Exercise 21(E)

1. Prove the following identities:

(i) $\frac{1}{\cos A + \sin A} + \frac{1}{\cos A - \sin A} = \frac{2\cos A}{2\cos^2 A - 1}$

(ii) $\operatorname{cosec} A - \cot A = \frac{\sin A}{1 + \cos A}$

(iii) $1 - \frac{\sin^2 A}{1 + \cos A} = \cos A$

(iv) $\frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A} = 2\operatorname{cosec} A$

(v) $\frac{\cot A}{1 - \tan A} + \frac{\tan A}{1 - \cot A} = 1 + \tan A + \cot A$

(vi) $\frac{\cos A}{1 + \sin A} + \tan A = \sec A$

(vii) $\frac{\sin A}{1 - \cos A} - \cot A = \operatorname{cosec} A$

(viii) $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{\cos A}{1 - \sin A}$

(ix) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{\cos A}{1 - \sin A}$

(x) $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$

(xi) $\frac{1 + (\sec A - \tan A)^2}{\operatorname{cosec} A(\sec A - \tan A)} = 2\tan A$

(xii) $\frac{(\operatorname{cosec} A - \cot A)^2 + 1}{\sec A(\operatorname{cosec} A - \cot A)} = 2\cot A$

(xiii) $\cot^2 A\left(\frac{\sec A - 1}{1 + \sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right) = 0$

(xiv) $\frac{(1 - 2\sin^2 A)^2}{\cos^4 A - \sin^4 A} = 2\cos^2 A - 1$

(xv) $\sec^4 A(1 - \sin^4 A) - 2\tan^2 A = 1$

(xvi) $\operatorname{cosec}^4 A(1 - \cos^4 A) - 2\cot^2 A = 1$

(xvii) $(1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A) = 2$ Solution:

(i) Taking LHS,
1/ (cos A + sin A) + 1/ (cos A – sin A)

$= \frac{\cos A + \sin A + \cos A - \sin A}{(\cos A + \sin A)(\cos A - \sin A)}$

$= \frac{2\cos A}{\cos^2 A - \sin^2 A}$

$= \frac{2\cos A}{\cos^2 A - (1 - \cos^2 A)}$

$= \frac{2\cos A}{2\cos^2 A - 1}$

= RHS

- Hence Proved

(ii) Taking LHS, cosec A – cot A

$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \frac{1 - \cos A}{\sin A}$

$= \frac{1 - \cos A}{\sin A} \times \frac{1 + \cos A}{1 + \cos A}$

$= \frac{1 - \cos^2 A}{\sin A(1 + \cos A)}$

$= \frac{\sin^2 A}{\sin A(1 + \cos A)}$

$= \frac{\sin A}{1 + \cos A}$

= RHS

- Hence Proved

(iii) Taking LHS, $1 - \sin^2 A/ (1 + \cos A)$

$= \frac{1 + \cos A - \sin^2 A}{1 + \cos A}$

$= \frac{\cos A + \cos^2 A}{1 + \cos A}$

$= \frac{\cos A(1 + \cos A)}{1 + \cos A}$

$= \cos A$

= RHS

- Hence Proved

(iv) Taking LHS,

(1 - cos A)/ sin A + sin A/ (1 – cos A)

$= \frac{(1-\cos A)^2 + \sin^2 A}{\sin A(1-\cos A)}$

$= \frac{1+\cos^2 A - 2\cos A + \sin^2 A}{\sin A(1-\cos A)}$

$= \frac{2-2\cos A}{\sin A(1-\cos A)}$

$= \frac{2(1-\cos A)}{\sin A(1-\cos A)}$

$= 2\operatorname{cosec} A$

= RHS

- Hence Proved

(v) Taking LHS, cot A/ (1 – tan A) + tan A/ (1 – cot A)

$= \frac{\frac{1}{\tan A}}{1-\tan A} + \frac{\tan A}{1-\frac{1}{\tan A}}$

$= \frac{1}{\tan A(1-\tan A)} + \frac{\tan^2 A}{\tan A - 1}$

$= \frac{1-\tan^3 A}{\tan A(1-\tan A)}$

$= \frac{(1-\tan A)(1+\tan A+\tan^2 A)}{\tan A(1-\tan A)}$

$= \frac{1+\tan A+\tan^2 A}{\tan A}$

$= \cot A + 1 + \tan A$

= RHS

- Hence Proved

(vi) Taking LHS, cos A/ (1 + sin A) + tan A

$= \frac{\cos A}{1+\sin A} + \frac{\sin A}{\cos A}$

$= \frac{\cos^2 A + \sin A + \sin^2 A}{(1+\sin A)\cos A}$

$= \frac{1+\sin A}{(1+\sin A)\cos A}$

$= \frac{1}{\cos A}$

$= \sec A$

= RHS

- Hence Proved

(vii) Consider LHS,

= (sin A/(1 – cos A)) – cot A

We know that, cot A = cos A/sin A

So,

= ($\sin^2$ A – cos A + $\cos^2$ A)/(1 – cos A) sin A

= (1 – cos A)/(1 – cos A) sin A

= 1/sin A

= cosec A

(viii) Taking LHS, (sin A – cos A + 1)/ (sin A + cos A – 1)

$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{\sin A - (\cos A - 1)}{\sin A - (\cos A - 1)}$$

$$= \frac{(\sin A - \cos A + 1)^2}{\sin^2 A - (\cos A - 1)^2}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 - 2\sin A\cos A - 2\cos A + 2\sin A}{\sin^2 A - \cos^2 A - 1 + 2\cos A}$$

$$= \frac{1 + 1 - 2\sin A\cos A - 2\cos A + 2\sin A}{-\cos^2 A - \cos^2 A + 2\cos A}$$

$$= \frac{2(1 - \cos A) + 2\sin A(1 - \cos A)}{2\cos A(1 - \cos A)}$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= \frac{1 + \sin A}{\cos A} \times \frac{1 - \sin A}{1 - \sin A}$$

$$= \frac{\cos^2 A}{\cos A(1 - \sin A)}$$

$$= \frac{\cos A}{1 - \sin A}$$

= RHS

- Hence Proved

(ix) Taking LHS,

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 - \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{1 - \sin^2 A}{(1 - \sin A)^2}}$$

$$= \sqrt{\frac{\cos^2 A}{(1 - \sin A)^2}}$$

$$= \frac{\cos A}{1 - \sin A}$$

= RHS

- Hence Proved

(x) Taking LHS,

$$\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A}$$

= RHS

- Hence Proved

(xi) Taking LHS,

$$\frac{1+(\sec A - \tan A)^2}{\operatorname{cosec} A(\sec A - \tan A)}$$

$$= \frac{(\sec^2 A - \tan^2 A) + (\sec A - \tan A)^2}{\operatorname{cosec} A(\sec A - \tan A)}$$

$$= \frac{(\sec A - \tan A)(\sec A + \tan A) + (\sec A - \tan A)^2}{\operatorname{cosec} A(\sec A - \tan A)}$$

$$= \frac{(\sec A + \tan A) + (\sec A - \tan A)}{\operatorname{cosec} A}$$

$$= \frac{2\sec A}{\operatorname{cosec} A}$$

$$= 2\frac{\frac{1}{\cos A}}{\frac{1}{\sin A}}$$

$$= 2\tan A$$

= RHS

- Hence Proved

(xii) Taking LHS,

$$\frac{(\operatorname{cosec}A - \cot A)^2 + 1}{\sec A(\operatorname{cosec}A - \cot A)}$$

$$= \frac{(\operatorname{cosec}A - \cot A)^2 + (\operatorname{cosec}^2 A - \cot^2 A)}{\sec A(\operatorname{cosec}A - \cot A)}$$

$$= \frac{(\operatorname{cosec}A - \cot A)^2 + (\operatorname{cosec}A - \cot A)(\operatorname{cosec}A + \cot A)}{\sec A(\operatorname{cosec}A - \cot A)}$$

$$= \frac{(\operatorname{cosec}A - \cot A) + (\operatorname{cosec}A + \cot A)}{\sec A}$$

$$= \frac{2\operatorname{cosec}A}{\sec A}$$

$= 2\cot A$

= RHS

- Hence Proved

(xiii) Taking LHS,

$$\cot^2 A\left(\frac{\sec A - 1}{1 + \sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$

$$= \cot^2 A\left(\frac{\sec A - 1}{1 + \sin A} \times \frac{\sec A + 1}{\sec A + 1}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$

$$= \cot^2 A\left[\frac{\sec^2 A - 1}{(1 + \sin A)(\sec A + 1)}\right] + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$

$$= \cot^2 A\left[\frac{\tan^2 A}{(1 + \sin A)(\sec A + 1)}\right] + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$

$$= \frac{1}{(1 + \sin A)(\sec A + 1)} + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$

$$= \frac{1 + \sec^2 A(\sin A - 1)(1 + \sin A)}{(1 + \sin A)(\sec A + 1)}$$

$$= \frac{1 + \sec^2 A(\sin^2 A - 1)}{(1 + \sin A)(\sec A + 1)}$$

$$= \frac{1 + \sec^2 A(-\cos^2 A)}{(1 + \sin A)(\sec A + 1)}$$

$$= \frac{1 - 1}{(1 + \sin A)(\sec A + 1)}$$

$= 0$

= RHS

- Hence Proved

(xiv) Taking LHS,

$\frac{(1-2\sin^2 A)^2}{\cos^4 A - \sin^4 A}$

$= \frac{(1-2\sin^2 A)^2}{(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)}$

$= \frac{(1-2\sin^2 A)^2}{1-\sin^2 A - \sin^2 A}$

$= \frac{(1-2\sin^2 A)^2}{1-2\sin^2 A}$

$= 1 - 2\sin^2 A$

$= 1 - 2(1-\cos^2 A)$

$= 2\cos^2 A - 1$

= RHS

- Hence Proved

(xv) Taking LHS,

$\sec^4 A (1 - \sin^4 A) - 2\tan^2 A$

$= \sec^4 A(1 - \sin^2 A)(1 + \sin^2 A) - 2\tan^2 A$

$= \sec^4 A(\cos^2 A)(1 + \sin^2 A) - 2\tan^2 A$

$= \sec^2 A + \sin^2 A/ \cos^2 A - 2\tan^2 A$

$= \sec^2 A - \tan^2 A$

= 1 = RHS

- Hence Proved

(xvi) $\text{cosec}^4 A(1 - \cos^4 A) - 2\cot^2 A$

$= \text{cosec}^4 A (1 - \cos^2 A)(1 + \cos^2 A) - 2\cot^2 A$

$= \text{cosec}^4 A (\sin^2 A)(1 + \cos^2 A) - 2\cot^2 A$

$= \text{cosec}^2 A (1 + \cos^2 A) - 2\cot^2 A$

$= \text{cosec}^2 A + \cos^2 A/\sin^2 A - 2\cot^2 A$

$= \text{cosec}^2 A + \cot^2 A - 2\cot^2 A$

$= \text{cosec}^2 A - \cot^2 A$

= 1 = RHS

- Hence Proved

(xvii) (1 + tan A + sec A) (1 + cot A – cosec A)

= 1 + cot A – cosec A + tan A + 1 – sec A + sec A + cosec A – cosec A sec A

= 2 + cos A/sin A+ sin A/cos A – 1/(sin A cos A)

$= 2 + (\cos^2 A + \sin^2 A)/ \sin A \cos A - 1/(\sin A \cos A)$

= 2 + 1/(sin A cos A) - 1/(sin A cos A)

= 2 = RHS

- Hence Proved

2. If sin A + cos A = p
and sec A + cosec A = q, then prove that: $q(p^2 - 1) = 2p$

Solution:

Taking the LHS, we have

$q(p^2 - 1) = (\sec A + \text{cosec } A)\,[(\sin A + \cos A)^2 - 1]$
$= (\sec A + \text{cosec } A)\,[\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1]$
$= (\sec A + \text{cosec } A)\,[1 + 2 \sin A \cos A - 1]$
$= (\sec A + \text{cosec } A)\,[2 \sin A \cos A]$
$= 2\sin A + 2 \cos A$
$= 2p$

3. If x = a cos θ and y = b cot θ, show that:
$a^2/x^2 - b^2/y^2 = 1$
Solution:

Taking LHS,

$a^2/x^2 - b^2/y^2$

$$= \frac{a^2}{a^2\cos^2\theta} - \frac{b^2}{b^2\cot^2\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\cos^2\theta}$$

$= 1$

4. If sec A + tan A = p, show that:
$\sin A = (p^2 - 1)/(p^2 + 1)$
Solution:

Taking RHS, $(p^2 - 1)/(p^2 + 1)$

$$= \frac{(\sec A + \tan A)^2 - 1}{(\sec A + \tan A)^2 + 1}$$

$$= \frac{\sec^2 A + \tan^2 A + 2\tan A\sec A - 1}{\sec^2 A + \tan^2 A + 2\tan A\sec A + 1}$$

$$= \frac{\tan^2 A + \tan^2 A + 2\tan A\sec A}{\sec^2 A + \sec^2 A + 2\tan A\sec A}$$

$$= \frac{2\tan^2 A + 2\tan A\sec A}{2\sec^2 A + 2\tan A\sec A}$$

$$= \frac{2\tan A(\tan A + \sec A)}{2\sec A(\tan A + \sec A)}$$

$= \sin A$

5. If tan A = n tan B and sin A = m sin B, prove that:
$\cos^2 A = m^2 - 1/ n^2 - 1$
Solution:

Given,
tan A = n tan B
n = tan A/ tan B
And, sin A = m sin B
m = sin A/ sin B
Now, taking RHS and substitute for m and n
$m^2 - 1/ n^2 - 1$

$$= \frac{\left(\frac{\sin A}{\sin B}\right)^2 - 1}{\left(\frac{\tan A}{\tan B}\right)^2 - 1}$$

$$= \frac{\tan^2 B(\sin^2 A - \sin^2 B)}{\sin^2 B(\tan^2 A - \tan^2 B)}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 B\left(\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}\right)}$$

$$= \frac{\cos^2 A(\sin^2 A - \sin^2 B)}{\sin^2 A\cos^2 B - (1 - \cos^2 B)\cos^2 A}$$

$$= \frac{\cos^2 A(1 - \cos^2 A - 1 + \cos^2 B)}{\cos^2 B(\sin^2 A + \cos^2 A) - \cos^2 A}$$

$$= \frac{\cos^2 A(\cos^2 B - \cos^2 A)}{\cos^2 B - \cos^2 A}$$

$$= \cos^2 A$$

6. (i) If 2 sin A – 1 = 0, show that:
$\sin 3A = 3 \sin A - 4 \sin^3 A$
(ii) If $4 \cos^2 A - 3 = 0$, show that:
$\cos 3A = 4 \cos^2 A - 3 \cos A$
Solution:

(i) Given, 2 sin A – 1 = 0
So, sin A = ½
We know, sin 30° = 1/2
Hence, A = 30°
Now, taking LHS
sin 3A = sin 3(30°) = sin 30° = 1
RHS = $3 \sin 30° - 4 \sin^3 30° = 3 (1/2) - 4 (1/2)^3 = 3 - 4(1/8) = 3/2 - ½ = 1$

Therefore, LHS = RHS

(ii) Given, $4\cos^2 A - 3 = 0$
$4\cos^2 A = 3$
$\cos^2 A = 3/4$
$\cos A = \sqrt{3}/2$
We know, $\cos 30^\circ = \sqrt{3}/2$
Hence, $A = 30^\circ$
Now, taking
LHS = $\cos 3A = \cos 3(30^\circ) = \cos 90^\circ = 0$
RHS = $4\cos^3 A - 3\cos A = 4\cos^3 30^\circ - 3\cos 30^\circ = 4(\sqrt{3}/2)^3 - 3(\sqrt{3}/2)$
$= 4(3\sqrt{3}/8) - 3\sqrt{3}/2$
$= 3\sqrt{3}/2 - 3\sqrt{3}/2$
$= 0$
Therefore, LHS = RHS

7. Evaluate:

(i) $2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$

(ii) $\sec 26^\circ \sin 64^\circ + \frac{\operatorname{cosec} 33^\circ}{\sec 57^\circ}$

(iii) $\frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$

(iv) $\cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$

(v) $\sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$

(vi) $\frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(vii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

(viii) $\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$

Solution:

(i)
$$2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$$
$$= 2\left(\frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ}\right)^2 + \left(\frac{\cot(90^\circ - 35^\circ)}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec(90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ}\right)$$
$$= 2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\tan 35^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ}\right)$$

$= 2(1)^2 + 1^2 - 3$
$= 2 + 1 - 3 = 0$

(ii)

$$\sec 26^\circ \sin 64^\circ + \frac{\text{coe}\sec 33^\circ}{\sec 57^\circ}$$
$$= \sec(90^\circ - 64^\circ)\sin 64^\circ + \frac{\text{coe}\sec(90^\circ - 57^\circ)}{\sec 57^\circ}$$
$$= \text{cos}\,\text{ec}64^\circ \sin 64^\circ + \frac{\sec 57^\circ}{\sec 57^\circ}$$

= 1 + 1 = 2

(iii)

$$\frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$$
$$= \frac{5\sin(90^\circ - 24^\circ)}{\cos 24^\circ} - \frac{2\cot(90^\circ - 5^\circ)}{\tan 5^\circ}$$
$$= \frac{5\cos 24^\circ}{\cos 24^\circ} - \frac{2\tan 5^\circ}{\tan 5^\circ}$$
$$= 5 - 2 = 3$$

(iv) cos 40º cosec 50º + sin 50º sec 40º
= cos (90 – 50)º cosec 50º + sin (90 – 50)º sec 40º
= sin 50º cosec 50º + cos 40º sec 40º
= 1 + 1 = 2

(v) sin 27º sin 63º – cos 63º cos 27º
= sin (90 - 63)º sin 63º – cos 63º cos (90 - 63)º
= cos 63º sin 63º – cos 63º sin 63º
= 0

(vi)

$$\frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\text{cos}\,\text{ec}58^\circ}$$
$$= \frac{3\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\text{cos}\,\text{ec}58^\circ}$$
$$= \frac{3\cos 18^\circ}{\cos 18^\circ} - \frac{\text{cos}\,\text{ec}58^\circ}{\text{cos}\,\text{ec}58^\circ}$$
$$= 3 - 1 = 2$$

(vii) 3 cos 80º cosec 10º + 2 cos 59º cosec 31º
= 3 cos (90 - 10)º cosec 10º + 2 cos (90 - 31)º cosec 31º
= 3 sin 10º cosec 10º + 2 sin 31º cosec 31º
= 3 + 2 = 5

(viii)

$$\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$$
$$= \frac{\cos(90^\circ - 15^\circ)}{\sin 15^\circ} + \frac{\sin(90^\circ - 78^\circ)}{\cos 78^\circ} - \frac{\cos(90^\circ - 72^\circ)}{\sin 72^\circ}$$
$$= \frac{\sin 15^\circ}{\sin 15^\circ} + \frac{\cos 78^\circ}{\cos 78^\circ} - \frac{\sin 72^\circ}{\sin 72^\circ}$$
$$= 1 + 1 - 1 = 1$$

8. Prove that:
(i) tan (55º + x) = cot (35º - x)
(ii) sec (70º - θ) = cosec (20º + θ)
(iii) sin (28º + A) = cos (62º – A)
(iv) 1/ (1 + cos (90º - A)) + 1/(1 – cos (90º - A)) = 2 cosec² (90º - A)
(v) 1/ (1 + sin (90º - A)) + 1/(1 – sin (90º - A)) = 2 sec² (90º - A)
Solution:

(i) tan (55º + x) = tan [90º – (35º - x)] = cot (35º - x)
(ii) sec (70º - **θ**) = sec [90º – (20º + **θ**)] = cosec (20º + **θ**)
(iii) sin (28º + A) = sin [90º – (62º - A)] = cos (62º - A)

(iv) $$\frac{1}{1+\cos(90^\circ - A)} + \frac{1}{1-\cos(90^\circ - A)}$$
$$= \frac{1}{1+\sin A} + \frac{1}{1-\sin A}$$
$$= \frac{1-\sin A + 1 + \sin A}{(1+\sin A)(1-\sin A)}$$
$$= \frac{2}{1-\sin^2 A}$$
$$= \frac{2}{\cos^2 A}$$
$$= 2\sec^2 A$$
$$= 2\operatorname{cosec}^2(90^\circ - A)$$

(v) $$\frac{1}{1+\sin(90^\circ - A)} + \frac{1}{1-\sin(90^\circ - A)}$$
$$= \frac{1}{1+\cos A} + \frac{1}{1-\cos A}$$
$$= \frac{1-\cos A + 1 + \cos A}{(1+\cos A)(1-\cos A)}$$
$$= \frac{2}{1-\cos^2 A}$$
$$= 2\operatorname{cosec}^2 A$$
$$= 2\sec^2(90^\circ - A)$$