NCERT Solutions for Class 9 Maths Chapter 9 Circles Ex 9.6

Question 1.

Prove that the line of centers of two intersecting circles subtends equal angles at the two points of intersection.

Solution.

Let two circles with centers as O and O’ intersect each other at point A and B respectively Join OO’.

Now,

In ΔAOO’ and BOO’

O’A = O’B (radius of same circle)

OA = OB (Radius of same circle)

OO’ = OO’ (Common)

Hence, by SSS congruence

ΔOAO’ ≅ΔOBO’

∠OAO’ = ∠OBO’ (By CPCT)

Therefore,

Line of centers of two intersecting circles subtends equal angles at the two points of intersection

Question 2.

Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Solution.

Draw OM perpendicular AB and ON perpendicular CD

Join OB and OD

BM = $\frac{AB}{2} = \frac{5}{2}$ (The perpendicular drawn from the center bisects the chord)

$$ND = \frac{CD}{2} = \frac{11}{2}$$

Let ON be x

Therefore, OM will be 6 –x

In ΔMOD,

$OM^2 + MB^2 = OB^2$

$(6 - x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$

$36 + x^2 - 12x + \frac{25}{4} = OB^2$ (i)

In ΔNOD,

$ON^2 + ND^2 = OD^2$

$(x)^2 + \left(\frac{11}{2}\right)^2 = OD^2$

$(x)^2 + \frac{121}{4} = OD^2$ (ii)

We have OB = OD (radii of the same circle)

Therefore,

From (i) and (ii),

$36 + x^2 - 12x + \frac{25}{4} = (x)^2 + \frac{121}{2}$

$36 - 12x + \frac{25}{4} = \frac{121}{2}$

$x = 1$

From (ii),

$(1)^2 + \frac{121}{4} = OD^2$

$OD = \frac{5}{2}\sqrt{5}$ cm

Question 3.

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Solution.

Let AB and CD be two parallel chords in a circle centered at O

Join OB and OD

Distance of the smaller chord AB from the centre of the circle = 4cm

OM = 4cm

$MB = \frac{AB}{2} = \frac{6}{2} = 3cm$

In ΔOMB,

$OM^2 + MB^2 = OB^2$

$(4)^2 + (3)^2 = OB^2$

$16 + 9 = OB^2$

OB = 5cm

In ΔOND,

$ON^2 + ND^2 = OD^2$

$(ON)^2 + (4)^2 = 5^2$

$(ON)^2 = 9$

ON = 3 cm

Question 4.

Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Solution.

In ΔAOD and ΔCOE,

OA = OC (radii)

OD = OE (Radii)

AD = CE (Given)

Therefore,

By SSS congruence,

ΔAOD ≅ΔCOE

∠OAD = ∠ OCE (By CPCT) (i)

∠ODA = ∠OEC (By CPCT) (ii)

$\angle OAD = \angle ODA$ (iii)

Using (i), (ii) and (iii), we get

$\angle OAD = \angle OCE = \angle ODA = \angle OEC = x$

Now,

In triangle ODE,

$OD = OE$

Hence, ABCD is a quadrilateral

$\angle CAD + \angle DEC = 180^o$

$a + x + x + y = 180^o$

$2x + a + y = 180^o$ (iv)

$\angle DOE = 180^o – 2y$

And,

$\angle COA = 180^o - 2a$

$\angle DOE - \angle COA = 2a – 2y$

$= 2a – 2 (180^o - 2x – a)$

$= 4a + 4x – 360^o$ (v)

$\angle BAC = 180^o – (a + x)$

And

$\angle ACB = 180 – (a - x)$

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180^o$ (Angle sum property of triangle)

$\angle ABC = 2a + 2x – 180^o$

$\angle ABC = ½ (4a + ax – 360^0)$ [using (v)]

Question 5.

Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Solution.

Let ABCD be a rhombus in which diagonals intersect at point O and a circle is drawn is drawn taking side CD as diameter

Now,

$\angle COD = 90^0$

(Since, diagonals of rhombus intersect at 90^0)

Hence,

$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^0$

Hence, point O has to lie on the circle

Question 6.

ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

Solution.

It is visible that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral, the sum of opposite angle is 180^0

$\angle AEC = \angle CBA = 180$

∠AED = ∠CBA (i)

We know that: Opposite angles of a parallelogram are equal

Hence,

∠ADE = ∠CBA (ii)

Using (i) and (ii)

∠ADE = ∠AED

AE = AD (angles opposite to equal sides are equal)

Question 7.

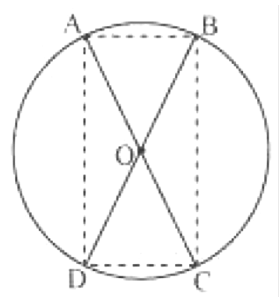
AC and BD are chords of a circle which bisect each other. Prove that:

(i) AC and BD are diameters

(ii) ABCD is a rectangle.

Solution.

Let two chords AB and CD are intersecting each other at point O



In ΔAOB and ΔCOD,

OA = OC (Given)

OB = OD (Given)

∠AOB = ∠COD (Vertically opposite angles)

$\Delta AOB \cong \Delta COD$ (SAS congruence rule)

AB = CD (By CPCT)

Similarly, it can be proved that ΔAOD ΔCOB

AD = CB (By CPCT)

Since in quadrilateral ACBD, opposite sides are equal in length, ACBD is a parallelogram

$\angle A = \angle C$

However,

$\angle A + \angle C = 180°$ (ABCD is a cyclic quadrilateral)

$\angle A + \angle A = 180°$

$2\angle A = 180°$

$\angle A = 90°$

As ACBD is a parallelogram and one of its interior angles is 90°, therefore, it is a rectangle

A is the angle subtended by chord BD and BD should be the diameter of the circle

Similarly, AC is the diameter of the circle

Question 8.

Bisectors of angles A, B and C of a triangle ABC intersect its circum circle at D, E and F respectively. Prove that the angles of the triangle DEF are $90^0 – 1/2$ A, $90^0 – 1/2$ B and $90^0 – 1/2$ C.

Solution.

It is given in the question that,

$\angle ABE = \frac{B}{2}$

However,

$\frac{B}{2} = \angle ADE$

Like this, similarly

$\angle ACF = \angle ADF = \frac{C}{2}$

$D = \angle ADE + \angle ADF$

$= \frac{B}{2} + \frac{C}{2}$

$= \frac{1}{2} (\angle B + \angle C)$

$= \frac{1}{2} (180^o - \angle A)$

$= 90^o - \frac{1}{2} \angle A$

Similarly,

$E = 90^o - \frac{1}{2} \angle B$

And,

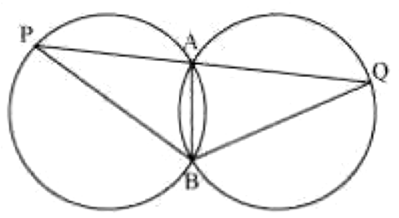
$F = 90^o - \frac{1}{2} \angle C$

Question 9.

Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Solution.

AB is common chord in the given both triangles



So,

∠APB = ∠AQB

Now, in triangle BPQ

∠APB = ∠AQB

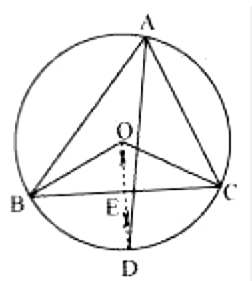
BQ = BP (Angles opposite to equal sides are equal)

Question 10.

In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circum circle of the triangle ABC.

Solution.

Let perpendicular bisector of side BC and angle bisector of A meet at point D. Let the perpendicular bisector of side BC intersect it at E



Perpendicular bisector of side BC will pass through circum centre O of the circle

BOC and BAC are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively

$\angle BOC = 2\ \angle BAC = 2\ \angle A$ (i)

In ΔBOE and ΔCOE,

OE = OE (Common)

OB = OC (Radii of same circle)

$\angle OEB = \angle OEC$ (Each 90° as OD perpendicular to BC)

$\Delta BOE \cong COE$ (RHS congruence rule)

$\angle BOE = \angle COE$ (By CPCT) (ii)

However,

$\angle BOE + \angle COE = \angle BOC$

$\angle BOE + \angle BOE = 2\ \angle A$ [From (i) and (ii)]

$2\ \angle BOE = 2\ \angle A$

$\angle BOE = \angle A$

$\angle BOE = \angle COE = \angle A$

The perpendicular bisector of side BC and angle bisector of A meet at point D

$\angle BOD = \angle BOE = \angle A$ (iii)

Since AD is the bisector of angle A

$\angle BAD = \frac{A}{2}$

$2\ \angle BAD = A$ (IV)

From (iii) and (iv), we get

$\angle BOD = 2\angle BAD$

Hence, proved