EXERCISE 23.16

1. Determine the distance between the following pair of parallel lines:
(i) 4x – 3y – 9 = 0 and 4x – 3y – 24 = 0
(ii) 8x + 15y – 34 = 0 and 8x + 15y + 31 = 0
Solution:
(i) 4x – 3y – 9 = 0 and 4x – 3y – 24 = 0
Given:
The parallel lines are
4x − 3y − 9 = 0 … (1)
4x − 3y − 24 = 0 … (2)
Let d be the distance between the given lines.
So,

$$d = \left|\frac{-9 + 24}{\sqrt{4^2 + (-3)^2}}\right| = \frac{15}{5} = 3 \text{ units}$$

∴ The distance between givens parallel line is 3units.

(ii) 8x + 15y – 34 = 0 and 8x + 15y + 31 = 0
Given:
The parallel lines are
8x + 15y − 34 = 0 … (1)
8x + 15y + 31 = 0 … (2)
Let d be the distance between the given lines.
So,

$$d = \left|\frac{-34 - 31}{\sqrt{8^2 + 15^2}}\right| = \frac{65}{17} \text{units}$$

∴ The distance between givens parallel line is 65/17 units.

2. The equations of two sides of a square are 5x – 12y – 65 = 0 and 5x – 12y + 26 = 0. Find the area of the square.
Solution:
Given:
Two side of square are 5x – 12y – 65 = 0 and 5x – 12y + 26 = 0
The sides of a square are
5x − 12y − 65 = 0 … (1)
5x − 12y + 26 = 0 … (2)
We observe that lines (1) and (2) are parallel.
So, the distance between them will give the length of the side of the square.

Let d be the distance between the given lines.

$$d = \left|\frac{-65 - 26}{\sqrt{5^2 + (-12)^2}}\right| = \frac{91}{13} = 7$$

∴ Area of the square = 7^2 = 49 square units

3. Find the equation of two straight lines which are parallel to x + 7y + 2 = 0 and at unit distance from the point (1, -1).

Solution:

Given:

The equation is parallel to x + 7y + 2 = 0 and at unit distance from the point (1, -1) The equation of given line is

x + 7y + 2 = 0 … (1)

The equation of a line parallel to line x + 7y + 2 = 0 is given below:

x + 7y + λ = 0 … (2)

The line x + 7y + λ = 0 is at a unit distance from the point (1, − 1).

So,

$$1 = \left|\frac{1-7+\lambda}{\sqrt{1+49}}\right|$$

$\lambda - 6 = \pm 5\sqrt{2}$

$\lambda = 6 + 5\sqrt{2}, 6 - 5\sqrt{2}$

now, substitute the value of λ back in equation x + 7y + λ = 0, we get

$x + 7y + 6 + 5\sqrt{2} = 0$ and $x + 7y + 6 - 5\sqrt{2}$

∴ The required lines:

$x + 7y + 6 + 5\sqrt{2} = 0$ and $x + 7y + 6 - 5\sqrt{2}$

4. Prove that the lines 2x + 3y = 19 and 2x + 3y + 7 = 0 are equidistant from the line 2x + 3y = 6.

Solution:

Given:

The lines A, 2x + 3y = 19 and B, 2x + 3y + 7 = 0 also a line C, 2x + 3y = 6.

Let d_1 be the distance between lines 2x + 3y = 19 and 2x + 3y = 6,

While d_2 is the distance between lines 2x + 3y + 7 = 0 and 2x + 3y = 6

$$d_1 = \left|\frac{-19 - (-6)}{\sqrt{2^2 + 3^2}}\right| \text{ and } d_2 = \left|\frac{7 - (-6)}{\sqrt{2^2 + 3^2}}\right|$$

$$d_1 = \left|-\frac{13}{\sqrt{13}}\right| = \sqrt{13} \text{ and } d_2 = \left|\frac{13}{\sqrt{13}}\right| = \sqrt{13}$$

Hence proved, the lines 2x + 3y = 19 and 2x + 3y + 7 = 0 are equidistant from the line 2x + 3y = 6

5. Find the equation of the line mid-way between the parallel lines 9x + 6y – 7 = 0 and 3x + 2y + 6 = 0.

Solution:

Given:

9x + 6y – 7 = 0 and 3x + 2y + 6 = 0 are parallel lines

The given equations of the lines can be written as:

3x + 2y – 7/3 = 0 … (1)

3x + 2y + 6 = 0 … (2)

Let the equation of the line midway between the parallel lines (1) and (2) be

3x + 2y + λ = 0 … (3)

The distance between (1) and (3) and the distance between (2) and (3) are equal.

$$\left|\frac{-\frac{7}{3}-\lambda}{\sqrt{3^2+2^2}}\right| = \left|\frac{6-\lambda}{\sqrt{3^2+2^2}}\right|$$

$$\left|-\lambda+\frac{7}{3}\right| = |6-\lambda|$$

$$6-\lambda = \lambda+\frac{7}{3}$$

$$\lambda = \frac{11}{6}$$

Now substitute the value of λ back in equation 3x + 2y + λ = 0, we get

3x + 2y + 11/6 = 0

By taking LCM

18x + 12y + 11 = 0

∴ The required equation of line is 18x + 12y + 11 = 0