EXERCISE 8.3

1. One angle of a quadrilateral is of 108º and the remaining three angles are equal. Find each of the three equal angles.

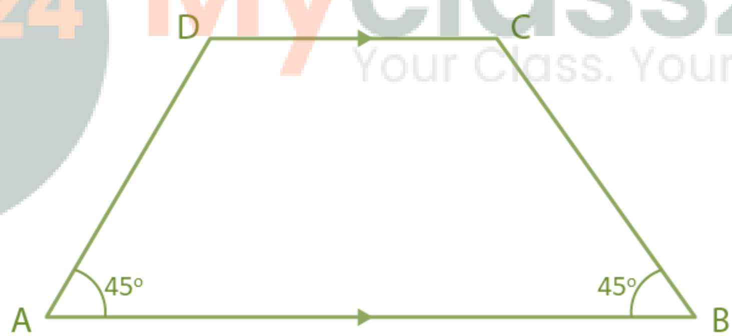
Solution:

Let the remaining three equal angles be x.
We know,
Sum of all interior angles of a quadrilateral is = 360º
$108^\circ + x + x + x = 360^\circ$
$108^\circ + 3x = 360^\circ$
$3x = 360^\circ - 108^\circ$
$3x = 252^\circ$
$x = 252/3$
$x = 84^\circ$
Each of three equal angles, $x = 84^\circ$.

2. ABCD is a trapezium in which AB || DC and ∠A = ∠B = 45º. Find angles C and D of the trapezium.

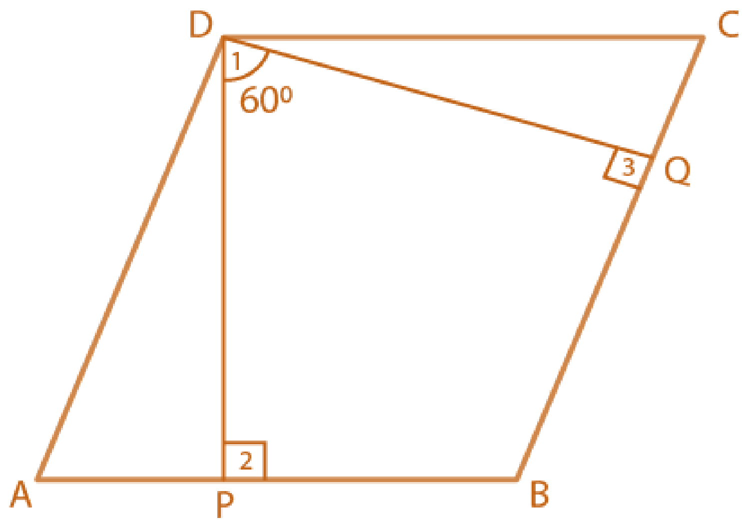
Solution:

According to the question,
ABCD is a trapezium
$\angle A = \angle B = 45^\circ$



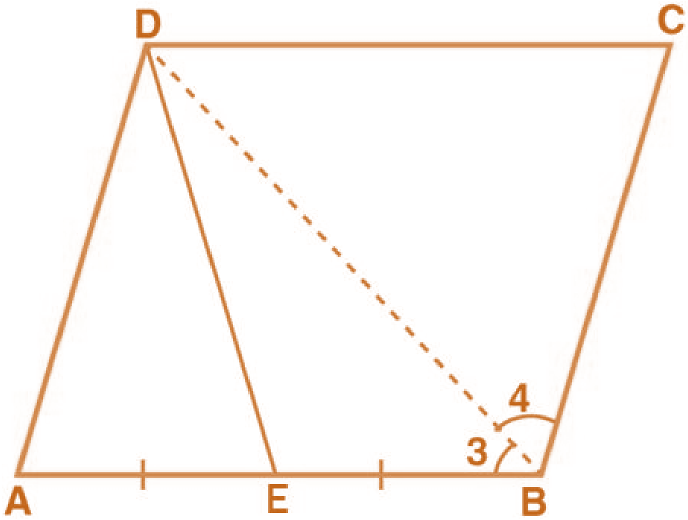
We know that,
Angles opposite to each other in quadrilateral are supplementary.
Then, we have,
$\angle A + \angle C = 180^\circ$
$45^\circ + \angle C = 180^\circ$
$\angle C = 180^\circ - 45^\circ$
$\angle C = 135^\circ$
Similarly,
We have,
$\angle B + \angle D = 180^\circ$
$45^\circ + \angle D = 180^\circ$
$\angle D = 180^\circ - 45^\circ$
$\angle D = 135^\circ$

3. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º. Find the angles of the parallelogram.
Solution:



According to the question,
ABCD is parallelogram,
DP ⊥ AB
DQ ⊥ BC.
$\angle PDQ = 60^\circ$
In quad. DPBQ,
Using angle sum property of a quadrilateral,
We have,
$\angle PDQ + \angle Q + \angle P + \angle B = 360^\circ$
$60^\circ + 90^\circ + 90^\circ + \angle B = 360^\circ$
$240^\circ + \angle B = 360^\circ$
$\angle B = 360^\circ - 240^\circ$
$\angle B = 120^\circ$
Since, opposite angles in parallelogram are equal,
We have,
$\angle B = \angle D = 120^\circ$
Since, opposite sides are parallel in parallelogram,
We have,
AB||CD
Also, since sum of adjacent interior angles is 180º,
We have,
$\angle B + \angle C = 180^\circ$
$120^\circ + \angle C = 180^\circ$
$\angle C = 180^\circ - 120^\circ$
$\angle C = 60^\circ$
Since, opposite angles in parallelogram are equal,
We have,
$\angle C = \angle A = 60^\circ$

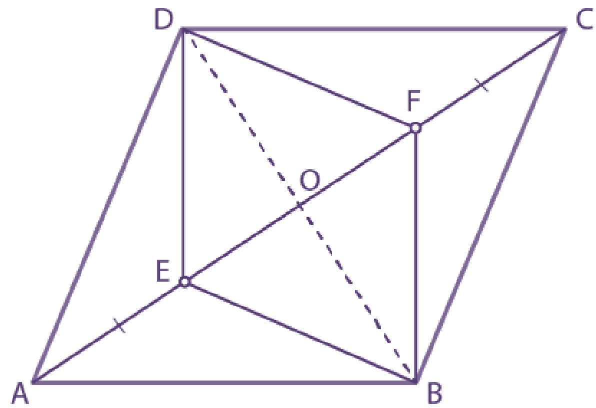
4. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.
Solution:



According to the question,
We have,
ABCD is a rhombus.
DE is the altitude on AB then AE = EB.
In ΔAED and ΔBED,
We have,
DE = DE (common line)
∠AED = ∠BED (right angle)
AE = EB (DE is an altitude)
∴ ΔAED ≅ ΔBED by SAS property.
∴ AD = BD (by C.P.C.T)
But AD = AB (sides of rhombus are equal)
⇒ AD = AB = BD
∴ ABD is an equilateral triangle.
∴ ∠A = 60°
Since, opposite angles of rhombus are equal, we get,
⇒ ∠A = ∠C = 60°
We also know that,
Sum of adjacent angles of a rhombus = supplementary.
So,
∠ABC + ∠BCD = 180°
∠ABC + 60° = 180°
∠ABC = 180° – 60° = 120°
Since, opposite angles of rhombus are equal, we get,
∠ABC = ∠ADC = 120°
Hence, Angles of rhombus are:
∠A = 60°, ∠C = 60°, ∠B = 120°, ∠D = 120°

5. E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.

Solution:



Construction:

Join BD, meeting AC at O.

According to the question,

Since diagonals of a parallelogram bisect each other,

We get,

OA = OC and OD = OB.

And,

OA = OC and AE = CF,

OA – AE = OC – CF

OE = OF

So, BFDE is a quadrilateral whose diagonals bisect each other.

Hence, BFDE is a parallelogram.