_2 + R_3]$$

$$= 3(x + y) \begin{vmatrix} 1 & 1 & 1 \\ x + 2y & x & x + y \\ x + y & x + 2y & x \end{vmatrix} \text{ [R}_1' = R_1/3(x + y)]$$

$$= 3(x + y) \begin{vmatrix} 1 & 1 & 1 \\ y & -2y & y \\ x + y & x + 2y & x \end{vmatrix} \text{ [R}_2' = R_2 - R_3]$$

$$= 3y(x + y) \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ x + y & x + 2y & x \end{vmatrix} \text{ [R}_2' = R_2/y]$$

$$= 3y(x + y) \begin{vmatrix} 0 & 3 & 0 \\ 1 & -2 & 1 \\ x + y & x + 2y & x \end{vmatrix} [R_1' = R_1 - R_2]$$

$$= 3y(x + y)[0 + 3(x + y) - x + 0] \text{ [expansion by first row]}$$

$$= 3y(x + y)(3y) = \mathbf{9y^2(x + y)}$$

#### 14. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 3x & -x + y & -x + z \\ x - y & 3y & z - y \\ x - z & y - z & 3z \end{vmatrix} = 3(x + y + z)(xy + yz + zx).$$

#### Answer

$$\begin{vmatrix} 3x & -x + y & -x + z \\ x - y & 3y & z - y \\ x - z & y - z & 3z \end{vmatrix}$$

$$= \begin{vmatrix} x + y + z & -x + y & -x + z \\ x + y + z & 3y & z - y \\ x + y + z & y - z & 3z \end{vmatrix} [C_1' = C_1 + C_2 + C_3]$$

$$= (x + y + z) \begin{vmatrix} 1 & -x + y & -x + z \\ 1 & 3y & z - y \\ 1 & y - z & 3z \end{vmatrix} [C_1' = C_1 / (x + y + z)]$$

$$= (x + y + z) \begin{vmatrix} 1 & 1 & 1 \\ -x + y & 3y & y - z \\ -x + z & z - y & 3z \end{vmatrix} \text{ [transforming row and column]}$$

$$= (x + y + z) \begin{vmatrix} 0 & 0 & 1 \\ -x - 2y & 2y + z & y - z \\ -x + y & -y - 2z & x \end{vmatrix} [C_1' = C_1 - C_2 \text{ \& } C_2' = C_2 - C_3]$$

$$= (x + y + z)[0 + 0 + (-x - 2y)(-y - 2z) - (-x + y)(2y + z)] \text{ [expansion by first row]}$$

$$= (x + y + z)(xy + 2y^2 + 2xz + 4yz + 2xy - 2y^2 + xz - yz)$$

$$= (x + y + z)(3xy + 3yz + 3xz)$$

$$= \mathbf{3(x + y + z)(xy + yz + zx)}$$

#### 15. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x - y)(y - z)(z - x).$$

#### Answer

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} [C_1' = C_1/x, C_2' = C_2/y \text{ \& } C_3' = C_3/z]$$

$$\begin{aligned}
&= xyz \begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix} [C_1' = C_1 - C_2 \text{ \& } C_2' = C_2 - C_3] \\
&= xyz \begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ (x+y)(x-y) & (y+z)(y-z) & z^2 \end{vmatrix} \\
&= xyz(x-y)(y-z) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & z \\ x+y & y+z & z^2 \end{vmatrix} [C_1' = C_1/(x-y) \text{ \& } C_2' = C_2/(y-z)] \\
&= xyz(x-y)(y-z)(0+0+y+z-x-y) \text{ [expansion by first row]} \\
&= \mathbf{xyz(x-y)(y-z)(z-x)}
\end{aligned}$$

### 16. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3.$$

### Answer

$$\begin{aligned}
&\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} \\
&= \begin{vmatrix} 2(a+b+c) & 0 & a+b+c \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} [R_1' = R_1 + R_2 + R_3] \\
&= (a+b+c) \begin{vmatrix} 2 & 0 & 1 \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} [R_1' = R_1/(a+b+c)] \\
&= (a+b+c)[2(b-c)c - b(c-a) + (c+a)(c-a) - (a+b)(b-c)] \text{ [expansion by first row]} \\
&= (a+b+c)(2bc - 2c^2 - bc + ab + c^2 - a^2 - ab - b^2 + ac + bc) \\
&= (a+b+c)(ab + bc + ac - a^2 - b^2 - c^2) \\
&= \mathbf{3abc - a^3 - b^3 - c^3}
\end{aligned}$$

### 17. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

### Answer

$$\begin{aligned}
&\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \\
&= \begin{vmatrix} 2(b+c) & 2(a+c) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix} [R_1' = R_1 + R_2 + R_3]
\end{aligned}$$

$$\begin{aligned}
&= 2 \begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} [R_1' = R_1/2] \\
&= 2 \begin{vmatrix} c & 0 & a \\ b-c & a & -a \\ c & c & a+b \end{vmatrix} [R_1' = R_1 - R_2 \text{ \& } R_2' = R_2 - R_3] \\
&= 2[c\{a(a+b) - (-ac)\} + 0 + a\{c(b-c) - ac\}] [\text{expansion by first row}] \\
&= 2(a^2c + abc + ac^2 + abc - ac^2 - a^2c) \\
&= 4abc
\end{aligned}$$

### 18. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = -a^3.$$

### Answer

$$\begin{aligned}
&\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} \\
&= \left(\frac{1}{3}\right) \begin{vmatrix} 3a & 3a+6b & 3a+6b+9c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_1' = 3R_1] \\
&= \left(\frac{1}{3}\right) \begin{vmatrix} 0 & -a & -2a-b \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_1' = R_1 - R_2] \\
&= \left(\frac{1}{6}\right) \begin{vmatrix} 0 & -a & -2a-b \\ 6a & 8a+12b & 10a+14b+18c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_2' = 2R_2] \\
&= \left(\frac{1}{6}\right) \begin{vmatrix} 0 & -a & -2a-b \\ 0 & -a & -a-b \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_2' = R_2 - R_3] \\
&= (1/6)[0 + 0 + 6a\{a(a+b) - a(2a+b)\}] [\text{expansion by first column}] \\
&= -a^3
\end{aligned}$$

### 19. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

### Answer

$$\begin{aligned}
&\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} \\
&= \begin{vmatrix} a+b & a+b & -(a+b) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} [R_1' = R_1 + R_2]
\end{aligned}$$

$$\begin{aligned}
&= (a + b) \begin{vmatrix} 1 & 1 & -1 \\ -c & a + b + c & -a \\ -b & -a & a + b + c \end{vmatrix} [R_1' = R_1/(a + b)] \\
&= (a + b) \begin{vmatrix} 1 & 1 & -1 \\ -c - b & b + c & b + c \\ -b & -a & a + b + c \end{vmatrix} [R_2' = R_2 + R_3] \\
&= (a + b) \begin{vmatrix} 1 & 1 & -1 \\ -(b + c) & b + c & b + c \\ -b & -a & a + b + c \end{vmatrix} \\
&= (a + b)(b + c) \begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -b & -a & a + b + c \end{vmatrix} [R_2' = R_1/(b + c)] \\
&= (a + b)(b + c) \begin{vmatrix} 0 & 2 & 0 \\ -1 & 1 & 1 \\ -b & -a & a + b + c \end{vmatrix} [R_1' = R_1 + R_2] \\
&= (a + b)(b + c)\{0 + 2(-b + a + b + c) + 0\}[\text{expansion by first row}] \\
&= \mathbf{2(a + b)(b + c)(c + a)}
\end{aligned}$$

## 20. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a & b & ax + by \\ b & c & bx + cy \\ ax + by & bx + cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2 + 3bxy + cy^2).$$

## Answer

$$\begin{aligned}
&\begin{vmatrix} a & b & ax + by \\ b & c & bx + cy \\ ax + by & bx + cy & 0 \end{vmatrix} \\
&= \left(\frac{1}{xy}\right) \begin{vmatrix} ax & bx & ax^2 + bxy \\ by & cy & bxy + cy^2 \\ ax + by & bx + cy & 0 \end{vmatrix} [R_1' = xR_1 \text{ \& } R_2' = yR_2] \\
&= \left(\frac{1}{xy}\right) \begin{vmatrix} 0 & 0 & ax^2 + 2bxy + cy^2 \\ by & cy & bxy + cy^2 \\ ax + by & bx + cy & 0 \end{vmatrix} [R_1' = R_1 + R_2 - R_3] \\
&= (1/xy)[0 + 0 + (ax^2 + 2bxy + cy^2)\{by(bx + cy) - cy(ax + by)\}[\text{expansion by first row}]] \\
&= (1/xy)(ax^2 + 2bxy + cy^2)(b^2xy + bcy^2 - acxy - bcy^2) \\
&= \mathbf{(b^2 - ac)(ax^2 + 2bxy + cy^2)}
\end{aligned}$$

## 21. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4(a-b)(b-c)(c-a)$$

## Answer

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} \\
= \begin{vmatrix} a^2 & b^2 & c^2 \\ a^2 + 2a + 1 & b^2 + 2b + 1 & c^2 + 2c + 1 \\ a^2 - 2a + 1 & b^2 - 2b + 1 & c^2 - 2c + 1 \end{vmatrix} \\
= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ a^2 - 2a + 1 & b^2 - 2b + 1 & c^2 - 2c + 1 \end{vmatrix} [R_2' = R_2 - R_3] \\
= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ a^2 - 2a + 1 & b^2 - 2b + 1 & c^2 - 2c + 1 \end{vmatrix} [R_2' = R_2/4] \\
= 4 \begin{vmatrix} a^2 & a & a^2 - 2a + 1 \\ b^2 & b & b^2 - 2b + 1 \\ c^2 & c & c^2 - 2c + 1 \end{vmatrix} [\text{transforming row and column}] \\
= 4 \begin{vmatrix} a^2 - b^2 & a - b & (a^2 - b^2) - 2(a - b) \\ b^2 - c^2 & b - c & (b^2 - c^2) - 2(b - c) \\ c^2 & c & c^2 - 2c + 1 \end{vmatrix} [R_1' = R_1 - R_2 \ \& \ R_2' = R_2 - R_3] \\
= 4 \begin{vmatrix} (a-b)(a+b) & a-b & (a-b)(a+b-2) \\ (b-c)(b+c) & b-c & (b-c)(b+c-2) \\ c^2 & c & c^2 - 2c + 1 \end{vmatrix} \\
= 4(a-b)(b-c) \begin{vmatrix} a+b & 1 & a+b-2 \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2 - 2c + 1 \end{vmatrix} [R_1' = R_1/(a-b) \ \& \ R_2' = R_2/(b-c)] \\
= 4(a-b)(b-c) \begin{vmatrix} a-c & 0 & a-c \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2 - 2c + 1 \end{vmatrix} [R_1' = R_1 - R_2] \\
= 4(a-b)(b-c)(a-c) \begin{vmatrix} 1 & 0 & 1 \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2 - 2c + 1 \end{vmatrix} [R_1' = R_1/(a-c)] \\
= 4(a-b)(b-c)(a-c)(c^2 - 2c + 1 - bc - c^2 + 2c + 0 + bc + c^2 - c^2) [\text{expansion by first row}] \\
= \mathbf{4(a-b)(b-c)(c-a)}
\end{vmatrix}$$

## 22. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix} = -8$$

### Answer

$$\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix} \\
= \begin{vmatrix} x^2 - 4x + 4 & x^2 - 2x + 1 & x^2 \\ x^2 - 2x + 1 & x^2 & x^2 + 2x + 1 \\ x^2 & x^2 + 2x + 1 & x^2 + 4x + 4 \end{vmatrix} \\
= \begin{vmatrix} -2x + 3 & -2x + 1 & -2x - 1 \\ -2x + 1 & -2x - 1 & -2x - 3 \\ x^2 & x^2 + 2x + 1 & x^2 + 4x + 4 \end{vmatrix} [R_1' = R_1 - R_2 \ \& \ R_2' = R_2 - R_3]$$

$$\begin{aligned}
&= \begin{vmatrix} 2 & 2 & 2 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix} [R_1' = R_1 - R_2] \\
&= 2 \begin{vmatrix} 1 & 1 & 1 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix} [R_1' = R_1/2] \\
&= 2 \begin{vmatrix} 1 & -2x+1 & x^2 \\ 1 & -2x-1 & x^2+2x+1 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix} [\text{transforming row and column}] \\
&= 2 \begin{vmatrix} 0 & 2 & -2x-1 \\ 0 & 2 & -2x-3 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix} [R_1' = R_1 - R_2 \text{ \& } R_2' = R_2 - R_3] \\
&= 2 \begin{vmatrix} 0 & 0 & 2 \\ 0 & 2 & -2x-3 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix} [R_1' = R_1 - R_2] \\
&= 2\{0 + 0 + 2(0 - 2)\} [\text{expansion by first row}] \\
&= -8
\end{aligned}$$

### 23. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (m+n)^2 & l^2 & mn \\ (n+l)^2 & m^2 & ln \\ (l+m)^2 & n^2 & lm \end{vmatrix} = (l^2 + m^2 + n^2)(l-m)$$

$$(m-n)(n-l).$$



### Answer

$$\begin{aligned}
&\begin{vmatrix} (m+n)^2 & l^2 & mn \\ (n+l)^2 & m^2 & ln \\ (l+m)^2 & n^2 & lm \end{vmatrix} \\
&= \left(\frac{1}{2}\right) \begin{vmatrix} m^2 + 2mn + n^2 & l^2 & 2mn \\ n^2 + 2nl + l^2 & m^2 & 2ln \\ l^2 + 2lm + m^2 & n^2 & 2lm \end{vmatrix} [C_3' = 2C_3] \\
&= \left(\frac{1}{2}\right) \begin{vmatrix} m^2 + n^2 & l^2 & 2mn \\ n^2 + l^2 & m^2 & 2ln \\ l^2 + m^2 & n^2 & 2lm \end{vmatrix} [C_1' = C_1 - C_3] \\
&= \left(\frac{1}{2}\right) \begin{vmatrix} l^2 + m^2 + n^2 & l^2 & 2mn \\ l^2 + m^2 + n^2 & m^2 & 2ln \\ l^2 + m^2 + n^2 & n^2 & 2lm \end{vmatrix} [C_1' = C_1 + C_2] \\
&= \left(\frac{1}{2}\right) (l^2 + m^2 + n^2) \begin{vmatrix} 1 & l^2 & 2mn \\ 1 & m^2 & 2ln \\ 1 & n^2 & 2lm \end{vmatrix} [C_1' = C_1/(l^2 + m^2 + n^2)] \\
&= \left(\frac{1}{2}\right) (l^2 + m^2 + n^2) \begin{vmatrix} 1 & 1 & 1 \\ l^2 & m^2 & n^2 \\ 2mn & 2ln & 2lm \end{vmatrix} [\text{transforming row and column}] \\
&= \left(\frac{1}{2}\right) (l^2 + m^2 + n^2) \begin{vmatrix} 0 & 0 & 1 \\ l^2 - m^2 & m^2 - n^2 & n^2 \\ -2n(l-m) & -2l(m-n) & 2lm \end{vmatrix} [C_1' = C_1 - C_2 \text{ \& } C_2' = C_2 - C_3]
\end{aligned}$$

$$\begin{aligned}
&= (l^2 + m^2 + n^2)(l-m)(m-n) \begin{vmatrix} 0 & 0 & 1 \\ 1+m & m+n & n^2 \\ -n & -1 & lm \end{vmatrix} [C_1' = C_1/(l-m) \text{ \& } R_2' = C_2/(l-m)] \\
&= (l^2 + m^2 + n^2)(l-m)(m-n)\{0 + 0 - l(l+m) + n(m+n)\} \text{ [expansion by first row]} \\
&= (l^2 + m^2 + n^2)(l-m)(m-n)\{0 + 0 - l(l+m) + n(m+n)\} \\
&= (l^2 + m^2 + n^2)(l-m)(m-n)(-l^2 - ml + mn + n^2) \\
&= (l^2 + m^2 + n^2)(l-m)(m-n)\{(n^2 - l^2) + m(n-l)\} \\
&= \mathbf{(l^2 + m^2 + n^2)(l-m)(m-n)(n-l)(l+m+n)}
\end{aligned}$$

#### 24. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2 + b^2 + c^2)(a-b)(b-c)(c-a)(a+b+c).$$

#### Answer

$$\begin{aligned}
&\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} \\
&= \left(\frac{1}{2}\right) \begin{vmatrix} b^2 + 2bc + c^2 & a^2 & 2bc \\ c^2 + 2ac + a^2 & b^2 & 2ca \\ a^2 + 2ab + b^2 & c^2 & 2ab \end{vmatrix} [C_3' = 2C_3] \\
&= \left(\frac{1}{2}\right) \begin{vmatrix} b^2 + c^2 & a^2 & 2bc \\ c^2 + a^2 & b^2 & 2ca \\ a^2 + b^2 & c^2 & 2ab \end{vmatrix} [C_1' = C_1 - C_3] \\
&= \left(\frac{1}{2}\right) \begin{vmatrix} a^2 + b^2 + c^2 & a^2 & 2bc \\ a^2 + b^2 + c^2 & b^2 & 2ca \\ a^2 + b^2 + c^2 & c^2 & 2ab \end{vmatrix} [C_1' = C_1 + C_2] \\
&= \left(\frac{1}{2}\right)(a^2 + b^2 + c^2) \begin{vmatrix} 1 & a^2 & 2bc \\ 1 & b^2 & 2ca \\ 1 & c^2 & 2ab \end{vmatrix} [C_1' = C_1/(a^2 + b^2 + c^2)] \\
&= \left(\frac{1}{2}\right)(a^2 + b^2 + c^2) \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ 2bc & 2ca & 2ab \end{vmatrix} \text{ [transforming row and column]} \\
&= \left(\frac{1}{2}\right)(a^2 + b^2 + c^2) \begin{vmatrix} 0 & 0 & 1 \\ a^2 - b^2 & b^2 - c^2 & c^2 \\ -2c(a-b) & -2a(b-c) & 2ab \end{vmatrix} [C_1' = C_1 - C_2 \text{ \& } C_2' = C_2 - C_3] \\
&= (a^2 + b^2 + c^2)(a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ a+b & b+c & c^2 \\ -c & -a & ab \end{vmatrix} [C_1' = C_1/(a-b) \text{ \& } C_2' = C_2/(b-c)] \\
&= (a^2 + b^2 + c^2)(a-b)(b-c)\{0 + 0 - a(a+b) + c(b+c)\} \text{ [expansion by first row]} \\
&= (a^2 + b^2 + c^2)(a-b)(b-c)\{0 + 0 - a(a+b) + c(b+c)\} \\
&= (a^2 + b^2 + c^2)(a-b)(b-c)(-a^2 - ba + bc + c^2) \\
&= (a^2 + b^2 + c^2)(a-b)(b-c)\{(c^2 - a^2) + b(c-a)\} \\
&= \mathbf{(a^2 + b^2 + c^2)(a-b)(b-c)(c-a)(a+b+c)}
\end{aligned}$$

## 25. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} = 4a^2b^2c^2.$$

## Answer

$$\begin{aligned} & \begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} \\ &= \begin{vmatrix} 2(b^2 + c^2) & 2(c^2 + a^2) & 2(a^2 + b^2) \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} [R_1' = R_1 + R_2 + R_3] \\ &= 2 \begin{vmatrix} (b^2 + c^2) & (c^2 + a^2) & (a^2 + b^2) \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} [R_1' = R_1/2] \\ &= 2 \begin{vmatrix} c^2 & 0 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} [R_1' = R_1 - R_2] \\ &= 2[c^2\{(c^2 + a^2)(a^2 + b^2) - b^2c^2\} + 0 + a^2\{b^2c^2 - c^2(c^2 + a^2)\}] \text{ [expansion by first row]} \\ &= 2[c^2(c^2a^2 + a^4 + b^2c^2 + a^2b^2 - b^2c^2) + a^2(b^2c^2 - c^4 - a^2c^2)] \\ &= 2[a^2c^4 + a^4c^2 + a^2b^2c^2 + a^2b^2c^2 - a^2c^4 - a^4c^2] \\ &= 4a^2b^2c^2 \end{aligned}$$

## 26. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

## Answer

Operating  $R_1 \rightarrow R_1 + bR_3$ ,  $R_2 \rightarrow R_2 - aR_3$

$$\begin{aligned} & \begin{vmatrix} 1+a^2-b^2+2b^2 & 2ab-2ab & -2b+b-a^2b-b^3 \\ 2ab-2ab & 1-a^2+b^2+2a^2 & 2a-a+a^3+ab^2 \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \\ &= \begin{vmatrix} 1+a^2+b^2 & 0 & -b-a^2b-b^3 \\ 0 & 1+a^2+b^2 & a+a^3+ab^2 \\ 2b & -2a & 1-a^2+b^2 \end{vmatrix} \\ &= \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \end{aligned}$$

Taking  $(1+a^2+b^2)$  from  $R_1$  and  $R_2$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Operating  $R_3 \rightarrow R_3 - 2bR_1 + 2aR_2$

$$= (1 + a^2 + b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1 + a^2 + b^2 \end{vmatrix}$$

Taking  $(1+a^2+b^2)$  from  $R_3$

$$(1 + a^2 + b^2)^3 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding with respect to  $C_1$

$$= (1+a^2+b^2)^3 1 \times [1-0]$$

$$= (1+a^2+b^2)^3$$

Hence proved

## 27. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2).$$

## Answer

Operating  $C_1 \rightarrow aC_1$

$$\frac{1}{a} \begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & a+b & c \end{vmatrix}$$



Operating  $C_1 \rightarrow C_1 + bC_2 + cC_3$

$$= \frac{1}{a} \begin{vmatrix} a^2 + b^2 - bc + c^2 + bc & b-c & c+b \\ a^2 + ac + b^2 + c^2 - ac & b & c-a \\ a^2 - ab + ab + b^2 + c^2 & a+b & c \end{vmatrix}$$

$$= \frac{1}{a} \begin{vmatrix} a^2 + b^2 + c^2 & b-c & c+b \\ a^2 + b^2 + c^2 & b & c-a \\ a^2 + b^2 + c^2 & a+b & c \end{vmatrix}$$

Taking  $(a^2+b^2+c^2)$  common from  $C_1$

$$= \frac{1}{a} (a^2 + b^2 + c^2) \begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix}$$

Operating  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$

$$= \frac{1}{a} (a^2 + b^2 + c^2) \begin{vmatrix} 0 & -c-a & b \\ 0 & -a & -a \\ 1 & a+b & c \end{vmatrix}$$

Operating  $C_2 \rightarrow C_2 - C_3$

$$= \frac{1}{a} (a^2 + b^2 + c^2) \begin{vmatrix} 0 & -(a+b+c) & b \\ 0 & 0 & -a \\ 1 & (a+b+c) & c \end{vmatrix}$$

Taking  $(a+b+c)$  common from  $C_2$

$$= \frac{1}{a}(a^2 + b^2 + c^2)(a + b + c) \begin{vmatrix} 0 & -1 & b \\ 0 & 0 & -a \\ 1 & 1 & c \end{vmatrix}$$

Expanding with respect to  $C_1$

$$= \frac{1}{a}(a^2 + b^2 + c^2)(a + b + c) \times 1 \times (0 - (-a))$$

$$= \frac{1}{a}(a^2 + b^2 + c^2)(a + b + c)(a)$$

$$= (a^2 + b^2 + c^2)(a + b + c)$$

## 28. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0.$$

## Answer

Expanding with  $R_1$

$$= b^2c^2(a^2c + abc - abc - a^2b) - bc(a^3c^2 + a^2bc^2 - a^2b^2c - a^3b^2) + (b+c)(a^3bc^2 - a^3b^2c)$$

$$= a^2b^3c^2 - a^2b^3c^2 - a^3bc^2 - a^2b^3c^2 + a^2b^3c^2 + a^3b^3c + a^3b^2c^2 - a^3b^3c + a^3bc^3 - a^3b^2c^2$$

$$= 0$$

## 29. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

## Answer

$$= \begin{vmatrix} b^2 + c^2 + 2bc & ab & ac \\ ab & a^2 + c^2 + 2ac & bc \\ ac & bc & a^2 + b^2 + 2ab \end{vmatrix}$$

Operating  $R_1 \rightarrow aR_1$ ,  $R_2 \rightarrow bR_2$ ,  $R_3 \rightarrow cR_3$

$$= \frac{1}{abc} \begin{vmatrix} a(b^2 + c^2 + 2bc) & a^2b & a^2c \\ ab^2 & b(a^2 + c^2 + 2ac) & b^2c \\ ac^2 & bc^2 & c(a^2 + b^2 + 2ab) \end{vmatrix}$$

Taking  $a$ ,  $b$ ,  $c$  common from  $C_1$ ,  $C_2$ ,  $C_3$  respectively

$$= \frac{abc}{abc} \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Operating  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (a+c)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (a+c+b)(a+c-b) & b^2 \\ (c-a-b)(c+a+b) & (c-a-b)(c+a+b) & (a+b)^2 \end{vmatrix}$$

Taking  $(a+b+c)$  common from  $R_1, R_2$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$

Operating  $R_3 \rightarrow R_3 - R_1 - R_2$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & a^2 + b^2 + 2ab - a^2 - b^2 \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

Operating  $C_1 \rightarrow aC_1, C_2 \rightarrow bC_2$

$$\frac{(a+b+c)^2}{ab} \begin{vmatrix} a(b+c-a) & 0 & a^2 \\ 0 & b(a+c-b) & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix}$$

Operating  $C_1 \rightarrow C_1 + C_3, C_2 \rightarrow C_2 + C_3$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} a(b+c) & a^2 & a^2 \\ b^2 & b(a+c) & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$$

Taking  $a, b, 2ab$  from  $R_1, R_2, R_3$

$$= \frac{(a+b+c)^2 a \cdot b \cdot 2ab}{ab} \begin{vmatrix} b+c & a & a \\ b & a+c & b \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding with  $R_3$

$$= 2ab(a+b+c)^2 \times 1 \times (ab + ac + bc + c^2 - ab)$$

$$= 2ab(a+b+c)^2 (c(a+b+c))$$

$$= 2abc(a+b+c)^3$$

### 30. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix} = 0.$$

**Answer**

$$= \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$$

Taking (b-a) common from  $C_1, C_3$

$$= (b-a)^2 \begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$$

Operating  $R_2 \rightarrow R_2 - R_1 + R_3$

$$= \begin{vmatrix} b & b-c-b+c & c \\ a & a-b-a+b & b \\ c & c-a-c+a & a \end{vmatrix}$$

$$= (b-a)^2 \begin{vmatrix} b & 0 & c \\ a & 0 & b \\ c & 0 & a \end{vmatrix}$$

[Properties of determinants say that if 1 row or column has only 0 as its elements, the value of the determinant is 0]

$$= 0$$

Hence Proved

### 31. Question

Using properties of determinants prove that:

$$\begin{vmatrix} -a(b^2 + c^2 - a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2 + a^2 - b^2) & 2c^3 \\ 2a^3 & ab^3 & -c(a^2 + b^2 + c^2) \end{vmatrix} = (abc)(a^2 + b^2 + c^2)^3.$$

**Answer**

Taking a, b, c from  $C_1, C_2, C_3$

$$= abc \begin{vmatrix} -b^2 - c^2 + a^2 & 2b^2 & 2c^2 \\ 2a^2 & b^2 - c^2 - a^2 & 2c^2 \\ 2a^2 & 2b^2 & -a^2 - b^2 + c^2 \end{vmatrix}$$

Operating  $R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$$= abc \begin{vmatrix} -b^2 - c^2 - a^2 & 0 & a^2 + b^2 + c^2 \\ 0 & -(a^2 + b^2 + c^2) & a^2 + b^2 + c^2 \\ 2a^2 & 2b^2 & -a^2 - b^2 + c^2 \end{vmatrix}$$

Taking  $(a^2 + b^2 + c^2)$  common from  $R_1, R_2$

$$= abc(a^2 + b^2 + c^2)^2 \begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -a^2 - b^2 + c^2 \end{vmatrix}$$

Operating  $R_3 \rightarrow R_3 + R_1 + R_2$

$$= abc(a^2 + b^2 + c^2)^2 \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 2a^2 & 2b^2 & a^2 + b^2 + c^2 \end{vmatrix}$$

Taking  $(a^2 + b^2 + c^2)$  common from  $C_3$  ♦

$$= abc(a^2 + b^2 + c^2)^3 \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 2a^2 & 2b^2 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$= abc(a^2 + b^2 + c^2)^3 \times 1 \times (1-0)$$

$$= abc (a^2 + b^2 + c^2)^3$$

Hence proved

### 32. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0, \text{ where } \alpha, \beta, \gamma \text{ are in AP.}$$

### Answer

Given that  $\alpha, \beta, \gamma$  are in an AP, which means  $2\beta = \alpha + \gamma$

Operating  $R_3 \rightarrow R_3 - 2R_2 + R_1$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1-2x+4+x-3 & x-2-2x+6+x-4 & x-\gamma-2x+2\beta+x-\alpha \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & -\gamma+2\beta-\alpha \end{vmatrix} \text{ [we know that } 2\beta = \alpha + \gamma]$$

Operating  $R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & -\gamma + \alpha + \gamma - \alpha \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & 0 \end{vmatrix}$$

[By the properties of determinants, we know that if all the elements of a row or column is 0, then the value of the determinant is also 0]

$$= 0$$

Hence proved

### 33. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

### Answer

Operating  $R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} (a+1)(a+2) - (a+2)(a+3) & a+2 - a - 3 & 0 \\ (a+2)(a+3) - (a+3)(a+4) & a+3 - a - 4 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a+2)(a+1 - a - 3) & -1 & 0 \\ (a+3)(a+2 - a - 4) & -1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -2(a+2) & -1 & 0 \\ -2(a+3) & -1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$= (2(a+2) - 2(a+3))$$

$$= (2a+4-2a-6)$$

$$= -2$$

### 34. Question

If  $x \neq y \neq z$  and  $\begin{vmatrix} x & x^3 & x^4 - 1 \\ y & y^3 & y^4 - 1 \\ z & z^3 & z^4 - 1 \end{vmatrix} = 0$ , prove that  $xyz(xy + yz + zx) = (x + y + z)$ .

### Answer

By properties of determinants, we can split the given determinant into 2 parts

$$\rightarrow 0 = \begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Taking  $x, y, z$  common from  $R_1, R_2, R_3$  respectively

$$\rightarrow 0 = xyz \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Operating  $R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$$\rightarrow 0 = xyz \begin{vmatrix} 0 & x^2 - z^2 & x^3 - z^3 \\ 0 & y^2 - z^2 & y^3 - z^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x - z & x^3 - z^3 & 0 \\ y - z & y^3 - z^3 & 0 \\ z & z^3 & 1 \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} x - z & (x - z)(x^2 + xz + z^2) & 0 \\ y - z & (y - z)(y^2 + yx + z^2) & 0 \\ z & z^3 & 1 \end{vmatrix} = xyz \begin{vmatrix} 0 & (x - z)(x + z) & (x - z)(x^2 + xz + z^2) \\ 0 & (y - z)(y + z) & (y - z)(y^2 + yz + z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

Taking  $(x-z)$  and  $(y-z)$  common from  $R_1, R_2$

$$\rightarrow (x - z)(y - z) \begin{vmatrix} 1 & (x^2 + xz + z^2) & 0 \\ 1 & (y^2 + yz + z^2) & 0 \\ z & z^3 & 1 \end{vmatrix} = (x - z)(y - z) \begin{vmatrix} 0 & x + z & (x^2 + xz + z^2) \\ 0 & y + z & (y^2 + yz + z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

Expanding with  $R_3$

$$\rightarrow y^2 + yz + z^2 - x^2 - xz - z^2 = xyz(xy^2 + xyz + xz^2 + zy^2 + yz^2 + z^3 - x^2y - xyz - yz^2 - x^2z - xz^2 - z^3)$$

$$\rightarrow (y-x)(y+x) + z(y-x) = xyz(xy^2 + zy^2 - x^2y - x^2z)$$

$$\rightarrow (y-x)(x+y+z) = xyz(xy(y-x) + z(y^2 - x^2))$$

$$\rightarrow (y-x)(x+y+z) = xyz(xy(y-x) + z(x+y)(y-x))$$

$$\rightarrow (y-x)(x+y+z) = xyz(xy(y-x) + (xz + yz)(y-x))$$

$$\rightarrow (y-x)(x+y+z) = xyz(y-x)(xy + xz + yz)$$

$$\rightarrow x + y + z = xyz(xy + xz + yz)$$

Hence Proved

### 35. Question

$$\text{Prove that } \begin{vmatrix} 1 & a^2 + bc & a^3 \\ 1 & b^2 + ca & b^3 \\ 1 & c^2 + ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2 + b^2 + c^2).$$

### Answer

Operating  $R_1 \rightarrow R_1 - R_2$ ,  $R_2 \rightarrow R_2 - R_3$

$$\begin{aligned} &= \begin{vmatrix} 0 & a^2 + bc - b^2 - ac & a^3 - b^3 \\ 0 & b^2 + ca - c^2 - ab & b^3 - c^3 \\ 1 & c^2 + ab & c^3 \end{vmatrix} \\ &= \begin{vmatrix} 0 & (a-b)(a+b) - c(a-b) & (a-b)(a^2 + ab + b^2) \\ 0 & (b-c)(b+c) - a(b-c) & (b-c)(b^2 + bc + c^2) \\ 1 & c^2 + ab & c^3 \end{vmatrix} \end{aligned}$$

Taking (a-b), (b-c) common from  $R_1$ ,  $R_2$  respectively

$$= (a-b)(b-c) \begin{vmatrix} 0 & a+b-c & a^2 + ab + b^2 \\ 0 & b+c-a & b^2 + bc + c^2 \\ 1 & c^2 + ab & c^3 \end{vmatrix}$$

Operating  $R_1 \rightarrow R_1 - R_2$

$$\begin{aligned} &= (a-b)(b-c) \begin{vmatrix} 0 & 2a-2c & a^2 + ab - bc - c^2 \\ 0 & b+c-a & b^2 + bc + c^2 \\ 1 & c^2 + ab & c^3 \end{vmatrix} \\ &= (a-b)(b-c) \begin{vmatrix} 0 & 2(a-c) & (a+c)(a-c) + b(a-c) \\ 0 & b+c-a & b^2 + bc + c^2 \\ 1 & c^2 + ab & c^3 \end{vmatrix} \end{aligned}$$

Taking (a-c) common from  $R_1$

$$= (a-c)(a-b)(b-c) \begin{vmatrix} 0 & 2 & a+b+c \\ 0 & b+c-a & b^2 + bc + c^2 \\ 1 & c^2 + ab & c^3 \end{vmatrix}$$

Expanding with  $C_1$

$$\begin{aligned} &= (a-c)(a-b)(b-c) \times (2b^2 + 2bc + 2c^2 - ab - b^2 - bc - ac - bc - c^2 + a^2 + ab + ac) \\ &= -(c-a)(b-c)(a-b)(a^2 + b^2 + c^2) \end{aligned}$$

Hence Proved

### 36. Question

Without expanding the determinant, prove that:

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

### Answer

Operating  $R_1 \rightarrow R_1 - R_2$ ,  $R_2 \rightarrow R_2 - R_3$

$$\rightarrow \begin{vmatrix} 0 & a-b & bc-ac \\ 0 & b-c & ac-ab \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} 0 & a-b & -c(a-b) \\ 0 & b-c & -a(b-c) \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & (a-b)(a+b) \\ 0 & b-c & (b-c)(b+c) \\ 1 & c & c^2 \end{vmatrix}$$

Taking (a-b) and (b-c) from R<sub>1</sub>, R<sub>2</sub>

$$\rightarrow (a-b)(b-c) \begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c) \begin{vmatrix} 0 & 1 & (a+b) \\ 0 & 1 & (b+c) \\ 1 & c & c^2 \end{vmatrix}$$

Method 1:

For the two determinants to be equal, their difference must be 0.

$$= \begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix} - \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0-0 & 1-1 & -(a+b+c) \\ 0-0 & 1-1 & -(a+b+c) \\ 1-1 & c-c & ab-c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & -(a+b+c) \\ 0 & 0 & -(a+b+c) \\ 0 & 0 & ab-c^2 \end{vmatrix}$$

Since 2 columns have only 0 as their elements, by properties of determinants

=0

Method 2:

Expanding both with C<sub>1</sub>

LHS

$$=(a-b)(b-c)(-a+c)$$

RHS

$$=(a-b)(b-c)(b+c-a-b)$$

$$=(a-b)(b-c)(-a+c)$$

∴ LHS = RHS

### 37. Question

Without expanding the determinant, prove that:

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix}$$

### Answer

Operating R<sub>1</sub>→R<sub>1</sub>-R<sub>3</sub>, R<sub>2</sub>→R<sub>2</sub>-R<sub>3</sub>

$$\begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & bc-ab & b+c-a-b \\ 0 & ac-ab & c+a-a-b \\ 1 & ab & a+b \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} 0 & a-c & (a-c)(a+c) \\ 0 & b-c & (b-c)(b+c) \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & -b(a-c) & -(a-c) \\ 0 & -a(b-c) & -(b-c) \\ 1 & ab & a+b \end{vmatrix}$$

Taking (a-c) and (b-c) common from  $R_1, R_2$

$$\rightarrow (a-c)(b-c) \begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} = (a-c)(b-c) \begin{vmatrix} 0 & -b & -1 \\ 0 & -a & -1 \\ 1 & ab & a+b \end{vmatrix}$$

Method 1:

If the determinants are equal, their difference must also be equal.

(a-c) and (b-c) get cancelled.

$$= \begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 0 & -b & -1 \\ 0 & -a & -1 \\ 1 & ab & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 0-0 & 1+b & a+c+1 \\ 0-0 & 1+a & b+c+1 \\ 1-1 & c-ab & c^2+a+b \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1+b & a+c+1 \\ 0 & 1+a & b+c+1 \\ 0 & c-ab & c^2+a+b \end{vmatrix}$$

Since all elements of  $C_1$  are 0, by properties of determinants,

=0

$\therefore$  The 2 determinants are equal.

Method 2:

Expanding with  $C_1$

$$\rightarrow (a-c)(b-c)(b+c-a-c) = (a-c)(b-c)(b-a)$$

$$\rightarrow (a-c)(b-c)(b-a) = (a-c)(b-c)(b-a)$$

$\therefore$  RHS and LHS are equal

### 38. Question

Show that  $x = 2$  is a root of the equation  $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix} = 0$ .

### Answer

Operating  $R_1 \rightarrow R_1 - R_2$

$$0 = \begin{vmatrix} x-2 & -6+3x & -1-x+3 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

$$0 = \begin{vmatrix} x-2 & 3(x-2) & -(x-2) \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

Taking (x-2) common from  $R_1$

$$0 = (x-2) \begin{vmatrix} 1 & 1 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

Here, we can see that x-2 is a factor of the determinant.

We can say that when x-2 is put in the equation, we get 0.

$$x-2=0$$

$$\rightarrow x=2$$

### 39. Question

Solve the following equations:

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = 0$$

### Answer

Operating  $R_1 \rightarrow R_1 - R_2$ ,  $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 0 & x-b & x^3-b^3 \\ 0 & b-c & b^3-c^3 \\ 1 & c & c^3 \end{vmatrix} = 0$$

$$0 = \begin{vmatrix} 0 & x-c & (x-b)^3 + 3xb(x-b) \\ 0 & b-c & (b-c)^3 + 3bc(b-c) \\ 1 & c & c^3 \end{vmatrix}$$

$$0 = (x-c)(b-c) \begin{vmatrix} 0 & 1 & (x-b)^2 + 3xb \\ 0 & 1 & (b-c)^2 + 3bc \\ 1 & c & c^3 \end{vmatrix}$$

Expanding with  $C_1$

$$0 = (x-c)(b-c)(b^2 - 2bc + c^2 + 3bc - x^2 + 2xb - b^2 - 3xb)$$

$$0 = (x-c)(b-c)(bc + c^2 - x^2 - xb)$$

$$0 = (x-c)(b-c)(-b(-c+x) - (c-x)(-c-x))$$

$$0 = (x-c)^2(b-c)(-b-c-x)$$

Either  $x-c=0$  or  $b-c=0$  or  $(-b-c-x)=0$

$$\therefore x=c \text{ or } b=c \text{ or } x=-(b+c)$$

If  $b=c$ ,  $x=b$

$$\therefore x=c \text{ or } x=b \text{ or } x=-(b+c)$$

### 40. Question

Solve the following equations:

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ b & b & x+c \end{vmatrix} = 0$$

### Answer

Operating  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

Taking  $(x+a+b+c)$  common from  $C_1$

$$(x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

Operating  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$

$$(x+a+b+c) \begin{vmatrix} 0 & 0 & -x \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix} = 0$$

Expanding with  $C_1$

$$0 = (x+a+b+c)(0+x^2)$$

$$0 = x^2(x+a+b+c)$$

Either  $x^2=0$  or  $(x+a+b+c)=0$

$$\therefore x=0 \text{ or } x=-(a+b+c)$$

#### 41. Question

Solve the following equations:

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

#### Answer

Operating  $C_1 \rightarrow C_1 + C_2 + C_3$

$$0 = \begin{vmatrix} 3x-8+3+3 & 3 & 3 \\ 3+3x-8+3 & 3x-8 & 3 \\ 3+3+3x-8 & 3 & 3x-8 \end{vmatrix}$$

$$0 = \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix}$$

Taking  $(3x-2)$  common from  $C_1$

$$0 = (3x-2) \begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Operating  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$

$$0 = (3x-2) \begin{vmatrix} 0 & 0 & -(3x-11) \\ 0 & 3x-11 & -3x+11 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Expanding with  $C_1$

$$0 = (3x-2)(0+(3x-11)^2)$$

$$0 = (3x-2)(3x-11)^2$$

Either  $3x-2=0$  or  $3x-11=0$

$$\therefore x = \frac{2}{3} \text{ or } x = \frac{11}{3}$$

#### 42. Question

Solve the following equations:

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

**Answer**Operating  $C_1 \rightarrow C_1 + C_2 + C_3$ 

$$0 = \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix}$$

Taking  $(x+9)$  common from  $C_1$ 

$$0 = (x+9) \begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix}$$

Operating  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$ 

$$0 = (x+9) \begin{vmatrix} 0 & 0 & 1-x \\ 0 & x-1 & 1-x \\ 1 & 3 & x+4 \end{vmatrix}$$

$$0 = (x+9)(0-x+x^2+1-x)$$

$$0 = (x+9)(x^2-2x+1)$$

$$0 = (x+9)(x-1)^2$$

 $\therefore$  Either  $x+9=0$  or  $x-1=0$ 

$$x=-9, x=1$$

**43. Question**

Solve the following equations:

$$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

**Answer**Operating  $R_1 \rightarrow R_1 + R_2 + R_3$ 

$$0 = \begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

Taking  $(x+9)$  common from  $R_1$ 

$$0 = (x+9) \begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

Operating  $C_1 \rightarrow C_1 - C_3$ ,  $C_2 \rightarrow C_2 - C_3$ 

$$0 = (x+9) \begin{vmatrix} 0 & 0 & 1 \\ 0 & x-2 & 2 \\ 7-x & 6-x & x \end{vmatrix}$$

Expanding with  $R_1$ 

$$0 = (x+9)(0-(x-2)(7-x))$$

$$0 = (x+9)(7-x)(2-x)$$

Either  $x+9=0$  or  $7-x=0$  or  $2-x=0$ 

$$\therefore x=-9 \text{ or } x=7 \text{ or } x=2$$

**44. Question**

Solve the following equations:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

**Answer**

Expanding with R1

$$0 = x(-3x^2 - 6x - 2x^2 + 6x) + 6(2x + 4 + 3x - 9) - 1(4x - 9x)$$

$$0 = x(-5x^2) + 6(5x - 5) - 1(-5x)$$

$$0 = -5x^3 + 30x - 30 + 5x$$

$$0 = -5x^3 + 35x - 30$$

$$x^3 - 7x + 6 = 0$$

$$x^3 - x - 6x + 6 = 0$$

$$x(x^2 - 1) - 6(x - 1) = 0$$

$$x(x-1)(x+1) - 6(x-1) = 0$$

$$(x-1)(x^2 + x - 6) = 0$$

$$(x-1)(x^2 + 3x - 2x - 6) = 0$$

$$(x-1)(x(x+3) - 2(x+3)) = 0$$

$$(x-1)(x+3)(x-2) = 0$$

Either  $x-1=0$  or  $x+3=0$  or  $x-2=0$

$\therefore x=1$  or  $x=-3$  or  $x=2$



**45. Question**

Prove that

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2 + b^2 + c^2)$$

**Answer**

Operating  $C_1 \rightarrow aC_1$

$$= \frac{1}{a} \begin{vmatrix} a^2 & b-c & c+b \\ a^2 + ac & b & c-a \\ a^2 - ab & b+a & c \end{vmatrix}$$

Operating  $C_1 \rightarrow C_1 + bC_2 + cC_3$

$$= \frac{1}{a} \begin{vmatrix} a^2 + b^2 + c^2 & b-c & c+b \\ a^2 + b^2 + c^2 & b & c-a \\ a^2 + b^2 + c^2 & b+a & c \end{vmatrix}$$

Taking  $(a^2 + b^2 + c^2)$

$$= \frac{a^2 + b^2 + c^2}{a} \begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$$

Operating  $C_2 \rightarrow C_2 - bC_1$ ,  $C_3 \rightarrow C_3 - cC_1$

$$= \frac{a^2 + b^2 + c^2}{a} \begin{vmatrix} 1 & -c & b \\ 1 & 0 & -a \\ 1 & a & 0 \end{vmatrix}$$

Expanding with  $R_3$

$$= \frac{a^2 + b^2 + c^2}{a} (ac - 0 + a^2 + ab)$$

$$= \frac{a^2 + b^2 + c^2}{a} a(a+b+c)$$

$$= (a^2 + b^2 + c^2)(a+b+c)$$

Hence Proved

## Exercise 6C

### 1 A. Question

Find the area of the triangle whose vertices are:

A(3, 8), B(-4, 2) and C(5, -1)

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$= \frac{1}{2} [(4 - 10) - (-3 - 40) + (6 + 32)]$$

$$= \frac{1}{2} [-6 + 43 + 38]$$

$$= \frac{75}{2}$$

= 37.5 sq. units

### 1 B. Question

Find the area of the triangle whose vertices are:

A(-2, 4), B(2, -6) and C(5, 4)

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$\begin{aligned}
&= \frac{1}{2} [(8 + 30) - (-8 - 20) + (12 - 8)] \\
&= \frac{1}{2} [38 + 28 + 4] \\
&= \frac{68}{2} \\
&= 34 \text{ sq. units}
\end{aligned}$$

### 1 C. Question

Find the area of the triangle whose vertices are:

A(-8, -2), B(-4, -6) and C(-1, 5)

### Answer

$$\begin{aligned}
\text{Area of a triangle} &= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \\
&= \frac{1}{2} \begin{vmatrix} -8 & -2 & 1 \\ -4 & -6 & 1 \\ -1 & 5 & 1 \end{vmatrix}
\end{aligned}$$

Expanding with  $R_3$

$$\begin{aligned}
&= \frac{1}{2} [(-20 - 6) - (-40 - 2) + (48 - 8)] \\
&= \frac{1}{2} [-26 + 42 + 40] \\
&= \frac{56}{2}
\end{aligned}$$

= 28 sq. units



### 1 D. Question

Find the area of the triangle whose vertices are:

P(0, 0), Q(6, 0) and R(4, 3)

### Answer

$$\begin{aligned}
\text{Area of a triangle} &= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \\
&= \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}
\end{aligned}$$

Expanding with  $R_1$

$$= \frac{1}{2} [18]$$

= 9 sq. units

### 1 E. Question

Find the area of the triangle whose vertices are:

P(1, 1), Q(2, 7) and R(10, 8)

### Answer

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 2 & 7 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Operating  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$

$$= \frac{1}{2} \begin{vmatrix} -9 & -7 & 0 \\ -8 & -1 & 0 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$= \frac{1}{2} [9 - 56]$$

$$= \frac{1}{2} [-47]$$

$$= \frac{-47}{2}$$

$$= -23.5 \text{ sq. units} = 23.5 \text{ sq units}$$

## 2 A. Question

Use determinants to show that the following points are collinear.

A(2, 3), B(-1, -2) and C(5, 8)

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$= \frac{1}{2} [(-8 + 10) - (16 - 15) + (-4 + 3)] = \frac{1}{2} [2 - 1 - 1]$$

$$= 0$$

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

## 2 B. Question

Use determinants to show that the following points are collinear.

A(3, 8), B(-4, 2) and C(10, 14)

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 10 & 14 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$\begin{aligned}
&= \frac{1}{2} [ (-56 - 20) - (42 - 80) + (6 + 32) ] \\
&= \frac{1}{2} [ -76 + 38 + 38 ] \\
&= 0
\end{aligned}$$

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

### 2 C. Question

Use determinants to show that the following points are collinear.

P(-2, 5), Q(-6, -7) and R(-5, -4)

**Answer**

$$\begin{aligned}
\text{Area of a triangle} &= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \\
&= \frac{1}{2} \begin{vmatrix} -2 & 5 & 1 \\ -6 & -7 & 1 \\ -5 & -4 & 1 \end{vmatrix}
\end{aligned}$$

Expanding with  $C_3$

$$\begin{aligned}
&= \frac{1}{2} [ (24 - 35) - (8 + 25) + (14 + 30) ] = \frac{1}{2} [ -11 - 33 + 44 ] \\
&= 0
\end{aligned}$$

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

### 3. Question

Find the value of k for which the points A(3, -2), B(k, 2) and C(8, 8) are collinear.

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2} \begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$\rightarrow 0 = \frac{1}{2} [ (8k - 16) - (24 + 16) + (6 + 2k) ]$$

$$\rightarrow 0 = \frac{1}{2} [ 10k - 50 ]$$

$$\rightarrow 10k - 50 = 0$$

$$\rightarrow 10k = 50$$

$$\therefore k = 5$$

### 4. Question

Find the value of k for which the points P(5, 5), Q(k, 1) and R(11, 7) are collinear.

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2} \begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$\rightarrow 0 = (7k-11)-(35-55)+(5-5k)$$

$$\rightarrow 0 = 2k-14$$

$$\rightarrow 2k=14$$

$$\therefore k=7$$

### 5. Question

Find the value of k for which the points A(1, -1), B(2, k) and C(4, 5) are collinear.

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2} \begin{vmatrix} 1 & -1 & 1 \\ 2 & k & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$\rightarrow 0 = (10-4k)-(5+4)+(k+2)$$

$$\rightarrow 0 = -3k+3$$

$$\rightarrow 3k=3$$

$$\therefore k=1$$

### 6. Question

Find the value of k for which the area of aABC having vertices A(2, -6), B(5, 4) and C(k, 4) is 35 sq units.

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$35 = \frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

Expanding with  $C_3$

$$\rightarrow 70 = (20-4k)-(8+6k)+(8+30)$$

$$\rightarrow 70 = -10k+50$$

$$\rightarrow 20 = -2k$$

$$\rightarrow k = -2$$

### 7. Question



If A(-2, 0), B(0, 4) and C(0, k) be three points such that area of a ABC is 4 sq units, find the value of k.

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$4 = \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding with  $C_1$

$$\rightarrow 8 = -2(4-k)$$

$$\rightarrow -4 = 4-k$$

$$\rightarrow k = 8$$

**8. Question**

If the points A(a, 0), B(0, b) and C(1, 1) are collinear, prove that  $\frac{1}{a} + \frac{1}{b} = 1$ .

**Answer**

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since the points are collinear, the area they enclose is 0

$$0 = \frac{1}{2} \begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding with  $C_1$

$$\rightarrow 0 = a(b-1) + (-b)$$

$$\rightarrow 0 = ab - a - b$$

$$\rightarrow a + b = ab$$

$$\rightarrow \frac{a+b}{ab} = 1$$

$$\rightarrow \frac{1}{a} + \frac{1}{b} = 1$$

Hence proved

**Objective Questions**

**1. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix} = ?$$

A. 1

B. 0

C.  $\cos 50^\circ$

D.  $\sin 50^\circ$



## Answer

To find: Value of  $\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix}$

Formula used: (i)  $\cos \theta = \sin (90 - \theta)$

We have,  $\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix}$

On expanding the above,

$$\Rightarrow \{\cos 70^\circ\} \{\cos 20^\circ\} - \{\sin 70^\circ\} \{\sin 20^\circ\}$$

On applying formula  $\cos \theta = \sin (90 - \theta)$

$$\Rightarrow \{\sin (90 - 70)\} \{\sin (90 - 20)\} - \{\sin 70^\circ\} \{\sin 20^\circ\}$$

$$\Rightarrow \{\sin 20^\circ\} \{\sin 70^\circ\} - \{\sin 70^\circ\} \{\sin 20^\circ\}$$

$$= 0$$

## 2. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix} = ?$$

A. 1

B.  $\frac{1}{2}$

C.  $\frac{\sqrt{3}}{2}$

D. none of these



## Answer

To find: Value of  $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$

Formula used: (i)  $\cos (A + B) = \cos A \cos B - \sin A \sin B$

We have,  $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$

On expanding the above,

$$\Rightarrow \{\cos 15^\circ\} \{\cos 15^\circ\} - \{\sin 15^\circ\} \{\sin 15^\circ\}$$

On applying formula  $\cos (A + B) = \cos A \cos B - \sin A \sin B$

$$= \cos (15 + 15)$$

$$= \cos (30^\circ)$$

$$= \frac{\sqrt{3}}{2}$$

## 3. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \sin 23^\circ & -\sin 7^\circ \\ \cos 23^\circ & \cos 7^\circ \end{vmatrix} = ?$$

A.  $\frac{\sqrt{3}}{2}$

B.  $\frac{1}{2}$

C.  $\sin 16^\circ$

D.  $\cos 16^\circ$

**Answer**

To find: Value of  $\begin{vmatrix} \sin 23^\circ & -\sin 7^\circ \\ \cos 23^\circ & \cos 7^\circ \end{vmatrix}$

Formula used: (i)  $\sin(A + B) = \sin A \cos B + \cos A \sin B$

We have,  $\begin{vmatrix} \sin 23^\circ & -\sin 7^\circ \\ \cos 23^\circ & \cos 7^\circ \end{vmatrix}$

On expanding the above,

$$\Rightarrow (\sin 23^\circ)(\cos 7^\circ) - (\cos 23^\circ)(-\sin 7^\circ)$$

$$\Rightarrow (\sin 23^\circ)(\cos 7^\circ) + (\cos 23^\circ)(\sin 7^\circ)$$

On applying formula  $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$= \sin(23 + 7)$$

$$= \sin(30^\circ)$$

$$= \frac{1}{2}$$

**4. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a + ib & c + id \\ -c + id & a - id \end{vmatrix} = ?$$

A.  $(a^2 + b^2 - c^2 - d^2)$

B.  $(a^2 - b^2 + c^2 - d^2)$

C.  $(a^2 + b^2 + c^2 + d^2)$

D. none of these

**Answer**

To find: Value of  $\begin{vmatrix} a + ib & c + id \\ -c + id & a - id \end{vmatrix}$

Formula used:  $i^2 = -1$

We have,  $\begin{vmatrix} a + ib & c + id \\ -c + id & a - id \end{vmatrix}$

On expanding the above,

$$\Rightarrow (a + ib)(a - id) - (-c + id)(c + id)$$



$$\begin{aligned} &\Rightarrow (a^2 - iab + iba - i^2b^2) - (-c^2 - icd + icd + i^2d^2) \\ &\Rightarrow \{a^2 - iab + iba - (-1)b^2\} - \{-c^2 - icd + icd + (-1)d^2\} \\ &\Rightarrow \{a^2 - iab + iba + 1b^2\} - \{-c^2 - icd + icd - 1d^2\} \\ &\Rightarrow a^2 + b^2 + c^2 + d^2 \end{aligned}$$

### 5. Question

Mark the tick against the correct answer in the following:

If  $\omega$  is a complex root of unity then  $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = ?$

- A. 1
- B. -1
- C. 0
- D. none of these

### Answer

To find: Value of  $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

Formula used:  $\omega^3 = 1$

We have,  $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$



On expanding the above along 1<sup>st</sup> column

$$\begin{aligned} &\Rightarrow 1 \begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix} \\ &\Rightarrow [1\{(\omega^2)(\omega) - (1)(1)\}] - [\omega\{(\omega)(\omega) - (\omega^2)(1)\}] + [\omega^2\{(\omega)(1) - (\omega^2)(\omega^2)\}] \\ &\Rightarrow [1\{\omega^3 - 1\}] - [\omega\{\omega^2 - \omega^2\}] + [\omega^2\{\omega - \omega^4\}] \dots (i) \end{aligned}$$

As  $\omega^3 = 1$ ,

$\Rightarrow \omega^3 \cdot \omega = 1 \cdot \omega$

$\Rightarrow \omega^4 = \omega$

Using the above obtained value of  $\omega^4$  in eqn. (i)

$$\begin{aligned} &\Rightarrow [1\{\omega^3 - 1\}] - [\omega\{\omega^2 - \omega^2\}] + [\omega^2\{\omega - \omega\}] \\ &\Rightarrow 1\{\omega^3 - 1\} \\ &\Rightarrow \omega^3 - 1 \\ &\Rightarrow 1 - 1 = 0 \end{aligned}$$

### 6. Question

Mark the tick against the correct answer in the following:

If  $\omega$  is a complex cube root of unity then the value of  $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$  is

- A. 2
- B. 4
- C. 0
- D. -3

**Answer**

To find: Value of  $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$

Formula used: (i)  $\omega^3 = 1$

(ii)  $1+\omega+\omega^2 = 0$

We have,  $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$

On expanding the above along 1<sup>st</sup> column

$$\Rightarrow 1 \begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega^2 \\ 1 & \omega \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix}$$

$$\Rightarrow [1\{(\omega^2)(\omega)-(1)(1)\}] - [\omega\{(\omega)(\omega)-(\omega^2)(1)\}] + [\omega^2\{(\omega)1-(\omega^2)(\omega^2)\}]$$

$$\Rightarrow [1\{\omega^3-1\}] - [\omega\{\omega^2-\omega^2\}] + [\omega^2\{\omega-\omega^4\}] \dots (i)$$

As  $\omega^3 = 1$ ,

$$\Rightarrow \omega^3, \omega=1, \omega$$

$$\Rightarrow \omega^4 = \omega$$

Using the above obtained value of  $\omega^4$  in eqn. (i)

$$\Rightarrow [1\{\omega^3-1\}] - [\omega\{\omega^2-\omega^2\}] + [\omega^2\{\omega-\omega\}]$$

$$\Rightarrow 1\{\omega^3-1\}$$

$$\Rightarrow \omega^3-1$$

$$\Rightarrow 1 - 1 = 0$$

**7. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = ?$$

- A. 8
- B. -8
- C. 16

**Answer**

To find: Value of  $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

We have,  $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying  $R_1 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 8 & 12 & 16 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 8 & 12 & 16 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$$

Taking 4 common from  $R_1$

$$\Rightarrow 4 \begin{vmatrix} 2 & 3 & 4 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow 4 \begin{vmatrix} -2 & 0 & 4 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$$

Taking -2 common from  $R_1$

$$\Rightarrow (4)(-2) \begin{vmatrix} 1 & 0 & -2 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying  $R_1 \rightarrow 9R_1$

$$\Rightarrow \frac{-8}{9} \begin{vmatrix} 9 & 0 & -18 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \frac{-8}{9} \begin{vmatrix} 9 & 0 & -18 \\ 4 & 3 & 0 \\ 0 & 16 & 43 \end{vmatrix}$$

Taking 9 common from  $R_1$

$$\Rightarrow -8 \begin{vmatrix} 1 & 0 & -2 \\ 4 & 3 & 0 \\ 0 & 16 & 43 \end{vmatrix}$$

Expanding along  $R_1$

$$\Rightarrow -8 [1[(3)(43)-(16)(0)] - 0 [(4)(43)-(0)(0)] - 2 [(4)(16)-(3)(0)]]$$



$$\Rightarrow -8 [[(129)-(0)] - 2 [(64)-(0)]]$$

$$\Rightarrow -8 [129 - 128]$$

$$\Rightarrow -8$$

### 8. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = ?$$

A. 2

B. 6

C. 24

D. 120

### Answer

To find: Value of  $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$

We have,  $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 6 \\ 2 & 6 & 24 \\ 6 & 24 & 120 \end{vmatrix}$$

Taking 2 common from  $R_2$

$$\Rightarrow 2 \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 12 \\ 6 & 24 & 120 \end{vmatrix}$$

Taking 6 common from  $R_3$

$$\Rightarrow 2 \times 6 \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 12 \\ 1 & 4 & 20 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow 12 \begin{vmatrix} 1 & 2 & 6 \\ 0 & 1 & 6 \\ 1 & 4 & 20 \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow 12 \begin{vmatrix} 1 & 2 & 6 \\ 0 & 1 & 6 \\ 0 & 2 & 14 \end{vmatrix}$$

Expanding column 1

$$\Rightarrow 12 [1\{(1)(14)-(6)(2)\}]$$

$$\Rightarrow 12 [1\{(14)-(12)\}]$$

$$\Rightarrow 12[2]$$



⇒ 24

### 9. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = ?$$

- A.  $(a + b + c)$
- B.  $3(a + b + c)$
- C.  $3abc$
- D. 0

### Answer

To find: Value of  $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

We have,  $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

Applying  $R_1 \rightarrow R_1 + R_2$

$$\Rightarrow \begin{vmatrix} a-b+b-c & b-c+c-a & c-a+a-b \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 + R_3$

$$\Rightarrow \begin{vmatrix} a-c+c-a & b-a+a-b & c-b+b-c \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

If every element of a row is 0 then the value of the determinant will be 0

### 10. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = ?$$

- A. 0
- B. 1
- C. -1
- D. none of these

### Answer



To find: Value of  $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

We have,  $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

Applying  $R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-2 \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - 3R_1$

$$\Rightarrow \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-2 \\ 0 & 3 & 3p-2 \end{vmatrix}$$

Expanding along  $C_1$

$$\Rightarrow [1\{(1)(3p-2)-(3)(p-2)\}]$$

$$\Rightarrow 1$$

### 11. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = ?$$



- A.  $(a - b)(b - c)(c - a)$
- B.  $-(a - b)(b - c)(c - a)$
- C.  $(a - b)(b - c)(c - a)(a + b + c)$
- D.  $abc(a - b)(b - c)(c - a)$

### Answer

To find: Value of  $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

We have,  $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

Applying  $C_2 \rightarrow C_2 - C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ a & b-a & c \\ a^3 & b^3-a^3 & c^3 \end{vmatrix}$$

Applying  $C_3 \rightarrow C_3 - C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$$

We know,  $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & (b-a)(b^2+ab+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

Taking (b-a) common from  $C_2$

$$\Rightarrow (b-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & c-a \\ a^3 & (b^2+ab+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

Taking (c-a) common from  $C_2$

$$\Rightarrow (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & (b^2+ab+a^2) & (c^2+ca+a^2) \end{vmatrix}$$

Expanding along  $C_1$

$$\Rightarrow (b-a)(c-a)[1\{(1)(c^2+ca+a^2) - (b^2+ab+a^2)(1)\}]$$

$$\Rightarrow (b-a)(c-a)[c^2+ca+a^2 - b^2 - ab - a^2]$$

$$\Rightarrow (b-a)(c-a)[c^2 - b^2 + ca - ab]$$

$$\Rightarrow (b-a)(c-a)[(c-b)(c+b) + a(c-b)]$$

$$\Rightarrow (b-a)(c-a)[(a+b+c)(c-b)]$$

$$\Rightarrow (a-b)(b-c)(c-a)(a+b+c)$$

## 12. Question

Mark the tick against the correct answer in the following: our Class, Your Page.

$$\begin{vmatrix} \sin \alpha & \cos \alpha & \sin(\alpha + \delta) \\ \sin \beta & \cos \beta & \sin(\beta + \delta) \\ \sin \gamma & \cos \gamma & \sin(\gamma + \delta) \end{vmatrix} = ?$$

- A. 0
- B. 1
- C.  $\sin(\alpha + \delta) + \sin(\beta + \delta) + \sin(\gamma + \delta)$
- D. none of these

## Answer

To find: Value of  $\begin{vmatrix} \sin \alpha & \cos \alpha & \sin(\alpha + \delta) \\ \sin \beta & \cos \beta & \sin(\beta + \delta) \\ \sin \gamma & \cos \gamma & \sin(\gamma + \delta) \end{vmatrix}$

Formula Used:  $\sin(A+B) = \sin A \cos B + \cos A \sin B$

We have,  $\begin{vmatrix} \sin \alpha & \cos \alpha & \sin(\alpha + \delta) \\ \sin \beta & \cos \beta & \sin(\beta + \delta) \\ \sin \gamma & \cos \gamma & \sin(\gamma + \delta) \end{vmatrix}$

Applying  $C_1 \rightarrow \cos(\delta)C_1$

$$\Rightarrow \begin{vmatrix} \sin \alpha \cos \delta & \cos \alpha & \sin(\alpha + \delta) \\ \sin \beta \cos \delta & \cos \beta & \sin(\beta + \delta) \\ \sin \gamma \cos \delta & \cos \gamma & \sin(\gamma + \delta) \end{vmatrix}$$

Applying  $C_2 \rightarrow \sin(\delta)C_2$

$$\Rightarrow \begin{vmatrix} \sin\alpha \cos\delta & \cos\alpha \sin\delta & \sin(\alpha+\delta) \\ \sin\beta \cos\delta & \cos\beta \sin\delta & \sin(\beta+\delta) \\ \sin\gamma \cos\delta & \cos\gamma \sin\delta & \sin(\gamma+\delta) \end{vmatrix}$$

We know,  $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\Rightarrow \begin{vmatrix} \sin\alpha \cos\delta & \cos\alpha \sin\delta & \sin\alpha \cos\delta + \cos\alpha \sin\delta \\ \sin\beta \cos\delta & \cos\beta \sin\delta & \sin\beta \cos\delta + \cos\beta \sin\delta \\ \sin\gamma \cos\delta & \cos\gamma \sin\delta & \sin\gamma \cos\delta + \cos\gamma \sin\delta \end{vmatrix}$$

Applying  $C_3 \rightarrow C_3 - C_1$

$$\Rightarrow \begin{vmatrix} \sin\alpha \cos\delta & \cos\alpha \sin\delta & \sin\alpha \cos\delta + \cos\alpha \sin\delta - \sin\alpha \cos\delta \\ \sin\beta \cos\delta & \cos\beta \sin\delta & \sin\beta \cos\delta + \cos\beta \sin\delta - \sin\beta \cos\delta \\ \sin\gamma \cos\delta & \cos\gamma \sin\delta & \sin\gamma \cos\delta + \cos\gamma \sin\delta - \sin\gamma \cos\delta \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \sin\alpha \cos\delta & \cos\alpha \sin\delta & \cos\alpha \sin\delta \\ \sin\beta \cos\delta & \cos\beta \sin\delta & \cos\beta \sin\delta \\ \sin\gamma \cos\delta & \cos\gamma \sin\delta & \cos\gamma \sin\delta \end{vmatrix}$$

$$= 0$$

When two columns are identical then the value of determinant is 0

### 13. Question

Mark the tick against the correct answer in the following:

If a, b, c be distinct positive real numbers then the value of  $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$  is

- A. positive
- B. negative
- C. a perfect square
- D. 0

### Answer

To find: Nature of  $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

We have,  $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} a+b+c & b+c+a & c+a+b \\ b & c & a \\ c & a & b \end{vmatrix}$$

Taking  $(a+b+c)$  common from  $R_1$

$$\Rightarrow (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

Expanding along  $R_1$

$$\Rightarrow (a+b+c)[1\{(b)(c)-(a)(a)\} - 1\{(b)(b)-(c)(a)\} + 1\{(a)(b)-(c)(c)\}]$$

$$\Rightarrow (a+b+c)[1\{bc-a^2\} - 1\{b^2-ca\} + 1\{ba - c^2\}]$$

$$\Rightarrow (a+b+c)[bc - a^2 - b^2 + ca + ab - c^2]$$

$$\Rightarrow -(a+b+c)[c^2 + a^2 + b^2 - ca - bc - ba]$$

$$\Rightarrow -\frac{1}{2}(a+b+c) 2[c^2 + a^2 + b^2 - ca - bc - ba]$$

$$\Rightarrow -\frac{1}{2}(a+b+c) [2c^2 + 2a^2 + 2b^2 - 2ca - 2bc - 2ba]$$

$$\Rightarrow -\frac{1}{2}(a+b+c) [c^2 + a^2 - 2ca + c^2 + b^2 - 2bc + a^2 + b^2 - 2ba]$$

$$\Rightarrow -\frac{1}{2}(a+b+c) [(c-a)^2 + (c-b)^2 + (a-b)^2]$$

Clearly, we can see that the answer is negative

#### 14. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = ?$$

- A. 0
- B.  $x^3$
- C.  $y^3$
- D. none of these



#### Answer

To find: Value of  $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

We have,  $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Applying  $R_2 \rightarrow 2R_2$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} x+y & x & x \\ 10x+8y & 8x & 4x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} x+y & x & x \\ 0 & 0 & x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying  $R_1 \rightarrow 8R_1$

$$\Rightarrow \frac{1}{2 \times 8} \begin{vmatrix} 8x+8y & 8x & 8x \\ 0 & 0 & x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \frac{1}{16} \begin{vmatrix} 8x+8y & 8x & 8x \\ 0 & 0 & x \\ 2x & 0 & -5x \end{vmatrix}$$

Expanding along  $R_2$

$$\Rightarrow \frac{1}{16} [x\{(2x)(8x) - (8x+8y)(0)\}]$$

$$\Rightarrow \frac{1}{16} [x\{16x^2\}]$$

$$\Rightarrow x^3$$

### 15. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = ?$$

- A.  $(a - 1)$
- B.  $(a - 1)^2$
- C.  $(a - 1)^3$
- D. none of these

### Answer

To find: Value of  $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

We have,  $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

Applying  $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Expanding along  $C_3$

$$\Rightarrow [1\{(a^2-1)(a-1) - (a-1)(2a-2)\}]$$

$$\Rightarrow [1\{(a-1)(a+1)(a-1) - (a-1)2(a-1)\}]$$

$$\Rightarrow [1\{(a+1)(a-1)^2 - 2(a-1)^2\}]$$

$$\Rightarrow [1\{(a-1)^2(a+1-2)\}]$$

$$\Rightarrow [1\{(a-1)^2(a-1)\}]$$

$$\Rightarrow (a-1)^3$$

### 16. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = ?$$

- A.  $a^3$
- B.  $-a^3$
- C. 0
- D. none of these

**Answer**

To find: Value of  $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$

We have,  $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$

Applying  $R_3 \rightarrow R_3 - 2R_2$

$$\Rightarrow \begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 0 & a & a+b \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - 3R_1$

$$\Rightarrow \begin{vmatrix} a & a+2b & a+2b+3c \\ 0 & a & 2a+b \\ 0 & a & a+b \end{vmatrix}$$

Expanding along  $C_1$

$$\Rightarrow [a\{(a)(a+b) - (a)(2a+b)\}]$$

$$\Rightarrow [a\{(a^2 + ab) - (2a^2 + ab)\}]$$

$$\Rightarrow [a\{a^2 + ab - 2a^2 - ab\}]$$

$$\Rightarrow [a\{-a^2\}]$$

$$\Rightarrow -a^3$$

**17. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = ?$$

- A.  $(a + b + c)(a - c)$
- B.  $(a + b + c)(b - c)$
- C.  $(a + b + c)(a - c)^2$
- D.  $(a + b + c)(b - c)^2$

**Answer**



To find: Value of  $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

We have,  $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} b+c+c+a+a+b & a+c+b & b+a+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 2(a+b+c) & a+b+c & a+b+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$$

$$\Rightarrow (a+b+c) \begin{vmatrix} 2 & 1 & 1 \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$$

Expanding along  $R_1$

$$\Rightarrow (a+b+c)[2\{(c)(c) - (b)(a)\} - 1\{(c+a)(c) - (a+b)(a)\} + 1\{(c+a)(b) - (a+b)(c)\}]$$

$$\Rightarrow (a+b+c)[2\{c^2 - ab\} - 1\{c^2 + ac - a^2 - ab\} + 1\{bc + ba - ac - bc\}]$$

$$\Rightarrow (a+b+c)[2c^2 - 2ab - c^2 - ac + a^2 + ab + ba - ac]$$

$$\Rightarrow (a+b+c)[c^2 + a^2 - 2ac]$$

$$\Rightarrow (a+b+c)(c - a)^2$$

### 18. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = ?$$

A.  $(x + y)$

B.  $(x - y)$

C.  $xy$

D. none of these

### Answer

To find: Value of  $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

We have,  $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying  $R_1 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} 0 & -x & 0 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

Expanding along  $R_1$



$$\Rightarrow [x\{(1)(1+y)-(1)(1)\}]$$

$$\Rightarrow [x\{1+y-1\}]$$

$$\Rightarrow xy$$

### 19. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = ?$$

- A.  $(a - b) (b - c) (c - a)$
- B.  $-(a - b) (b - c) (c - a)$
- C.  $(a + b) (b + c) (c + a)$
- D. None of these

### Answer

To find: Value of  $\begin{vmatrix} bc & b+c & 1 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$

We have,  $\begin{vmatrix} bc & b+c & 1 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$

Applying  $R_1 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} bc-ca & b-a & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} c(b-a) & b-a & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

Taking  $(b - a)$  common

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ ca-ab & c-b & 0 \\ ab & a+b & 1 \end{vmatrix}$$

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ a(c-b) & c-b & 0 \\ ab & a+b & 1 \end{vmatrix}$$

Taking  $(c - b)$  common

$$\Rightarrow (b-a)(c-b) \begin{vmatrix} c & 1 & 0 \\ a & 1 & 0 \\ ab & a+b & 1 \end{vmatrix}$$

Expanding along  $C_3$

$$\Rightarrow (b - a) (c - b) [1\{(c) (1) - (a) (1)\}]$$



$$\Rightarrow (b - a)(c - b)(c - a)$$

$$\Rightarrow (a - b)(b - c)(c - a)$$

### 20. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = ?$$

A.  $4abc$

B.  $2(a + b + c)$

C.  $(ab + bc + ca)$

D. none of these

### Answer

To find: Value of  $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

We have,  $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} b+c+b+c & a+c+a+c & a+b+a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 common

$$\Rightarrow 2 \begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow 2 \begin{vmatrix} c & 0 & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Expanding along  $R_1$

$$\Rightarrow 2 [c\{(c + a)(a + b) - (b)(c)\} + a\{(b)(c) - (c)(c + a)\}]$$

$$\Rightarrow 2 [c\{ac + cb + a^2 + ab - bc\} + a\{bc - c^2 - ac\}]$$

$$\Rightarrow 2 [c\{ac + a^2 + ab\} + a\{bc - c^2 - ac\}]$$

$$\Rightarrow 2 [ac^2 + ca^2 + abc + abc - ac^2 - a^2c]$$

$$\Rightarrow 2 [2abc]$$

$$\Rightarrow 4abc$$

### 21. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix} = ?$$

- A.  $a + b + c$   
 B.  $2(a + b + c)$   
 C.  $4abc$   
 D.  $a^2b^2c^2$

**Answer**

To find: Value of  $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

We have,  $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

Applying  $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} a & 1 & b+c \\ b-a & 0 & a-b \\ c & 1 & a+b \end{vmatrix}$$

Taking  $(a - b)$  common

$$\Rightarrow (a - b) \begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ c & 1 & a+b \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow (a - b) \begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ c-a & 0 & a-c \end{vmatrix}$$

Taking  $(c-a)$  common

$$\Rightarrow (b-a)(c-a) \begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ 1 & 0 & -1 \end{vmatrix}$$

Expanding along  $R_1$

$$\begin{aligned} &= (b - a)(c - a)[0 - 1(1 - (a - b)) + (b + c)(0)] \\ &= (b - a)(c - a)(-1 + a - b) \\ &= (b - a)(c - a)(a - b - 1) \\ &= (b - a)(ac - bc - c - a^2 + ab + a) \\ &= (abc - b^2c - bc - a^2b + ab^2 + ab - a^2c + abc + ac + a^3 + a^2b + a^2) \\ &= 4abc \end{aligned}$$

**22. Question**

Mark the tick against the correct answer in the following:



$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} = ?$$

- A. -2
- B. 2
- C.  $x^2 - 2$
- D.  $x^2 + 2$

**Answer**

To find: Value of  $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

We have,  $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

Applying  $R_1 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_3 - R_2$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ x+7 & x+10 & x+14 \end{vmatrix}$$

Expanding along  $R_1$

$$\begin{aligned} &\Rightarrow [2\{(5)(x+14) - (6)(x+10)\} - 3\{(4)(x+14) - (6)(x+7)\} + 4\{(4)(x+10) - (5)(x+7)\}] \\ &\Rightarrow [2\{5x + 70 - 6x - 60\} - 3\{4x + 56 - 6x - 42\} + 4\{4x + 40 - 5x - 35\}] \\ &\Rightarrow [2\{10 - x\} - 3\{14 - 2x\} + 4\{5 - x\}] \\ &\Rightarrow [20 - 2x - 42 + 6x + 20 - 4x] \\ &\Rightarrow -2 \end{aligned}$$

**23. Question**

Mark the tick against the correct answer in the following:

If  $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$  then  $x = ?$

- A. 0
- B. 6
- C. -6
- D. 9

**Answer**

To find: Value of  $x$



We have,  $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$

Applying  $R_1 \rightarrow 2R_1$

$$\Rightarrow \begin{vmatrix} 10 & 6 & -2 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$$

Applying  $R_1 \rightarrow R_1 - R_3$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$$

Expanding along  $R_1$

$$\Rightarrow [1\{(x)(-2) - (6)(2)\}] = 0$$

$$\Rightarrow [1\{-2x - 12\}] = 0$$

$$\Rightarrow -2x - 12 = 0$$

$$\Rightarrow -2x = 12$$

$$\Rightarrow x = -6$$

#### 24. Question

Mark the tick against the correct answer in the following:

The solution set of the equation  $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$  is

A.  $\{2, -3, 7\}$

B.  $\{2, 7, -9\}$

C.  $\{-2, 3, -7\}$

D. none of these

#### Answer

To find: Value of  $x$

We have,  $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Applying  $R_1 \rightarrow 2R_1$

$$\Rightarrow \begin{vmatrix} 2x & 6 & 14 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Applying  $R_1 \rightarrow R_1 - R_3$

$$\Rightarrow \begin{vmatrix} 2x-7 & 0 & 14-x \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Expanding along  $R_1$

$$\Rightarrow [(2x-7)\{(x)(x) - (6)(2)\} + (14-x)\{(2)(6) - (x)(7)\}] = 0$$

$$\Rightarrow [(2x-7)\{x^2-12\} + (14-x)\{12-7x\}] = 0$$

$$\Rightarrow [2x^3 - 24x - 7x^2 + 84 + 168 - 98x - 12x + 7x^2] = 0$$

$$\Rightarrow [2x^3 - 134x + 252] = 0$$

$$\Rightarrow [x^3 - 67x + 126] = 0$$

By Hit and trial  $x = -2, 3, -7$

### 25. Question

Mark the tick against the correct answer in the following:

The solution set of the equation  $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 2x-64 \end{vmatrix} = 0$  is

- A. {4}
- B. {2, 4}
- C. {2, 8}
- D. {4, 8}

### Answer

To find: Value of  $x$

We have,  $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

Applying  $C_2 \rightarrow C_2 - 2C_1$

$$\Rightarrow \begin{vmatrix} x-2 & 1 & 3x-4 \\ x-4 & -1 & 3x-16 \\ x-8 & -11 & 3x-64 \end{vmatrix} = 0$$

Applying  $C_3 \rightarrow C_3 - 3C_1$

$$\Rightarrow \begin{vmatrix} x-2 & 1 & 2 \\ x-4 & -1 & -4 \\ x-8 & -11 & -40 \end{vmatrix} = 0$$

Expanding along  $R_1$

$$\Rightarrow [x-2\{(-1)(-40) - (-4)(-11)\} - 1\{(x-4)(-40) - (-4)(x-8)\} + 2\{(x-4)(-11) - (-1)(x-8)\}] = 0$$

$$\Rightarrow [(x-2)\{40-44\} - 1\{(-40x + 160 + 4x - 32)\} + 2\{-11x + 44 + x - 8\}] = 0$$

$$\Rightarrow [(x-2)\{-4\} - 1\{-36x + 128\} + 2\{-10x+36\}] = 0$$

$$\Rightarrow [-4x + 8 + 36x - 128 - 20x + 72] = 0$$

$$\Rightarrow 12x - 48 = 0$$

$$\Rightarrow x = 4$$

### 26. Question

Mark the tick against the correct answer in the following:

The solution set of the equation  $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$  is

- A. {a, 0}
- B. {3a, 0}
- C. {a, 3a}
- D. None of these

**Answer**

To find: Value of x

We have, 
$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Applying  $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 2x & -2x & 0 \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \begin{vmatrix} 2x & -2x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Taking 2 common from  $R_1$

$$\Rightarrow 2 \begin{vmatrix} x & -x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Taking 2 common from  $R_2$

$$\Rightarrow 2 \times 2 \begin{vmatrix} x & -x & 0 \\ 0 & x & -x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Applying  $R_3 \rightarrow R_1 + R_3$

$$\Rightarrow 4 \begin{vmatrix} x & -x & 0 \\ 0 & x & -x \\ a & a-2x & a+x \end{vmatrix} = 0$$

Expanding along  $R_1$

$$\Rightarrow 4[x\{(x)(a+x) - (-x)(a-2x)\}] - (-x)\{(0)(a+x) - (-x)(a)\} = 0$$

$$\Rightarrow 4[x\{ax + x^2 + ax - 2x^2\}] - (-x)\{ax\} = 0$$

$$\Rightarrow 4[x\{2ax - x^2\}] + ax^2 = 0$$

$$\Rightarrow 4[2ax^2 - x^3 + ax^2] = 0$$

$$\Rightarrow -x^2 + 3ax = 0$$

$$\Rightarrow -x(x - 3a) = 0$$

$$\Rightarrow x = 0, \text{ or } x = 3a$$

**27. Question**

Mark the tick against the correct answer in the following:



The solution set of the equation  $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$  is

A.  $\left\{\frac{2}{3}, \frac{8}{3}\right\}$

B.  $\left\{\frac{2}{3}, \frac{11}{3}\right\}$

C.  $\left\{\frac{3}{2}, \frac{8}{3}\right\}$

D. None of these

**Answer**

To find: Value of x

We have,  $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Applying  $R_1 \rightarrow R_1 - R_2$

$\Rightarrow \begin{vmatrix} 3x-11 & 11-3x & 0 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Applying  $R_2 \rightarrow R_2 - R_3$

$\Rightarrow \begin{vmatrix} 3x-11 & 11-3x & 0 \\ 0 & 3x-11 & 11-3x \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Expanding along  $R_1$

$\Rightarrow (3x-11)\{(3x-11)(3x-8) - (3)(11-3x)\} - (11-3x)\{(0)((3x-8) - (11-3x)(3))\} = 0$

$\Rightarrow (3x-11)\{(3x-11)(3x-8+3)\} - (11-3x)\{- (11-3x)(3)\} = 0$

$\Rightarrow (3x-11)^2(3x-5) + (3x-11)\{(3x-11)(3)\} = 0$

$\Rightarrow (3x-11)^2(3x-5) + (3x-11)^2(3) = 0$

$\Rightarrow (3x-11)^2(3x-5+3) = 0$

$\Rightarrow (3x-11)^2(3x-2) = 0$

$\Rightarrow x = \frac{11}{3}, \text{ Or, } x = \frac{2}{3}$

**28. Question**

Mark the tick against the correct answer in the following:

The vertices of a  $\Delta ABC$  are  $A(-2, 4)$ ,  $B(2, -6)$  and  $C(5, 4)$ . The area of  $\Delta ABC$  is

A. 17.5 sq units

B. 35 sq units

C. 32 sq units

D. 28 sq units

**Answer**

To find: Area of ABC

Given: A(-2,4), B(2,-6) and C(5,4)

$$\text{Formula used: } \Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

We have, A(-2,4), B(2,-6) and C(5,4)

$$\Delta = \frac{1}{2} \begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$\Rightarrow \frac{1}{2} [-2\{(-6)(1)-(4)(1)\} - 4\{(2)(1)-(5)(1)\} + 1\{(2)(4)-(5)(-6)\}]$$

$$\Rightarrow \frac{1}{2} [-2\{-6-4\} - 4\{2-5\} + 1\{8+30\}]$$

$$\Rightarrow \frac{1}{2} [-2\{-10\} - 4\{-3\} + 1\{38\}]$$

$$\Rightarrow \frac{1}{2} [20 + 12 + 38]$$

$$\Rightarrow \frac{1}{2} [70]$$

$\Rightarrow 35$  sq. units

**29. Question**

Mark the tick against the correct answer in the following:

If the points A(3, -2), B(k, 2) and C(8, 8) are collinear then the value of k is

- A. 2
- B. -3
- C. 5
- D. -4

**Answer**

To find: Area of ABC

Given: A(3,-2), B(k,2) and C(8,8)

$$\text{The formula used: } \Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

We have, A(3,-2), B(k,2) and C(8,8)

$$\Delta = \frac{1}{2} \begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

Expanding along  $R_1$

$$\Rightarrow \frac{1}{2} [3\{(2)(1)-(8)(1)\} - (-2)\{(k)(1)-(8)(1)\} + 1\{(k)(8)-(2)(8)\}] = 0$$

$$\Rightarrow \frac{1}{2} [3\{2-8\} + 2\{k-8\} + 1\{8k-16\}] = 0$$

$$\Rightarrow -18 + 2k - 16 + 8k - 16 = 0$$

$$\Rightarrow 10k - 50 = 0$$

$$\Rightarrow k = 5$$

